# 圆柱壳动力屈曲中载荷端质量的影响

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**文摘**　圆柱薄壳受有轴向动力载荷作用的非线性响应计算中，以往忽略载荷作用端质量的影响。本文在推导和响应计算中，在系统动能T中把壳体载荷端质量的影响考虑进去，并且计算了载荷作用端不同质量对动力屈曲载荷的影响。同时包含了轴向惯性和边界效应的影响。

**主题词**　圆柱壳体，动力屈曲，轴向脉冲载荷，结构稳定性。

## 一、前　言

圆柱形薄壳结构承载性能优越，制造工艺比较简单，因此在现代工程中普遍地采用着，特别是在宇航工业和船艇工业中广泛应用。圆柱壳的动力屈曲问题的研究对火箭、导弹等结构的设计极为重要。

关于圆柱壳的弹性稳定问题已被许多学者研究过。最早由Donnell[1]、Karman和钱学森[2]研究过非线性问题，大量的实验结果和屈曲载荷之间存在很大差异，认为这个差异的主要原因是原始几何缺陷所致，因而大部份的研究限制在静态分析。目前对于受有动载荷作用的动态分析还是不多的。

本文径向挠度的模式取为包含反对称模式和轴对称模式，通过求解平衡方程得到轴向位移函数和周向位移函数，运用Lagrange运动方程推导出广义坐标中的四个自由度系统的非线性变系数微分方程组，采用Runge-Kutta法进行数值积分。在非线性响应计算中计及轴向惯性力项，考虑边界效应，更重要的是计及圆柱薄壳载荷作用端质量对动力屈曲载荷的影响。

## 二、运动方程的建立

### 运动平衡方程

根据文献[1]Donnell的位移和应变的非线性关系：

$$\varepsilon_x = u_{,x} + \frac{K}{2}(w_{,x})^2 \tag{1}$$

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$$\varepsilon_y = v_{,y} + \frac{K}{2}(w_{,y})^2 - (w/R) \tag{2}$$

$$r_{xy} = u_{,y} + v_{,x} + K(w_{,x}\, w_{,y}) \tag{3}$$

式中$K=1+\frac{\bar{w}}{w}\approx1+\frac{2\bar{a}_1}{a_1}=1+\frac{\bar{a}}{a_1}$，$w$和$\bar{w}$分别为薄壳的径向位移函数和原始几何缺陷。

$$w = h\Big[a_1(t)\sin\frac{\pi}{L_x}x\sin\frac{n}{R}y + a_2(t)\cos\frac{2\pi}{L_x}x + a_3(t)\cos\frac{2n}{R}y\Big] \tag{4}$$

$$\bar{w} = h\Big[\bar{a}_1(t)\sin\frac{\pi}{L_x}x\sin\frac{n}{R}y + \bar{a}_2(t)\cos\frac{2\pi}{L_x}x + \bar{a}_3(t)\cos\frac{2n}{R}y\Big] \tag{5}$$

其中$L_x$为轴向半波长，$L_y$为周向半波长，$n$为周向全波数。令$\alpha=\frac{n}{L_x}$，$\beta=\frac{n}{R}$，$m=\frac{L_y}{L_x}$。

运用变分原理得到下面的运动方程和平衡方程

$$(D/h)\nabla^4 w = -\rho w_{,tt} + (K\sigma_x w_{,x})_{,x} + (K\tau_{xy}w_{,x})_{,y} + (K\tau_{xy}w_{,y})_{,x} + (K\sigma_y w_{,y})_{,y} + (\sigma_y/R) \tag{6}$$

$$\sigma_{x,x} + \tau_{xy,y} = 0 \tag{7}$$

$$\tau_{xy,x} + \sigma_{y,y} = 0 \tag{8}$$

将(4)和(5)式运用虎克定律代入(7)式和(8)式得到

$$\frac{1}{(1-\nu^2)}u_{,xx} + \frac{1}{2(1-\nu)}v_{,xy} + \frac{1}{2(1+\nu)}u_{,yy} = -\frac{1}{(1-\nu^2)}\Big[Kw_{,x}w_{,xx} + \nu\Big(Kw_{,y}w_{,xy} - \frac{1}{R}w_{,x}\Big)\Big] - \frac{K}{2(1+\nu)}(w_{,xy}w_{,y} + w_{,x}w_{,yy}) \tag{9}$$

$$\frac{1}{2(1+\nu)}v_{,xx} + \frac{1}{2(1-\nu)}u_{,xy} + \frac{1}{(1-\nu^2)}v_{,yy} = -\frac{1}{(1-\nu^2)}\Big(Kw_{,y}w_{,yy} - \frac{1}{R}w_{,y} + \nu Kw_{,x}w_{,xy}\Big) - \frac{K}{2(1+\nu)}(w_{,xx}w_{,y} + w_{,x}w_{,xy}) \tag{10}$$

运用应力函数$F$($\sigma_x=F_{,yy}$，$\sigma_y=F_{,xx}$，$\tau_{xy}=-F_{,xy}$)代入(6)式可得到

$$(D/h)\nabla^4 w = -\rho w_{,tt} + KF_{,yy}w_{,xx} - 2KF_{,xy}w_{,xy} + KF_{,xx}w_{,yy} + \frac{1}{R}F_{,xx} \tag{11}$$

运用应力函数$F$和(1)、(2)和(3)式导出

$$\nabla^4 F = EK[(w_{,xy})^2 - (w_{,xx}w_{,yy})] - (E/R)w_{,xx} \tag{12}$$

将(4)式和(5)式代入(12)式得到

$$\begin{aligned}\nabla^4 F = &\left[2EKh^2\alpha^2\beta^2 a_1(a_2+a_3) + \frac{E\alpha^2 h}{R}a_1\right]\sin\alpha x\sin\beta y \\ &-2EKh^2\alpha^2\beta^2 a_1 a_2\sin 3\alpha x\sin\beta y \\ &-2EKh^2\alpha^2\beta^2 a_1 a_3\sin\alpha x\sin 3\beta y \\ &-16EKh^2\alpha^2\beta^2 a_2 a_3\cos 2\alpha x\cos 2\beta y \\ &+E\alpha^2((K/2)h^2\beta^2 a_1^2 + (4h/R)a_2)\cos 2\alpha x \\ &+(1/2)EKh^2\alpha^2\beta^2 a_1^2\cos 2\beta y\end{aligned} \tag{13}$$

其解为

$$\begin{aligned}F = &G_1\sin\alpha x\sin\beta y + G_2\sin 3\alpha x\sin\beta y + G_3\sin\alpha x\sin 3\beta y \\ &+G_4\cos 2\alpha x\cos 2\beta y + G_5\cos 2\alpha x + G_6\cos 2\beta y \\ &-(\sigma^2/2)y^2 + (g/2)x^2\end{aligned} \tag{14}$$

式中

$$G_1 = \frac{2EKh^2\alpha^2\beta^2 a_1(a_2+a_3) + (Eh/R)\alpha^2 a_1}{(\alpha^2+\beta^2)^2}$$

$$G_2 = -\frac{2EKh^2\alpha^2\beta^2 a_1 a_2}{(9\alpha^2+\beta^2)^2}$$

$$G_3 = -\frac{2EKh^2\alpha^2\beta^2 a_1 a_3}{(\alpha^2+9\beta^2)^2}$$

$$G_4 = -\frac{EKh^2\alpha^2\beta^2 a_2 a_3}{(\alpha^2+B^2)^2}$$

$$G_5 = \frac{E\alpha^2((K/2)h^2\beta^2 a_1^2 + (4h/R)a_2)}{16\alpha^4}$$

$$G_6 = \frac{EKh^2\alpha^2 a_1^2}{32\beta^2}$$

**求解平衡方程**

将(4)式代入(9)式和(10)式得到

$$\begin{aligned}&\frac{1}{(1-\nu^2)}u_{,xx} + \frac{1}{2(1-\nu)}v_{,xy} + \frac{1}{2(1+\nu)}u_{,yy} \\ &= A_1\sin 2\alpha x + A_2\sin 4\alpha x + A_3\sin 2\alpha x\cos 2\beta y + A_4\cos\alpha x\sin\beta y \\ &\quad + A_5\cos 3\alpha x\sin\beta y + A_6\cos\alpha x\sin 2\beta y\cos\beta y\end{aligned} \tag{15}$$

$$\begin{aligned}&\frac{1}{2(1+\nu)}v_{,xx} + \frac{1}{2(1-\nu)}u_{,xy} + \frac{1}{(1-\nu^2)}v_{,yy} \\ &= B_1\sin 2\beta y + B_2\sin 4\beta y + B_3\cos 2\alpha x\sin 2\beta y + B_4\sin\alpha x\cos\beta y \\ &\quad + B_5\sin\alpha x\cos 3\beta y + B_6\sin 2\alpha x\cos\alpha x\cos\beta y\end{aligned} \tag{16}$$

式中

$$A_1 = \frac{Kh^2\alpha(\alpha^2-\nu\beta^2)a_1^2}{4(1-\nu^2)} - \frac{2\nu h\alpha a_2}{(1-\nu^2)R}$$

$$A_2=-\frac{4Kh^2\alpha^3a_2^2}{(1-\nu^2)}$$

$$A_3=-\frac{Kh^2\alpha(\alpha^2+\beta^2)a_1^2}{4(1-\nu^2)}-\frac{4Kh^2\alpha\beta^2a_2a_3}{(1+\nu)}$$

$$A_4=\frac{Kh^2\alpha[6\alpha^2a_2-(1-\nu)\beta^2(a_2+4a_3)]a_1}{2(1-\nu^2)}$$

$$+\frac{\nu h\alpha a_1}{(1-\nu^2)R}$$

$$A_5=\frac{Kh^2\alpha[6\alpha^2+(1-\nu)\beta^2]a_1a_2}{2(1-\nu^2)}$$

$$A_6=Kh^2\alpha\beta^2\left[\frac{1}{2(1-\nu)}-\frac{2}{2(1+\nu)}\right]a_1a_3$$

$$B_1=\frac{Kh^2\beta(\beta^2-\nu\alpha^2)a_1^2}{4(1-\nu^2)}-\frac{2\nu h\beta a_3}{(1-\nu^2)R}$$

$$B_2=-\frac{4Kh^2\beta^3a_3^2}{(1-\nu^2)}$$

$$B_3=-\frac{Kh^2\beta(\alpha^2+\beta^2)a_1^2}{4(1-\nu^2)}-\frac{4Kh^2\alpha^2\beta a_2a_3}{(1+\nu)}$$

$$B_4=\frac{Kh^2\beta[6\beta^2a_3-(1-\nu)\alpha^2(a_3+4a_2)]a_1}{2(1-\nu^2)}$$

$$+\frac{\nu h\beta a_1}{(1-\nu^2)R}$$

$$B_5=\frac{Kh^2\beta[6\beta^2+(1-\nu)\alpha^2]a_1a_3}{2(1-\nu^2)}$$

$$B_6=Kh^2\alpha^2\beta\left[\frac{1}{2(1-\nu)}-\frac{2}{(1+\nu)}\right]a_1a_2$$

联立(15)式和(16)式，其特解为

$$\bar{u}=C_1\sin 2\alpha x+C_2\sin 4\alpha x+C_3\sin 2\alpha x\cos 2\beta y+C_4\cos\alpha x\sin\beta y$$
$$+C_5\cos 3\alpha x\sin\beta y+C_6\cos\alpha x\sin\beta y\cos 2\beta y \tag{17}$$

$$\bar{v}=D_1\sin 2\beta y+D_2\sin 4\beta y+D_3\cos 2\alpha x\sin 2\beta y+D_4\sin\alpha x\cos\beta y$$
$$+D_5\sin\alpha x\cos 3\beta y+D_6\sin\alpha x\cos 2\alpha x\cos\beta y \tag{18}$$

式中

$$C_1=-(1+\nu)Kh^2\alpha\left[\alpha^4+\frac{3}{4}(2-\nu)\alpha^2\beta^2-\nu\beta^4\right]a_1^2$$

$$+\frac{16\nu(1+\nu)}{R}h\alpha^3a_2$$

$$C_2=64(1+\nu)Kh^2\alpha^5a_2^2$$

$$C_3=(1+\nu)Kh^2\alpha\left[\alpha^4+\frac{(\nu^2-10\nu+7)}{4(1-\nu)}\alpha^2\beta^2+\frac{1}{2}\beta^4\right]a_1^2$$

$$+32(1-\nu^2)Kh^2\alpha\beta^2(2\alpha^2+\beta^2)a_2a_3$$

$$C_4=-(1+\nu)Kh^2\alpha\left[10\alpha^4-\frac{(14\nu^2-17\nu+15)}{2(1-\nu)}\alpha^2\beta^2\right.$$

$$\left.-(1-\nu)\beta^4\right]a_1a_2-(1+\nu)Kh^2\alpha\beta\left[(1+2\nu)\alpha^2\beta-\frac{(9\nu^2-8)}{(1-\nu)}\beta^3\right]a_1a_3$$

$$-\frac{(1+\nu)}{R}h\alpha\left[2\nu\alpha^2-\frac{(2\nu^2-2\nu+1)}{(1-\nu)}\beta^2\right]a_1$$

$$C_5=-(1+\nu)Kh^2\alpha\left[30\alpha^4+\frac{(14\nu^2-29\nu+17)}{2(1-\nu)}\alpha^2\beta^2+(1-\nu)\beta^4\right]a_1a_2$$

$$C_6=2(1+\nu)Kh^2\alpha\beta\left[(2\nu-5)\alpha^2\beta+\frac{4(5\nu-4)}{(1-\nu)}\beta^3\right]a_1a_3$$

$$D_1=-(1+\nu)Kh^2\beta\left[\beta^4+\frac{3}{4}(2-\nu)\alpha^2\beta^2-\nu\alpha^4\right]a_1^2$$

$$+\frac{16(1+\nu)\nu}{R}\beta^3a_3$$

$$D_2=64(1+\nu)Kh^2\beta^5a_3^2$$

$$D_3=(1+\nu)Kh^2\beta\left[\beta^4+\frac{(\nu^2-10\nu+7)}{4(1-\nu)}\alpha^2\beta^2+\frac{1}{2}\alpha^4\right]a_1^2$$

$$D_4=-(1+\nu)Kh^2\beta\left[10\beta^4-\frac{(14\nu^2-17\nu+15)}{2(1-\nu)}\alpha^2\beta^2-(1-\nu)\alpha^4\right]a_1a_3$$

$$-(1+\nu)Kh^2\alpha\beta\left[(1+2\nu)\alpha\beta^2-\frac{(9\nu^2-8)}{(1-\nu)}\alpha^3\right]a_1a_2$$

$$-\frac{(1+\nu)}{R}h\beta\left[2\beta^2+\frac{(1-2\nu)}{(1-\nu)}\alpha^2\right]a_1$$

$$D_5=-(1+\nu)Kh^2\beta\left[30\beta^4+\frac{(14\nu^2-29\nu+17)}{2(1-\nu)}\alpha^2\beta^2+(1-\nu)\alpha^4\right]a_1a_3$$

$$D_6=2(1+\nu)Kh^2\alpha\beta\left[(2\nu-5)\alpha\beta^2+\frac{4(5\nu-4)}{(1-\nu)}\alpha^3\right]a_1a_2$$

现在要找出能满足边界条件的解的形式，即可得出齐次解，齐次解为如下的形式

$$u_h=Ax+B+u_{BL}$$

$$v_h=v_{BL}$$

这里$u_{BL}$和$v_{BL}$是边界层解。$u$和$v$的全解为

$$u=\bar{u}+Ax+B+u_{BL} \tag{19}$$

$$v=\bar{v}+v_{BL} \tag{20}$$

在$x=0$和$x=L$处满足边界条件

当$x=0$，则$u=0$， $v=0$得到

$$u=(C_4+C_5)\sin\beta y+C_6\sin\beta y\cos2\beta y+B+u_{BL}(0,y)=0$$

$$v=(D_1+D_3)\sin2\beta y+D_2\sin4\beta y+v_{BL}(0,y)=0$$

边界层解选择为使其能够消去第一项和第二项，那末$B=0$。

当$x=L$，则$u=a_0(t)$，$v=0$得到

$$u=(C_4+C_5)\sin\beta y+C_6\sin\beta y\cos2\beta y+AL+u_{BL}(L,y)=a_0(t)$$

$$v=(D_1+D_3)\sin2\beta y+D_2\sin4\beta y+v_{BL}(L,y)=0$$

同理，边界层解选择为使其能消去第一项和第二项，因此得到

$$A=\frac{a_0(t)}{L}$$

$a_0(t)$为与时间有关的新变量，代表轴向运动。得到轴向位移函数和周向位移函数如下

$$u(x,y,t)=\bar{u}(x,y,t)-\frac{a_0(t)}{L}x$$

$$v(x,y,t)=\bar{v}(x,y,t)$$

**运用Lagrange运动方程推导出非线性微分方程组**

圆柱薄壳的薄膜应变能$V_1$， 弯曲应变能$V_2$，载荷势能$V_3$和系统动能$T$如下

$$V_1=\frac{h}{2E}\int_0^L\int_0^{2\pi R}[(\sigma_x+\sigma_y)^2-2(1+\nu)(\sigma_x\sigma_y-\tau^2_{xy})]dxdy \tag{21}$$

$$V_2=\frac{D}{2}\int_0^L\int_0^{2\pi R}\{(w_{,xx}+w_{,yy})^2-2(1-\nu)[w_{,xx}w_{,yy}-(w_{,xy})^2]\}dxdy \tag{22}$$

$$V_3=h\int_0^{2\pi R}\sigma(t)u(L,y)dy \tag{23}$$

$$T=\frac{1}{2}\rho h\int_0^L\int_0^{2\pi R}[(w_{,t})^2+(u_{,t})^2+(v_{,t})^2]dxdy+\frac{1}{2}M(u_{,t})^2_{x=L} \tag{24}$$

这里$D$为弯曲刚度等于$Eh^3/12(1-\nu^2)$；$\sigma(t)$为单位轴向脉冲载荷；$E$为弹性模量；$\nu$为泊桑比；$\rho$为壳体的质量密度。通过$a_0(t)$计及轴向惯性，忽略周向惯性，$M$为壳体载荷端的质量。

将$F$，$w$，$\bar{w}$，$u$和$v$的表达式代入各个能量表达式中，运用Lagrange运动方程推导得到下面四个非线性变系数微分方程组

$$\ddot{a}_1+4H_1a_1(a_2+a_3)^2+4\bar{a}H_1(a_2+a_3)^2+4H_2a_1(a_2+a_3)+2\bar{a}H_2(a_2+a_3)$$
$$+4H_3a_1a_2^2+4\bar{a}H_3a_2^2+(4H_4+H_9)a_1a_3^2+\bar{a}(4H_4+H_9)a_3^2-16\bar{a}H_1\frac{a_2^2a_3^2}{a_1^2}$$
$$-16\bar{a}^2H_1\frac{a_2^2a_3^2}{a_1^3}+\left(\frac{1}{16}H_5+\frac{1}{8}H_9\right)a_1^3+\bar{a}\left(\frac{3}{32}H_5+\frac{3}{16}H_9\right)a_1^2$$
$$+\left[\frac{\bar{a}^2}{32}H_5+H_6+H_7+\frac{\bar{a}^2}{16}H_9-(1+\nu)H_{11}\right]a_1+\frac{1}{2}H_8a_1a_2$$

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限于篇幅边界层解的具体表达式未列出。

$$+\frac{\bar{a}}{4}H_8a_2-4\bar{a}H_9\frac{a_3^4}{a_1^2}-4\bar{a}^2H_9\frac{a_3^4}{a_1^3}+4\bar{a}(1+\nu)H_{11}\frac{a_3^2}{a_1^2}-\frac{\bar{a}}{2}(1+\nu)H_{11}=0 \quad (25)$$

$$\ddot{a}_2+2H_1a_1^2(a_2+a_3)+4\bar{a}H_1a_1(a_2+a_3)+2\bar{a}^2H_1(a_2+a_3)+\left(H_2+\frac{1}{8}H_8\right)a_1^2$$

$$+\bar{a}\left(H_2+\frac{1}{8}H_8\right)a_1+2H_3a_1^2a_2+4\bar{a}H_3a_1a_2+\Big[2\bar{a}^2H_3$$

$$+\frac{16m^4}{12(1-\nu^2)}H_9+H_{10}\Big]a_2+8H_1a_2a_3^2+16\bar{a}H_1\frac{a_2a_3^2}{a_1}+8\bar{a}^2H_1\frac{a_2a_3^2}{a_1^2}=0 \quad (26)$$

$$\ddot{a}_3+2H_1a_1^2(a_2+a_3)+4\bar{a}H_1a_1(a_2+a_3)+2\bar{a}^2H_1(a_2+a_3)+H_2a_1^2$$

$$+\bar{a}H_2a_1+\left(2H_4+\frac{1}{2}H_9\right)a_1^2a_3+\bar{a}(4H_4+H_9)a_1a_3$$

$$+\left[2\bar{a}^2H_4+\frac{16}{12(1-\nu^2)}H_9+\frac{\bar{a}^2}{2}H_9-4(1+\nu)H_{11}\right]a_3+8H_1a_2^2a_3$$

$$+16\bar{a}H_1\frac{a_2^2a_3}{a_1}+8\bar{a}^2H_1\frac{a_2^2a_3}{a_1^2}+4H_9a_3^3-4\bar{a}^2H_9\frac{a_3^3}{a_1^2}$$

$$+8\bar{a}H_9\frac{a_3^3}{a_1}+4(1+\nu)\bar{a}H_{11}\frac{a_3}{a_1}=0 \quad (27)$$

$$\ddot{a}_0+\frac{Eh}{\left(\frac{1}{3}+\bar{M}\right)L}a_0-\frac{Eh^2R}{8\left(\frac{1}{3}+\bar{M}\right)L}\left(\frac{n}{R}\right)^2\left(1+\frac{\bar{a}}{a_1}\right)(a_1^2+8a_3^2)$$

$$+\frac{1}{\left(\frac{1}{3}+\bar{M}\right)}\left(\frac{R\nu}{L}-\frac{1}{h^2}\frac{1}{L\rho}\right)B_{12}=0 \quad (28)$$

式中

$$H_1=\frac{Eh^2}{\rho}\frac{m^4}{(m^2+1)^2}\left(\frac{n}{R}\right)^4$$

$$H_2=\frac{Eh}{R\rho}\frac{m^4}{(m^2+1)^2}\left(\frac{n}{R}\right)^2$$

$$H_3=\frac{Eh^2}{\rho}\frac{m^4[(9m^2+1)^2+18(1+\nu)m^2]}{(9m^2+1)^4}\left(\frac{n}{R}\right)^4$$

$$H_4=\frac{Eh^2}{\rho}\frac{m^4[(m^2+9)^2+18(1+\nu)m^2]}{(m^2+9)^4}\left(\frac{n}{R}\right)^4$$

$$H_5=\frac{Eh^2}{\rho}(1+m^4)\left(\frac{n}{R}\right)^4$$

$$H_6=\frac{E}{R^2\rho}\frac{m^4}{(m^2+1)^2}$$

$$H_7=\frac{D}{h\rho}(m^2+1)^2\left(\frac{n}{R}\right)^4,\quad H_8=\frac{Eh}{R\rho}\left(\frac{n}{R}\right)^2$$

$$H_9=\frac{Eh^2}{\rho}\left(\frac{n}{R}\right)^4,\quad H_{10}=\frac{Eh^2}{R^2\rho}$$

$$H_{11}=\frac{1}{\rho}\left(\frac{n}{R}\right)^2\sigma_0\sin\left(\frac{\pi}{\tau_0}t\right),\quad H_{12}=\sigma_0\sin\left(\frac{\pi}{\tau_0}t\right)$$

这里无量纲时间$\bar{\tau}=t\sqrt{\frac{E}{\rho}}\frac{1}{R}\times10^2$，无量纲质量$\bar{M}=\frac{M}{2\pi RhL\rho}$，无量纲单位脉冲载荷$\bar{\sigma}=\frac{\sigma R}{Eh}$，无量纲动力临界载荷$\bar{\sigma}_D=\bar{\sigma}/\sigma_{cl}$。求解这组非线性微分方程为数学上的初值问题，其解依赖于$\bar{a}$、$m$、$n$和$\sigma(t)$。

## 三、结 果 讨 论

**基本参数**

壳的几何参数：$R/h$等于1000，$L/R$等于2.5，壳体材料为纲。

载何参数：$\sigma(t)=\sigma_0\sin\left(\frac{\pi}{\tau_0}t\right)$，脉冲时间$\tau_0=3$ms，其脉冲载荷近似于三角波。

波长比取$m=1$，周向波数取受静载时按小挠度理论算得的值$n=29$，初始缺陷取$\bar{a}=0.3$。

采用Runge-Kutta法求解，初始位移取$a_1=0.15$和零初始速度。

**讨论**

圆柱薄壳承受轴向脉冲载荷作用的屈曲问题，计及轴向惯性的影响，明显地改善了动力屈曲载荷的理论值。本文计算壳体载荷作用端的质量在不同值时对动力响应值的影响，当$\bar{M}=0.033$，$\bar{M}=5$，$\bar{M}=10$和$\bar{M}=20$，响应中反对称项和轴对称项$a_1(t)$、$a_2(t)$、$a_3(t)$和$a_0(t)$的最大值随着$\bar{M}$的增加而减小。将$a_1(t)$的最大值发生突然急跳时的载荷值随$\bar{M}$的坛加而减小表明在图2中，当$\bar{M}=10$后$a_1(t)$的最大值突然急跳时的载荷值不再减小。

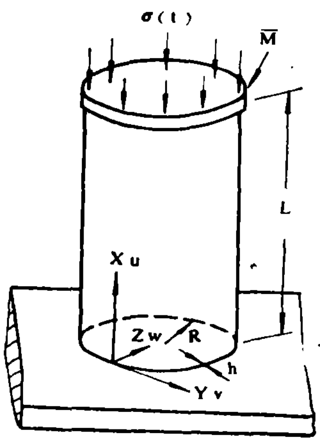


图1　壳的几何形状和坐标

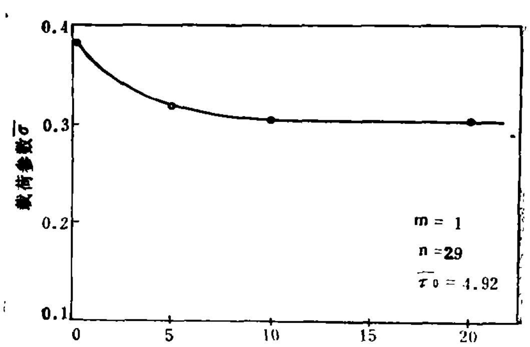


图2　质量对动力屈曲载荷的影响

## 四、结　论

本文探讨圆柱薄壳受有近似三角波的轴向脉冲载荷作用时，考虑原始缺陷$\bar{a}$取为0.3，脉冲载荷时间为3毫秒的情况，考虑轴向惯性和壳体载荷端质量而建立的非线性微分方程组。也考虑边界效应。得出如下结论：动力屈曲载荷定义为载荷参数有很小的变化时径向位移中起主要作用的反对称项模式的幅值出现急速跳跃，以此定义动力临界载荷；壳体载荷作用端上质量的增加使动力屈曲载荷降低，当$\bar{M}$增加到10时，动力屈曲载荷不再降低，说明壳体载荷作用端的质量对动力屈曲载荷有重要影响。

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# THE EFFECT OF MASS AT THE LOADING END OF THE SHELL ON DYNAMIC BUCKLING OF THIN CYLINDRICAL SHELL


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**ABSTRACT** In the past, in all the calculations of nonlinear response of thin cylindrical shell under axial dynamic load, the effect of mass at the loading end of the shell was neglected. In this paper, the effect of mass at loading end of the shell has been taken into account. The effect of different masses has been calculated. The influence of axial inertia and boundary effect has been included.



**SUBJECT TERMS** Cylindrical shell, Dynamic buckling, Axial pulsating load, Structural stability.