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## Wavelike patterns in one-dimensional coupled map lattices

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### Abstract

We investigate the existence of wavelike solution for the logistic coupled map lattices for which the spatiotemporal periodic patterns can be predicted by a simple two-dimensional mapping. The existence of such wavelike solutions is proved by the implicit function theorem with constraints. We also examine the stabilities of these wave solutions under perturbations of uniform small deformation type. We show that in some specific cases these perturbations are completely general. The technique used in this paper is also applicable to investigate other space–time regular patterns.

*Keywords:* Wavelike patterns; Coupled map lattices; Existence; Stability

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### 1. Introduction

Pattern dynamics in coupled map lattices (CML) have recently attracted considerable attention [1–3]. It has been found that CMLs exhibit a variety of space–time patterns: kink–antikinks, space–time periodic structures or wavelike patterns, space–time intermittency and spatiotemporal chaos [2], which are common to other spatially extended systems [3]. Among all space–time patterns, wavelike patterns play an important role. They have been observed as typical space–time patterns in numerical simulations, for instance, in the regime of “pattern selection” in CML [2]. Moreover, more complex situations, such as spatiotemporal intermittence, are reasonably considered as a result of nonlinear interactions of different wavelike patterns and (or) kinks [4].

Wavelike patterns, or simply waves, are defined to be spatially periodic structures even though their temporal motion may be steady, periodic or chaotic. Numerical simulations show that CMLs may exhibit wavelike patterns in several regions of parameter space and the ranges of their possible wavelengths depend on these parameters. This phenomenon is physically described as a wavelength selection from nonlinearity [2]. Furthermore, it is also known that there exists a nonlinear dispersion relation for such type of dynamics [5]. Recently, an expression has been obtained for the wave patterns [6] which is exact for the values of the parameters when the high-order harmonics vanish. In this case, the corresponding dynamics of the CML can be reduced to a simple two-dimensional map (SM), which plays the role of amplitude equations. To our surprise, the validity of the SM model is not restricted to

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such an extreme condition. It can also describe wavelike patterns in the more general situation as was shown in our numerical simulations. In this paper we will show how the SM model can represent dynamics of wavelike patterns in CML and reveal the mechanism of wavelength selection.

The validity of the SM model under the general conditions stated below indicates in which cases and in what exact conditions the low-dimensional model may catch the dynamics of patterns for spatial extended systems. The key problem is how to find a general expression for the addressed patterns and construct the corresponding low-dimension models. In such cases the SM will describe the dynamics of the CML for the corresponding set of initial conditions. Moreover, it is also necessary to verify the stability of the solutions and re-confirm the validity of the low-dimensional models mathematically. The present work suggests a more systematic approach to find the low-dimension mappings, and to investigate the wavelike patterns of CMLs.

The organization of this paper is as follows: after a brief presentation of the model of interest, the wavelike solution and the corresponding SM model are reformulated. We use the implicit function theorem with constraints to extend the existence regions of each type of solution so that the domain of the validity of the SM model is indirectly confirmed. After that, we discuss the stability of such wave solutions under a special type of the small perturbations that we call deformations. We shall show that they are completely general in some specific cases. The final section is the conclusion.

## 2. Existence of wavelike solutions

Our working model is a diffusive coupled map lattice with nearest neighbouring coupling and periodic boundary conditions, namely:

$$x_j^{t+1} = (1 - \epsilon)f(x_j^t) + \frac{1}{2}\epsilon\{f(x_{j-1}^t) + f(x_{j+1}^t)\}, \quad x_0^t = x_N^t, \quad x_1^t = x_{N+1}^t, \quad j = 1, 2, \dots, N, \quad (2.1)$$

where  $\epsilon$  is the coupling parameter with  $0 \leq \epsilon \leq 1$ ,  $t$  a discrete time step and  $j$  the  $j$ th lattice site.  $f$  denotes the local map, which is always taken in this work as the logistic map:

$$f(x) = 1 - ax^2 \quad (0 \leq a \leq 2).$$

### 2.1. Dynamics of wavelike patterns

For a wavelike pattern, we search for a solution of the following form in (2.1):

$$x_j^t = A^t + B^t \cos(j\omega + \phi) \quad (2.2)$$

where  $\omega = (2\pi/N)q$  is a wave number,  $q$  an integer to be determined later and  $\phi$  a constant phase. In order to simplify the notation, we write  $A = A^t$  and  $\bar{A} = A^{t+1}$ , with the same notation for  $B^t$  and other amplitudes  $C^t$ ,  $D^t$ ,  $E^t$  of the next section.

Substituting (2.2) into (2.1), we have

$$\bar{A} + \bar{B} \cos(j\omega + \phi) = \{f(A) - \frac{1}{2}aB^2\} + \{\alpha f'(A)B\} \cos(j\omega + \phi) + \{-\frac{1}{2}\beta aB^2\} \cos(2j\omega + 2\phi)$$

in which

$$\alpha = 1 - 2\epsilon \sin^2(\frac{1}{2}\omega) \quad \text{and} \quad \beta = 1 - 2\epsilon \sin^2 \omega.$$

If  $\omega$  (or  $q$ ) and  $\epsilon$  are chosen such that  $\beta = 0$ , then the SM model is immediately obtained:

$$\bar{A} = f(A) - \frac{1}{2}aB^2, \quad \bar{B} = \alpha f'(A)B. \quad (2.3)$$

Eq. (2.2) is the exact solution of the CML (2.1) in the case where  $\beta = 0$ , and (2.3), as a simple two-dimensional mapping, is a system of the amplitude equations corresponding to (2.1). It completely describes the dynamics of the CML for all the initial conditions in the basin of attraction of the patterns (2.2). When (2.3) has a periodic solution, the CML (2.1) exhibits a spatiotemporal periodic pattern. In fact, the condition  $\beta = 0$  is not necessary for the existence of the wavelike solution. We will modify (2.2) a little so that it can still describe the wavelike solution of (2.1) even for  $\beta \neq 0$ .

## 2.2. The implicit function theorem with constraints

Used in its more general form, the implicit function theorem may ensure the existence and uniqueness of the solutions of (2.1) in the neighbour of  $\beta = 0$  if the Jacobian matrix of (2.1) is invertible. However, it cannot guarantee that the obtained solution possesses the specified spatial pattern. In other words, it may as well be possible that the obtained solution is not of wave type. One way to deal with this problem is to use the norm with weights [7], by which the existence and stability of kinks have been proved [8]. Instead, the method which we shall use is to consider the specified spatial patterns as constraint mapping. The fixed point of (2.1) together with the constraint mapping is just the addressed pattern. Therefore, the implicit function theorem will work in the system of both (2.1) and constraint mapping.

## 2.3. Existence of the wavelike solution in the neighbourhood of $\beta = 0$

For the sake of simplicity, we first consider the steady wave, that is,  $(A, B)$  is a fixed point of (2.3):

$$A = -\frac{1}{2a\alpha}, \quad B = \sqrt{\frac{2}{a} \left( 1 + \frac{1}{2a\alpha} - \frac{1}{4a\alpha^2} \right)}. \quad (2.4)$$

Using the map (2.1),  $\mathcal{F} : \mathbb{R}^N \times \mathbb{R}^2 \rightarrow \mathbb{R}^N$  may be defined as

$$[x_j] \times (a, \epsilon) \mapsto \mathcal{F}([x_j], a, \epsilon) = [(1 - \epsilon)f(x_j) + \frac{1}{2}\epsilon(f(x_{j-1}) + f(x_{j+1}))] - [x_j],$$

and the constraint mapping  $\mathcal{G} : \mathbb{R}^N \times \mathbb{R}^2 \rightarrow \mathbb{R}^N$  of the wave-like patterns as

$$[B_j] \times (a, \epsilon) \mapsto \mathcal{G}([B_j], a, \epsilon) = [A + B_j \cos(j\omega_0 + \phi)],$$

where  $\mathbb{R}^N$  is an  $N$ -dimensional Euclidean space,  $[x_j] = (x_1, x_2, \dots, x_N)^T$  is an  $N$ -dimensional vector ( $T$  denotes transposition), and  $A$  is the same as in (2.4). Let us choose  $a_0$  and  $\alpha_0$  in such a way that (2.4) is a stable fixed point when  $\omega_0$  and  $\epsilon_0$  satisfy both  $\beta = 0$  and  $\alpha_0 = 1 - 2\epsilon_0 \sin^2(\frac{1}{2}\omega_0)$  [6]. Therefore, (2.1) and (2.2) give

$$\mathcal{F}([x_j], a, \epsilon) = 0, \quad [x_j] = \mathcal{G}([B_j], a, \epsilon),$$

or

$$\mathcal{F}(\mathcal{G}([B_j], a, \epsilon), a, \epsilon) = 0. \quad (2.5)$$

Obviously, the solution of (2.5) is a steady wave of the CML (2.1) since the  $[B_j]$  are close to  $[B]$ . The next task is to prove the existence of the solution of (2.5). We know that (2.5) holds for the given  $B$  and  $(a_0, \epsilon_0)$ , i.e.;

$$\mathcal{F}(\mathcal{G}([B], a_0, \epsilon_0), a_0, \epsilon_0) = 0.$$

At this point, the corresponding Jacobian matrix is

$$\mathcal{J}(\mathcal{F} \circ \mathcal{G})_{(B, a_0, \epsilon_0)} = \frac{\partial \mathcal{F}([x_j], a, \epsilon)}{\partial \mathcal{G}([B_j], a, \epsilon)} \frac{\partial \mathcal{G}([B_j], a, \epsilon)}{\partial (B_1, \dots, B_N)} = \{(I + \frac{1}{2}\epsilon E)D - I\}W, \tag{2.6}$$

where  $I$  is the identity matrix, and (diag denotes a diagonal matrix)

$$D = \text{diag}\{f'(A + B \cos(\omega + \phi)), \dots, f'(A + B \cos(N\omega + \phi))\},$$

$$W = \text{diag}\{\cos(\omega + \phi), \dots, \cos(N\omega + \phi)\},$$

$$E = \begin{pmatrix} -1 & 2 & \dots & 2 \\ 2 & -1 & 2 & \dots & 0 \\ 0 & 2 & -1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 2 & 0 & 0 & \dots & 2 & -1 \end{pmatrix}.$$

Here the initial phase  $\phi$  is chosen so that  $\cos(j\omega + \phi) \neq 0$  for each  $j$ . In other words, there are no nodes in the lattice sites. Therefore, the matrix  $W$  is invertible. By the same argument,  $\phi$  may at the same time be chosen such that  $D$  is also invertible. Then, (2.6) can be rewritten as

$$\mathcal{J}(\mathcal{F} \circ \mathcal{G})_{(B, a_0, \epsilon_0)} = (I - D^{-1} + \frac{1}{2}\epsilon E)DW.$$

Thus, the determinant of (2.6) is not zero if

$$\|\frac{1}{2}\epsilon E\| < \|I - D^{-1}\|, \tag{2.7}$$

where  $\|\cdot\|$  is a matrix norm. Eq. (2.7) is easy to be verified because  $(I - D^{-1})$  is diagonal. For example, if  $\|\cdot\|$  is taken as the rank norm, (2.7) becomes

$$\epsilon < \min\{1 - [f'(A + B \cos(j\omega + \phi))]^{-1}, j = 1, 2, \dots, N\}.$$

In this case, by the implicit function theorem, there exists a neighbourhood  $U$  of  $(a_0, \epsilon_0)$  such that (2.5) has a unique solution in  $\Omega$ :

$$B_j = B_j(\epsilon, a) = B + O(s). \tag{2.8}$$

In the second equation of (2.8),  $s$  is small, as a consequence of the implicit function theorem, since it is determined by the distance of the solution to the reference solution with  $\beta = 0$ , i.e.  $s$  is such that  $|B_j - B| < s$  when  $(a, \epsilon)$  is in  $U$ . Therefore, the CML (2.1) displays wavelike patterns even for  $\beta \neq 0$ , namely a steady solution of the wave type:

$$x_j^t = A^t + B_j^t \cos(j\omega + \phi). \tag{2.9}$$

The amplitudes  $B_j$  of (2.9) are close to  $B$  of (2.2) whereas its mean value and wave number are kept unchanged. Due to this modified amplitude, (2.9) is no longer a strictly spatial periodic structure. However, since the pairs of neighbouring sites where  $\{\cos(j\omega_0 + \phi)\}$  changes the sign itself are frozen inside  $U$  (they correspond to “nodes”, although these nodes are not located in the lattice sites), this configurations look like wave patterns. The lattice length between the nearest two such “nodes”, or simply the wavelength, is determined by  $\omega_0$  corresponding to  $\beta = 0$ . Therefore, this kind of approximate wavelength depends only on the values of the parameters, in the sense that the allowed wavelengths are such that they exist and are stable for the corresponding value of  $\alpha$  when  $\beta$  is zero. That is to say, they are completely determined by the SM. Sometimes they are called natural wavelengths. The

patterns between the corresponding nearest two “nodes” are called domains. In this case, the lengths of the domains do not change in time.

The validity of (2.8) in  $U$  means that, for each fixed  $a$ , the corresponding wave number  $\omega = (2\pi/N)q_0$  is kept constant in a neighbourhood  $I_1 = (\epsilon_0 - \delta, \epsilon_0 + \delta)$  of the point  $\epsilon_0$ , where  $\delta$  is a small positive real number. In other words, there is an interval  $I_1$  of the diffusive parameter for which the wave number is locked. This result is easily seen from the following discussion: if  $\beta = 0$ , the diffusive parameter takes only a discrete set of values  $\epsilon_q$ , and each one corresponds to one of the possible discrete wave number  $\omega_q = (2\pi/N)q$ ,  $q = 1, 2, \dots, N$ . By the implicit function theorem,  $\omega$  takes the same value as  $\omega_q$  in the neighbourhood of  $\epsilon_q$ , that is, the wave number is locked. This wave number locking leads to a “staircase” in the curve of the wave number versus the diffusive parameter, a phenomenon which is similar to the frequency locking.

Using the same argument, we can prove the existence of another wavelike pattern by constructing the corresponding mapping and computing its Jacobian. Namely, we consider the set of patterns of the form:

$$x_j^t = A_j^t + B^t \cos(j\omega + \phi), \quad (2.10)$$

where the mean value is modified instead of the amplitude. The difference is only that now  $W$  is replaced by  $I$  in expression (2.6) of the corresponding Jacobi matrix. Comparing the two solutions (2.9) and (2.10), we see that if  $W$  is invertible in (2.6), that is to say, no node is located at the lattice sites, these two wavelike patterns exist, and are equal at least in a common subrange of the parameter space (by uniqueness):

$$x_j^t = A^t + B_j^t \cos(j\omega + \phi) = A^t + (B^t + O(s)) \cos(j\omega + \phi) = A_j^t + B^t \cos(j\omega + \phi).$$

However, if  $W$  is not invertible, that is, if some nodes are located at the lattice sites, (2.10) always exists and (2.9) may not exist (of course, with the  $B_j$  close to the  $B$ ).

*Remark 1.* The initial phase  $\phi$  is only important to ensure that the nodes are not on the sites. Notice that, if  $\phi$  is time dependent, the solution is a travelling wave, a case that we have addressed in another paper [10].

*Remark 2.* Wavelike solutions do exist in the parameter region where the Jacobian matrix (2.6) is invertible.

*Remark 3.* In order to generalize (2.9), it is a natural idea to look for invariant subspaces of the configuration space as they may be expressed in terms of the amplitudes  $A_j$ ,  $B_j$  and so on. In this way, it is possible, even if rapidly cumbersome, to discuss more complex patterns. In the same way, but with the same limitations, it is also possible to treat the periodic (in time) solutions of the SM analysed in [6].

### 3. Stability of wavelike patterns

#### 3.1. Uniformly small deformations

Once a wavelike solution of CML (2.1) is obtained, its stability must be studied. The full analytical treatment of the stability problem of the wavelike patterns (even in the linear case) is, for the moment, out of our reach. One way to go further is of course to use a computer to work out the spectrum of the  $N \times N$  Jacobian matrix at the given solution. This was the method used in [6]. Instead we shall investigate here a different possibility, namely to consider only those perturbations that are compatible with a family of the given pattern fixed for proving the existence of solutions. This means that here we consider only a restricted class of perturbations that we call deformations.

This admissible perturbations must respect the type of solution under consideration, i.e. they shall change the original solutions (or patterns) only by some deformation. From the view point of pattern dynamics, these perturbations give rise to tiny changes of the original pattern so that the perturbed pattern looks very similar to the original one, the former being only a deformation of the latter by drift, modulation or torsion. We trust that these deformations are the realistic permissible perturbations for the case we treat.

*Definition.* Let  $[x_j^t] = X(A(j, t), w)$  be a solution of the CML (2.1), where  $A(j, t)$ ,  $w$  are space–time dependent, and space–time independent parameters, respectively. These parameters may be multi-dimensional. Let  $[\tilde{x}_j] = \tilde{X}$  be another solution of the CML (2.1) perturbed slightly from the solution by an amount  $[\delta x_j] = \delta X$  such that  $\tilde{x}_j = x_j + \delta x_j$ . Then:

- (1)  $\delta X$  is uniformly small on the lattice if there exists a small  $\delta$  independent of  $j$  such that  $\|[\delta x_j]\| < \delta$  for each  $j$ , where  $\|\cdot\|$  is a norm.
- (2)  $\delta X$  is a deformation if  $[\tilde{x}_j] = [x_j] + [\delta x_j] = X(A_\mu(j, t), w_\mu)$  for some  $\mu$  where  $X(A_\mu(j, t))$  is continuously differentiable as a function of  $\mu$  and  $[\tilde{x}_j]|_{\mu=0} = [x_j]$ . In other words, deformations are the perturbations of the structural parameters of the patterns.

A perturbation that satisfies both (1) and (2) is called a uniformly small deformation (USD). We shall also see that they are general in some specific case, that is to say that condition (2) of the above definition may be redundant in such a case.

For example,  $\cos[j(\omega + \mu)]$  is a deformation of the pattern  $\cos(j\omega)$  but it is not a uniformly small perturbation.

We now show how to handle the problem of the linear stability of the wave solution (2.2) under USD, the other families of solutions being treated in the same way.

### 3.2. Stability of wave solution

The wave (2.2) depends on three structural parameters: amplitude, phase and wave number. As seen in the examples of the Section 3.1, perturbations of the wave number can never be a USD, so that we may only consider the perturbations of the amplitude and phase. When the perturbations of the amplitudes and phases are independent of the spatial lattice sites  $j$ , it is readily found that the linear stability of (2.2) is equivalent to the stability of the solution (2.4) of the SM model (2.3). Therefore, in the following we shall consider only the case where the amplitude's perturbation is  $j$ -dependent. The perturbations of the phase will be ignored since they do not affect our conclusion.

Let us first consider the simplest perturbed wave, for which the perturbation introduces only a new wave number:

$$\begin{aligned}\tilde{x}_j^t &= (A^t + sD^t \cos(j\Omega)) + (B^t + 2sC^t \cos(j\Omega)) \cos(j\omega) \\ &= A^t + B^t \cos(j\omega) + s(C^t (\cos[j(\omega - \Omega)] + \cos[j(\omega + \Omega)]) + D^t \cos[j\Omega]),\end{aligned}$$

where  $A^t$ ,  $B^t$ ,  $C^t$  and  $D^t$  are in order of 1, and  $\Omega$  and  $s$  are the small parameters (of the same order) that fix the size of the perturbation. This kind of perturbation is indeed a USD, and  $\tilde{x}_j^t$  is a modulated wave. Substituting  $\tilde{x}_j^t$  into the CML (2.1), we obtain

$$\tilde{x}_j^{t+1} = A^{t+1} + B^{t+1} \cos(j\omega) + s(C^{t+1} (\cos[j(\omega - \Omega)] + \cos[j(\omega + \Omega)]) + D^{t+1} \cos[j\Omega]) + O(s^2),$$

where (using the same notations as in the last section)

$$\bar{A} = f(A) - \frac{1}{2}aB^2, \quad \bar{B} = \alpha f'(A)B, \quad (3.1)$$

and

$$\bar{C} = \alpha[f'(A)C + \frac{1}{2}f'(B)D], \quad \bar{D} = f'(B)C + f'(A)D. \tag{3.2}$$

The Jacobian matrices of (3.1) and (3.2) are

$$J(A, B) = \begin{pmatrix} f'(A) & \frac{1}{2}f'(B) \\ \alpha f'(B) & \alpha f'(A) \end{pmatrix}$$

and

$$J(C, D) = \begin{pmatrix} \alpha f'(A) & \frac{1}{2}\alpha f'(B) \\ f'(B) & f'(A) \end{pmatrix},$$

respectively. It is verified easily that the matrices  $J(A, B)$  and  $J(C, D)$  have the same trace and determinant:

$$\begin{aligned} \text{tr}[J(A, B)] &= \text{tr}[J(C, D)] = (\alpha + 1)f'(A), \\ \det[J(A, B)] &= \det[J(C, D)] = \alpha[f'(A)^2 - \frac{1}{2}f'(B)^2], \end{aligned}$$

where  $\text{tr}$  and  $\det$  represent the trace and the determinant of a matrix separately, so they have the same characteristic polynomial.

Therefore, the perturbations  $C^t$  and  $D^t$  will converge exponentially to zero as the time  $t$  goes to infinity if the eigenvalues of the coefficient matrix in (3.1) are less than unity. For example, if (2.4) is the stable fixed point of (3.1), the perturbations  $C^t$  and  $D^t$  will approach zero. In this sense, (2.2) is the stable steady wave of (2.1). For the same reason, if (3.1) has a stable periodic orbit,  $C^t$  and  $D^t$  will also approach zero. In this case, the stabilities of the periodic solutions of (3.1) provide the sufficient but not necessary conditions for ones of wavelike solutions of (2.1). This fact suggests that we may verify the stability of (3.1) directly instead of looking for wavelike solutions of (2.1).

Now, let us show that, up to the first order in the perturbation, such is the general case of a USD. Moreover we shall see that here, due to the special type of pattern considered, *all the small perturbations are of USD type*, then allowing to conclude in this case to the linear stability under all sufficiently small perturbations.

Suppose that

$$\begin{aligned} \bar{x}'_j &= A + s \sum_{l=1}^N D_l \cos(j\Omega_l) + \left( B + 2s \sum_{l=1}^N C_l \cos(j\Omega_l) \right) \cos(j\omega + \phi) \\ &= A + B \cos(j\omega) + s \sum_{l=1}^N (C_l [\cos(j\omega - j\Omega_l + \phi) + \cos(j\omega + j\Omega_l + \phi)] + D_l \cos(j\Omega_l)), \end{aligned}$$

where now  $s$  and the  $\Omega_l$  are the small parameters (of the same order). Again, introducing the above expression into (2.1) we obtain, up to second order,

$$\bar{x}'_j{}^{t+1} = \bar{A} + \bar{B} \cos(j\omega) + s \sum_{l=1}^N (\bar{C}_l [\cos(j\omega - j\Omega_l + \phi) + \cos(j\omega + j\Omega_l + \phi)] + \bar{D}_l \cos(j\Omega_l)),$$

where again

$$\bar{A} = f(A) - \frac{1}{2}\alpha B^2, \quad \bar{B} = \alpha f'(A)B, \tag{3.3}$$

and

$$\bar{C}_l = \alpha[f'(A)C_l + \frac{1}{2}f'(B)D_l], \quad \bar{D}_l = f'(B)C_l + f'(A)D_l. \tag{3.4}$$

Therefore, the same property as the above holds now for all the amplitude perturbations  $C_l$  and  $D_l$ . Finally, since we allow here arbitrary local small perturbations on the parameter  $A$ , this immediately implies that the only restriction on the perturbations is given by condition (1) of the definition of USD given above. Therefore, in this case, USD are the most general small perturbations on the solutions of the CML.

#### 4. Conclusion

In this paper we present a systematic (if not general!) approach to construct low-dimensional mappings representing the dynamics of regular space–time patterns of CMLs. We may summarize as follows:

- (1) find the solutions of regular space–time patterns for some special values of the parameters for which, indeed, the amplitude equations are exact, starting from initial conditions in the basin of attraction of the given pattern;
- (2) extend the region whose solutions exist by using the implicit function theorem with constraints, which fix the shape of the corresponding patterns;
- (3) study the linear stability of such solutions under special perturbations (that we call uniformly small deformations); however, as stated above, in some cases, these are all the possible small perturbations.

We used the above approach in the simplest case to construct the two-dimensional SM (2.3). When the diffusive parameter  $\epsilon$  satisfies the equality  $\beta = 0$ , (2.3) gives the exact description of the wavelike pattern in the CML (2.1), as well as their linear stability with respect to perturbations of a special type defined as uniformly small deformations. When the diffusive parameter  $\epsilon$  is such that  $\beta$  is in a neighbourhood of  $\beta = 0$ , the CML (2.1) exhibits a wavelike pattern (2.9) similar to (2.2). Therefore, the simple model (2.3) can not only represent the wavelike patterns of the CML (2.1), but also predict their stability to some extent. Similar conclusions can also be obtained for waves with higher temporal periods as computed in [6]. However, chaos will never occur in this way, for chaotic amplitudes ( $A'$ ,  $B'$ ) will force the perturbations to grow fast, and therefore we should be in presence of patterns completely different from the type of those studied here. They may instead probably be understood as modulated (in space as well as in time) waves with respect to our basic patterns.

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