RESEARCH NOTES

An improvement on the theoretical model for the large-scale coherent structures in the outer region of a turbulent boundary layer

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According to the experimental results, there exist large-scale coherent structures in the outer region of a turbulent boundary layer, which have been studied by many authors[1—4]. As experimental results, Antonia (1990)[5] showed the phase-averaged streamlines and isovorticity lines of the large-scale coherent structures in a turbulent boundary layer for different Reynolds numbers. Based on the hydrodynamic stability theory, the 2-D theoretical model for the large-scale structures was proposed by Luo and Zhou[6], in which the eddy viscosity was defined as a complex function of the position in the normal direction. The theoretical results showed in ref. [6] were in agreement with those in ref. [5]. However, there were two problems in the results. One is that in the experimental results, there were divergent focuses between two saddle points in the streamlines, but in the theoretical results, there were centers. The other is that the stretched parts of the isovorticity lines appear at the location of centers in the theoretical results, while in the experimental results they located somewhere between the focuses and saddle points. The reason is that the computations were based on a 2-D model.

In the present paper, a 3-D theoretical model is proposed, where a 2-D wave and a pair of 3-D waves are employed, which have the same streamwise wave number. Streamlines and isovorticity lines obtained by the 3-D model are in satisfactory agreement with those by experimental results.

1 Equations and the eddy viscocity

All quantities q about a flow can be decomposed into

$$q = \bar{q} + \tilde{q} + q',$$

where $\overline{q}$ is the long-time averaged value of q, $\widetilde{q}$ is the difference between the phase-averaged value of q and $\overline{q}$, which corresponds to those induced by the coherent structures, and q' is the value of small-scale addies, whose long-time averaged value and phase-averaged value are both equal to zero. By substituting all the variables into the Navier-Stokes equation, taking long-time average and phase average, and subtracting the former from the latter, the equations for $\widetilde{u}$, $\widetilde{v}$, $\widetilde{w}$ and $\widetilde{p}$ can be obtained, where $\widetilde{u}$, $\widetilde{v}$, $\widetilde{w}$ are the streamwise, normalwise and spanwise velocity components respectively, and $\widetilde{p}$ is the pressure. In addition, according to experimental results, the amplititude of the coherent structures in the outer region was not very large, so that the nonlinear terms originating from the coherent structures themselves were ignored. Using the continuity equation and eliminating the pressure term induced by the coherent structures, the equations for $\widetilde{u}$, $\widetilde{v}$, $\widetilde{w}$ are obtained as follows:

$$\left(\gamma\nabla^2 - \frac{\partial}{\partial t} - \overline{U}\frac{\partial}{\partial x}\right)\nabla^2\widetilde{v} + \overline{U}''\frac{\partial}{\partial x}\widetilde{v} = \frac{\partial^2}{\partial x^2}\frac{\partial}{\partial y}(\widetilde{\gamma}_{11} - \widetilde{\gamma}_{22}) + \frac{\partial^2}{\partial z^2}\frac{\partial}{\partial y}(\widetilde{\gamma}_{33} - \widetilde{\gamma}_{22})$$

$$+ \left(\frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial z^2}\right)\left(\frac{\partial}{\partial x}\widetilde{\gamma}_{12} + \frac{\partial}{\partial z}\widetilde{\gamma}_{23}\right) + 2\frac{\partial^3\widetilde{\gamma}_{31}}{\partial x\partial y\partial z}, \qquad (1)$$

$$\left(\gamma\nabla^2 - \frac{\partial}{\partial t} - \overline{U}\frac{\partial}{\partial x}\right)\left(\frac{\partial}{\partial z}\widetilde{u} - \frac{\partial}{\partial x}\widetilde{w}\right) - \overline{U}'\frac{\partial}{\partial z}\widetilde{v}$$

$$= \frac{\partial^2}{\partial x\partial z}(\widetilde{\gamma}_{33} - \widetilde{\gamma}_{11}) + \left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial z^2}\right)\widetilde{\gamma}_{13} + \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\widetilde{\gamma}_{23} - \frac{\partial}{\partial z}\widetilde{\gamma}_{12}\right), \qquad (2)$$

where x, y and z are the coordinates in the streamwise, normalwise, and spanwise direction respectively, and $\overline{U}(y)$ is the mean flow velocity in the streamwise direction. $\nabla^2 = \frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2} + \frac{\partial}{\partial z^2}$, $\widetilde{\gamma}_{ij} = -(\langle u'_i u'_j\rangle - \overline{u'_i u'_j})$ $(i, j = 1, 2, 3)$ are the stresses induced by small-scale eddies acting on the large-scale eddies. $\langle u'_i u'_j\rangle$ and $\overline{u'_i u'_j}$ are the phase average and long-time average values of $u'_i u'_j$ respectively. γ the kinematic viscosity of the fluid. In calculation, we assumed that the Reynolds stresses can be expressed by eddy viscosity model, such as $\widetilde{\gamma}_{ij} = \overline{\gamma}_T \left(\frac{\partial\widetilde{u}_i}{\partial x_j} + \frac{\partial u_j}{\partial u_i}\right)$, where $\overline{\gamma}_T$ is the eddy viscosity; subscripts 1, 2 and 3 represent x, y and z direction respectively.

Using the free stream velocity $\overline{U}_e$ and the boundary layer thickness δ as the reference quantites, we can non-dimensionalize eqs. (1) and (2). Divide the eddy viscosity $\overline{\gamma}_T$ by the kinematic viscosity, denote it by the same variable $\overline{\gamma}_T$, and write $\widetilde{u}$, $\widetilde{v}$, $\widetilde{w}$ in the form of a travelling wave such that $\widetilde{u} = u(y)e^{i(\alpha x + \beta z - \omega t)}$, $\widetilde{v} = v(y)e^{i(\alpha x + \beta z - \omega t)}$ and $\widetilde{w} = w(y)e^{i(\alpha x + \beta z - \omega t)}$, where α, β, ω are the streamwise, spanwise wave number and frequency, respectively. The equations satisfied by u, v, w can thus be obtained:

$$\{(D^2 - \alpha^2 - \beta^2)^2 - i\alpha R[(\overline{U} - C)(D^2 - \alpha^2 - \beta^2) - \overline{U}'']\}v$$

$$+ [\tilde{\gamma}_T''(D^2 + \alpha^2 + \beta^2) + 2\tilde{\gamma}_T'D(D^2 - \alpha^2 - \beta^2) + \tilde{\gamma}_T(D^2 - \alpha^2 - \beta^2)^2]v = 0, \quad (3)$$

$$[(D^2 - \alpha^2 - \beta^2) - i\alpha R(\overline{U} - C) + \tilde{\gamma}_T'D + \tilde{\gamma}_T(D^2 - \alpha^2 - \beta^2)]$$

$$\cdot (i\beta u - i\alpha w) = i\beta R\overline{U}'v, \quad (4)$$

$$i\alpha u + i\beta w = - Dv, \quad (5)$$

where the Reynolds number $R = \frac{U_e\delta}{\gamma}$, wave speed $C = \omega/\alpha$, and $D = \mathrm{d}/\mathrm{d}y$. As shown in ref. [6], the boundary conditions are $u = 0$, $v = Dv = 0$, at $y = 0.05\delta$ and $y \to \infty$. In the model, the wave numbers of the three waves satisfy $\alpha_M = \alpha_L = \alpha_R$, $\beta_M = 0$, $\beta_L = -\beta_R$, where M, L, R represent the 2-D wave, and the pair of 3-D waves, and the eddy viscosity can be expressed as

$$\overline{\gamma}_T = \gamma_T e^{i\Delta\theta}[1 - \mathrm{sech}(13y/\delta)],$$

where γ_T is the amplitude of $\overline{\gamma_T}$, and $\Delta\theta$ is the phase difference.

2 Results of calculation

The experiments showed that the scale of the coherent structures in the streamwise direction is about 2δ, so in the calculation, the streamwise wave number of the waves is $2\pi/\lambda = 3.14$. At present, there have been no experimental results of how to choose the spanwise wave number. In this paper, the calculation was done with different spanwise numbers. The results showed that when the spanwise number of 3-D wave was about half of the streamwise number, both the streamlines and the isovorticity lines obtained by computation were in good agreement with those of experimental observations. The wave numbers of the three waves are shown in table 1.

Table 1 Wave numbers of the three waves

	M	L	R
α	3.14	3.14	3.14
β	0	1.57	−1.57

We used temporal mode in our computations. In general, C has to be a complex number such that $C = C_r + iC_i$, where C_r is the phase speed of the wave, αC_i is the amplification rate of wave amplitude. Given R and α, β different wave speeds can be obtained, corresponding to different values of γ_T and $\Delta\theta$. In the outer region of a turbulent boundary layer, the rate of time variation of the intensity, or more precisely, the energy of the large-scale structures is small, which implies that the waves are nearly neutral. On the other hand, ref. [5] indicated that the convective velocity of the large-scale structures clustered around the value $0.8U_e$. So in this paper, we choose the values of γ_T and $\Delta\theta$ according to the condition that $C_i \approx 0$ and $C_r \approx 0.8U_e$. In figs. 1 and 2, the values of γ_T and $\Delta\theta$ for 2-D wave and 3-D wave satisfying the above condition are shown as the functions of R, where the solid lines are for 2-D wave, and the dotted lines for 3-D wave. The values of γ_T and $\Delta\theta$ for 3-D waves are smaller than those for 2-D wave, especially the variation of the phase difference $\Delta\theta$.

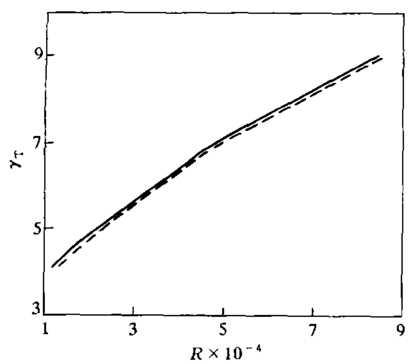

Fig. 1. Relation of γ_T to Reynolds number.

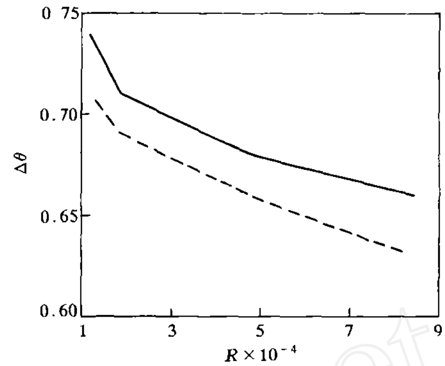

Fig. 2. Relation of $\Delta\theta$ to Reynolds number.

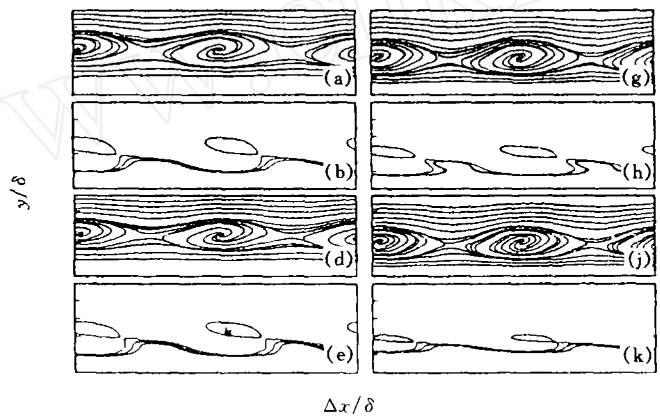

Fig. 3. Streamlines and spanwise vorticity contours (this paper).

The streamlines and isovorticity lines obtained by the calculation using the new model are shown in fig. 3 for four different Reynolds numbers. For easy comparison, the results in refs. [5, 6] are quoted in figs. 4 and 5, respectively, where (a), (d), (g), (j) are streamlines, and (b), (e), (h), (k) are isovorticity lines, corresponding to $R = 11\ 800$, 19 005, 47 949 and 84 427, respectively.

3 Conclusion

From these figures, we can find that there are divergent focuses between two saddle points in the patterns of streamlines obtained by 3-D model, instead of centers obtained by 2-D model. The patterns of streamlines in fig. 3 show that the streamlines on the left part of the top and the right part of the bottom of the large-scale eddies become concentrated because of the drow of the large-scale eddies, which are very similar to the patterns in fig. 4. Fig. 3 shows that the isovorticity lines at the bottom are smoother than those in fig. 5 and the stretched parts of the isovorticity lines become closed rings. All these show that the theoretical model employing the 3-D waves proposed in this paper is better than the 2-D model. The positions of the streamlines and isovorticity lines

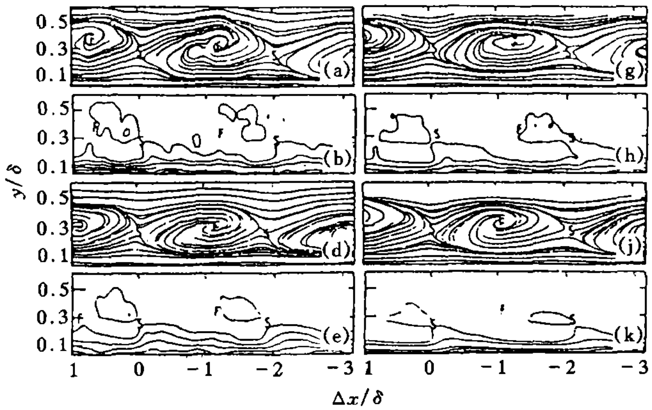

Fig. 4. Streamlines and spanwise vorticity contours (Antonia, 1990).

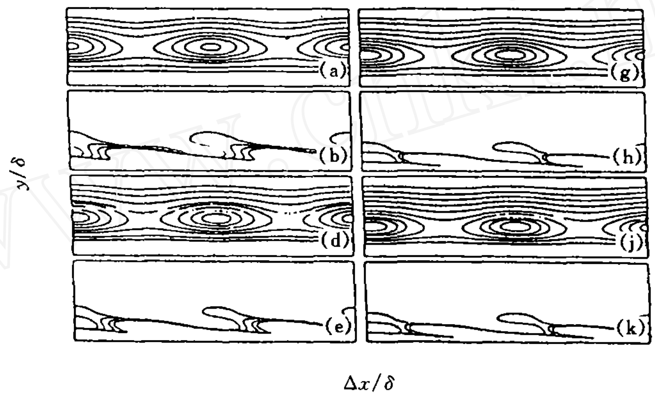

Fig. 5. Streamlines and spanwise vorticity contours (Luo, 1993).

in the streamwise direction obtained by this model are somewhat different from the experimental results. A possible reason is that the effect of bursts in the wall region of the boundary layer on the large-scale coherent structures in the outer region is not considered.

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