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# Suppressing Chaos and Transient in Parameter Perturbed Systems

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**Abstract**

*The systems with some system parameters perturbed are investigated. These systems might exist in nature or be obtained by perturbation or truncation theory. Chaos might be suppressed or induced. Some of these dynamical systems exhibit extraordinary long transients, which makes the temporal structure seem sensitively dependent on initial conditions in finite observation time interval.*

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With the development of the nonlinear science, the applications of the chaotic dynamics have received great attention. Two of these applications are the "Controlling Chaos"[1] and the "Synchronization".[2] A popular technique to achieve these applications is to perturb the system parameters tinily. In fact, for many natural dynamical systems, this perturbation on some system or environment parameters exists naturally rather than being added artificially.[3,4] This perturbation might also be caused by the process of modeling the underlying systems, e.g., some orders of perturbations or truncation.[5] On the other hand, it has been learned that the transients also play important role. The transient chaos introduced by Tél displays some chaotic behavior[6] and the characterizing quantities can also be extracted from transients.[7] The transients are rather short in all these considered systems. In Ref. [4], we had shown that only the transient might be observed in the dynamical systems with some parameters perturbed constantly. In this note, we will further discuss these dynamical systems with one or more system parameters perturbed as follows:

$$\dot{x} = \sigma(y - x)\,, \qquad \dot{y} = (r - z - f(r, z_1))x - y\,, \qquad \dot{z} = xy - bz\,, \tag{1}$$

$$\dot{x}_1 = \epsilon\sigma(y_1 - x_1)\,, \qquad \dot{y}_1 = \epsilon((r - z_1)x_1 - y_1)\,, \qquad \dot{z}_1 = \epsilon(x_1 y_1 - b z_1)\,, \tag{2}$$

where the parameter $\epsilon$ is very small. The other parameters are $\sigma = 10, b = 8/3, r = 125$ in this note. This system might be the zeroth- and first-order perturbations of a dynamical system. For this system, the characteristic time for the last three equations (called subsystem (2) hereafter) is $1/\epsilon$ longer than the first three (subsystem (1)). The part $f(r, z_1)$ can be regarded as the perturbation on the parameter $r$ in Eq. (1). Chaos in Eq. (1) might be suppressed or induced due to this perturbation (no matter how slowly it varies). Consequently, it should take special caution to throw away the very slowly varied part in a considered system, i.e., the high order of perturbation or truncation expansions. Moreover, the full system exhibits very long transient. If we further consider more perturbations with much longer characteristic time as follows:

$$\begin{aligned}
&\dot{x} = \sigma(y - x)\,, && \dot{y} = (r - z - f(r, z_1))x - y\,, && \dot{z} = xy - bz\,,\\
&\dot{x}_1 = \epsilon\sigma(y_1 - x_1)\,, && \dot{y}_1 = \epsilon((r - z_1 - f(r, z_2))x_1 - y_1)\,, && \dot{z}_1 = \epsilon(x_1 y_1 - b z_1)\,,\\
&\dot{x}_2 = \epsilon\sigma(y_2 - x_2)\,, && \dot{y}_2 = \epsilon((r - z_2 - f(r, z_2))x_2 - y_2)\,, && \dot{z}_2 = \epsilon(x_2 y_2 - b z_2)\,,\\
&\cdots
\end{aligned} \tag{3}$$

the system displays extraordinary long transient which also exhibits some chaotic behavior. The temporal structure is sensitively dependent[8] on initial conditions in a finite observation time interval.

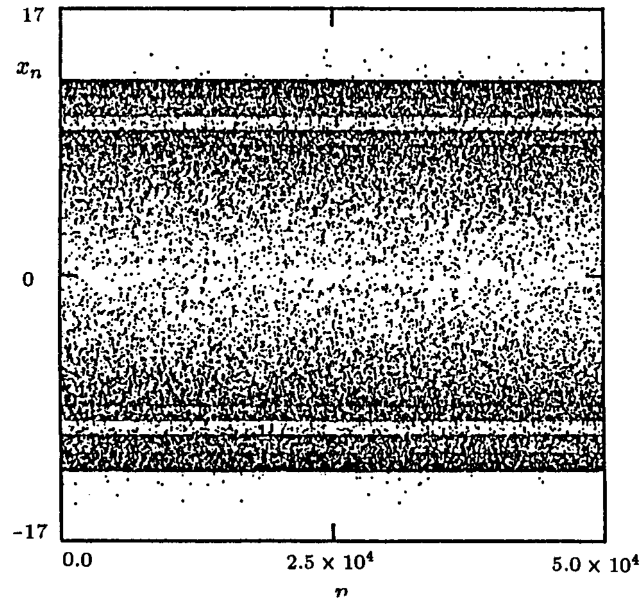


**Fig. 1.** $x$ vs. $n$ on the Poincare section $z = r - 1$ for the pure Lorenz equations at $r = 125$.

When $f(r, z_1) = 0$, the subsystems (1) and (2) are decomposed. Both equations (1) and (2) correspond to pure Lorenz equations. They exhibit chaotic attractor as shown in Fig. 1.

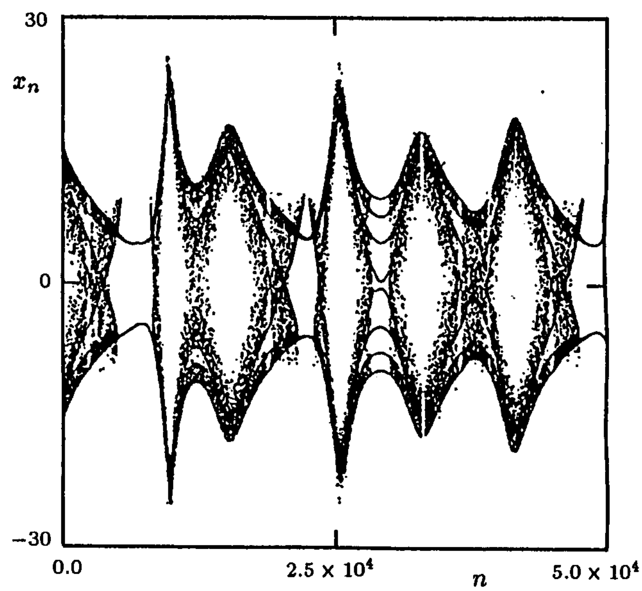


**Fig. 2.** $x$ vs. $n$ for the perturbed system (1) and (2) at $r = 125$.

When $f(r, z_1) \neq 0$, although $f(r, z_1)$ might be much smaller than $r$ and $\epsilon$ is very small, the behavior of the perturbed dynamical systems (1) and (2) might be qualitatively different from that of the pure Lorenz equations. Figure 2 shows the behavior for $f(r, z_1) = (z_1 - r + 1)/2$ and $\epsilon = 0.0001$. It is rather remarkable to find that in some time intervals the $x$ value changes "regularly", i.e., chaos is suppressed in these time intervals. However, in other time intervals, the $x$ value changes "chaotically". This phenomenon is different from the conventional intermittencies (types I, II and III[10]) and the intermittency observed recently.[9] Similar behavior has also observed for the cases when the pure Lorenz equations exhibit periodic motion (say, $r = 100$, the period 3 windows). In these cases, chaos is induced by perturbation on parameter.

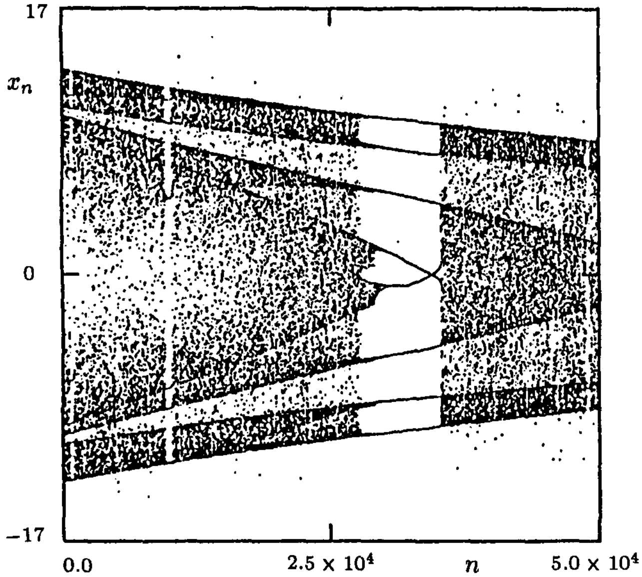


**Fig. 3.** The same as Fig. 2 but for $\epsilon = 0.00005$.

The quantity $\epsilon$ determines the relative characteristic time for each subsystem in the perturbed system (1) and (2). It plays very important role in the observation of the aforementioned phenomenon although it cannot affect the value range of the $x_1, y_1, z_1$. In Fig. 3, we also show 50000 points for this perturbed system with $\epsilon = 0.00005$. This figure is very similar to an enlarged part of Fig. 2. This makes it difficult to observe the full attractor for the full system (1) and (2) in a very large but limited time interval. Consequently, the temporal structure seems sensitively dependent on initial conditions (of $x_1, y_1$ and $z_1$) in a very large time interval. When $\epsilon$ is small enough, this intermittency phenomenon might not be observed in a limited time interval.

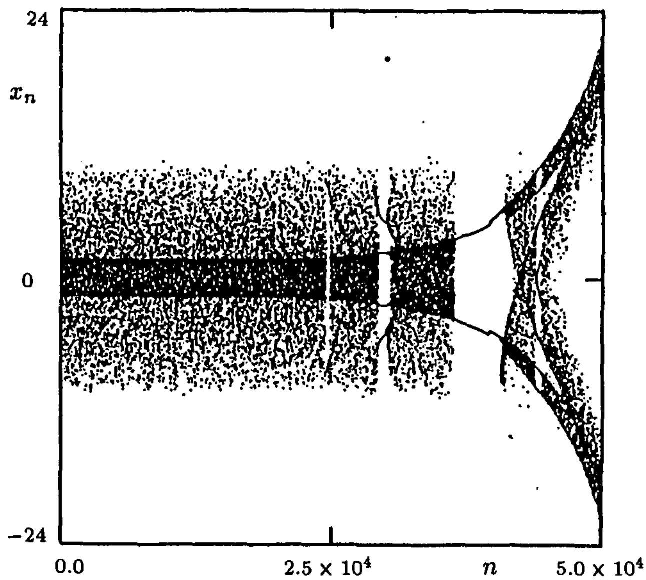


**Fig. 4.** The transient for the perturbed system (1) and (2) with $\epsilon = 0.00005$.

The above discussion is not particular to the systems taken in the form of (1) and (2). Subsystem (2) might be much complex, i.e., the $n$th-order perturbation or truncation. Subsystem (1), correspondingly, might consist of a large number of equations, such as all the first $0 \sim (n-1)$th orders. Since subsystem (2), though small and very slowly varied, can induce or suppress chaos of subsystem (1), the subsystem (1) might never be a good approach to the full system.

Now we further consider the behavior of the parameter perturbed system (1) and (2). Subsystem (2) has a much longer characteristic time than subsystem (1) so that the full system (1) and (2) has a very long transient ($1/\epsilon$ time longer than the pure Lorenz equations) and the transient (see, Fig. 4) exhibits behavior rather similar to that of a chaotic attractor. Furthermore, the perturbed system might take in the form of Eq. (3), which includes all orders of the perturbation or truncation expansions. This system will exhibit extraordinary long transient. Since the transient depends sensitively on the initial conditions, e.g., the temporal structure is sensitively dependent on initial conditions in a great but finite observation time interval. A detailed study on this idea and its application to understanding the behavior (i.e., sensitive dependence on initial conditions) of the spatio-temporal chaos observed on the coupled lattice maps and partial differential equations will be presented in an extended paper.

In conclusion, we have investigated the dynamical systems with one or more parameters perturbed chaotically. These parameter perturbed systems might exist in nature or be obtained by perturbation or truncation theory in which the parameters are perturbed by higher-order perturbations or truncations. Due to the perturbation (which might be very small and slowly varied), chaos might be suppressed or induced. This result indicates that it should take special caution to use the perturbation or truncation theory to dynamical systems. Some of these dynamical systems exhibit extraordinary long transients, which makes the temporal structure seem sensitively dependent on initial conditions in finite observation time interval. This observation might be helpful to understand the spatio-temporal chaos observed in the coupled lattice maps and the partial differential equations.

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[9] When our manuscript was just completed, we received a preprint (Z.L. QU, *et al.*, submitted to Phys. Rev. Lett.), in which the suppression of chaos in some time intervals were observed in a nonautonomous model with a tiny, constant frequency perturbation. However, their observation had definite period. In this note, the parameter is perturbed by chaotic signal ($z_1$) so that the behavior observed in this note is never recursive.

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