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Drop Migration of Middle Reynolds Number in a Vertical Temperature Gradient

The experimental investigation of the thermocapillary drop migration in a vertical temperature gradient was performed on ground. Silicon oil and pure soybean oil were used as experimental medium in drops and as continuous phases, respectively, in the present experiment. The drop migration, under the combined effects of buoyancy and thermocapillarity, was studied for middle Reynolds numbers in order of magnitude $O(10^1)$. The drop migration velocities depending on drop diameters were obtained. The present experimental results show relatively small migration velocity in comparison with the one suggested by Young et al. for linear theory of small Reynolds number. An example of flow patterns inside the drop was observed by PIV method.

1 Introduction

Marangoni migration of single drops or bubbles, studied in the case of the linear perturbation model by *Young*, *Goldstein*, and *Block* in 1959 as the YGB model [1], is an interesting subject in fluid mechanics. In addition, it has extensive applications in the field of chemical engineering with respect to heat and mass transfer processes. The gravity effect and thermocapillary effect are usually coupled together in the process on ground. The subject of Marangoni drop migration becomes more attractive in microgravity science because of the importance of interfacial phenomena under microgravity where the gravity effect is greatly reduced and the drop migration is dominated by the thermocapillary effect.

The linear theory of the YGB model omitted the inertia influence (small Reynolds number) and convective energy transport (small Marangoni number). The migration velocity may be expressed as follows for the case where the applied temperature gradient is parallel to the direction of gravity

$$V = V_g + V_T, \tag{1}$$

with

$$V_g = AR^2, \quad V_T = B\Gamma R, \tag{2}$$

and

$$A = \frac{2g(\varrho - \varrho')(\mu + \mu')}{3\mu(2\mu + 3\mu')},$$

$$B = \frac{\mathrm{d}\sigma}{\mathrm{d}T} \frac{2\kappa}{(2\mu + 3\mu')(2\kappa - \kappa')}. \tag{3}$$

R is the radius of the drop. σ, μ, ϱ, and κ are the interfacial tension, the viscosity, the density, and the thermal conductivity of continuous phase, respectively. Superscript (′) represents the quantities of the drop phase. $\mathrm{d}\sigma/\mathrm{d}T$ and $\Gamma = \mathrm{d}T/\mathrm{d}z$ are the vertical temperature gradients at infinite. The analytical solutions were obtained to include the influence of inertia [2, 3], or the influence of heat convection [4–6]. However for large Reynolds numbers and large Marangoni numbers, eq. (1) can only be solved by the method of numerical simulation [7].

A few microgravity experiments for thermocapillary drop migration on board microgravity sounding rockets and Spacelab have been performed. The results of small drops with 11 ± 1.5 μm in diameter agree with the theory of the YGB model [8]; however, the migration velocities of larger drops with 0.69–2.38 mm in diameter are smaller than the ones given by the YGB model [9]. More experiments have been completed on ground, and the migration velocity may be obtained by the coupled effects of gravity and thermocapillarity. In cases of small drops and small applied temperature gradients, the coupled effects may be separated, see [1, 10, 11].

In the present paper, the migration of drops with a few millimeters in diameter are studied experimentally for Reynolds number of order $O(10^1)$. Experimental results show that the coupled drop migration velocities are smaller than the one with linear modes. This conclusion agrees generally with the sounding rocket experiment [9], where only the influence of thermocapillarity is regarded. The liquid system used in the present experiment and the experimental conditions are discussed respectively in sects. 2 and 3. The measurements of migration velocities and the velocity patterns inside and outside the drop given by the PIV method are summarized in sect. 4. The discussion and conclusion are presented in sect. 5.

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2 Liquid System

In this experiment, 5cSt silicon oil and pure soybean oil were used as drop phase and continuous phase. The densities of two liquids are nearly equal at 0 °C. In order to avoid the uncertainty of the experimental medium, parameters of the physical properties of drop and continuous phases were measured in the present experiment. The viscosities of the liquids were obtained in a wide temperature range by using the *Haake VT 550* viscometer. More changes can be seen in the viscosity of soybean oil than for 5 cSt silicon oil in the measured temperature range. The interfacial tension measured by drop volume method has negative gradient within the temperature range between 25 and 75 °C. The measurement was conducted after putting the two liquids together at a certain temperature so that a relative saturation of miscibility could be reached. Therefore, the miscibility effects, which could alter the interfacial tension, are reduced. The measurements of $d\sigma/dT$ give a constant value: 0.0055 ± 0.0001 dyn/(cm · K) in the temperature range between 25 and 75 °C after fitting of the experimental data. Furthermore, the differential of interfacial tension to temperature $d\sigma/dT$ is extrapolated to the low temperature range. These data are the basis of our migration experiments. The measurement errors are within 2 % for σ, and smaller than 5 % for other physical parameters. In the present experiment, the pure aluminum particles were used as tracers to show the drop's inner flow field. The measurements of interfacial tension for the liquid systems with and without particles show that the pure aluminum particles do not induce obvious change of interfacial tension of the liquid system. Table 1 summarizes the major properties of the fluid system.

The silicon oil and pure soybean oil are nearly immiscible at room temperature. The drop will migrate in an environment with increasing temperature, and the liquids may be a bit miscible at higher temperature during migration. However, a large temperature gradient was applied in the present experiment, and the thermal expansion may have increased the drop volume during migration. Experimental results show that the drop diameters changed slightly when a drop moved upward from starting point to the top of the test cell, as shown in fig. 1, where D and D' represent the drop diameter at the starting position and top position, respectively. The experimental results show that both contributions of miscible effect and thermal expansion to the change of drop diameter are not important.

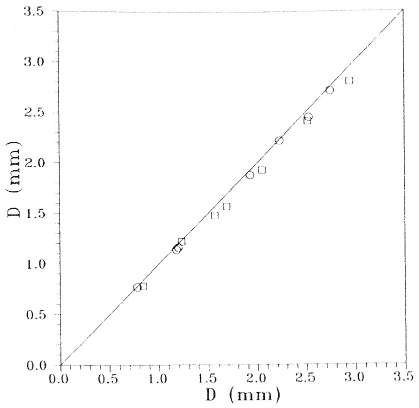

Fig. 1. The changes of drop diameters during migration. The temperature gradients are 7.5 K/mm (□) and 5.85 K/mm (○). D and D' represent the diameter of drops at the starting point and the top position, respectively

3 Experimental Procedure

The experimental hardware, as shown schematically in fig. 2, was developed to satisfy the scientific requriements. The cross-section of the test cell is 46 × 46 mm^2 and the height can be adjusted step by step from 24 to 70 mm. The upper and lower copper blocks are fixed with Teflon frames. Three sides of the frame consist of transparent windows which permitted optical observation and image record. The fourth side of the frame consists of a Teflon plane with holes for drop injection. All windows were glazed by two glass planes with an air gap in the middle for better heat insulation. The upper copper block of the test cell was heated by electrical coils and the lower block was put in a fluid bath which kept the temperature under 0 °C. In this case, a larger vertical temperature gradient could be obtained. 3 electrical coils for heating were used to supply a higher temperature at the upper copper block. 2 thermocouples were fixed in the upper and lower blocks and then connected with the *EUROTHERM 818* controllers to control the temperature difference between the two blocks. A

Table 1. Physical parameters of liquids. Superscript (') represents the qunatities of the drop phase

temperature liquid properties	0 °C	25 °C	57 °C	75 °C
μ' [cp = 10^{-2} dyn · s/cm^2]	5.977	4.268	2.668	2.014
κ' [mw/mK]	114.3	111	107.6	104.5
ρ' [g/cm^3]	0.9340	0.91847	0.8823	0.8642
β' [1/K]	0.00118	0.00118	0.00118	0.00118
μ' [cp = 10^{-2} dyn · s/cm^2]	165	56.5	20.1	12.3
κ [mW/mK]	177.5	175	172.4	170.8
ϱ [g/cm^3]	0.9340	0.91091	0.8983	0.8859
β [1/K]	0.00081	0.00081	0.00081	0.00081
σ [dyn/cm]	0.875[1]	0.745	0.545	0.445
$d\sigma/dT$ [dyn/(cm · K)]	−0.0055[1]	−0.0055	−0.0055	−0.0055

[1] The data are given by extrapolation.

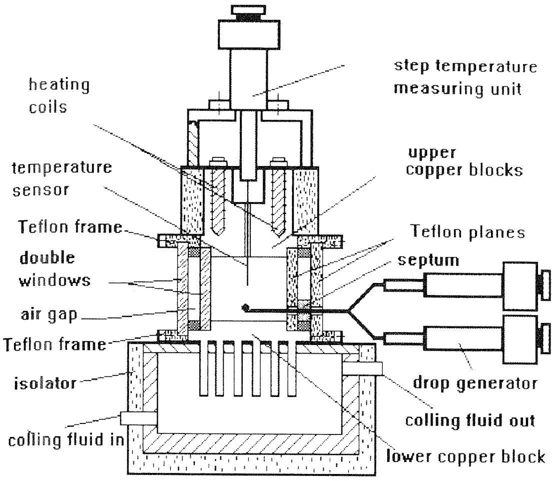

Fig. 2. Schematic diagram of the experimental apparatus

temperature measurement device was used to obtain vertical temperature distribution of the continuous phase liquid in the test cell. The temperature distribution of the continuous liquid phase could be measured point by point automatically in the vertical direction. A special device was developed to inject drops into the continuous phase liquid. The main part of the drop generator consisted of two slender co-axial steel pipes, and can be conveniently inserted into or drawn out from the test cell through the side Teflon planes. The generator was connected via flexible pipes with 2 syringes which were full of drop liquid and continuous phase liquid. The syringes were driven by step motors and could be controlled by hand or by computer program to support liquid drop in a range from 0.1 to more than 10 mm in diameter. In the present experiment, drops were generated and suspended successfully and the drop size was exactly controlled.

The experiments for larger Reynolds numbers require a larger radius and migration velocity, followed by a larger temperature gradient. The applied temperature differences between both sides of the test cell with 25 mm in height may be applied as high as 180 °C. The test cell was carefully cleaned by acetone and dried before each experiment. The drops were injected into the test cell using a computer aid system when the stable linear temperature distribution in the liquid was established. The drops remained at the starting position for a period of time before separating from the ejector to get thermal equilibrium with the surrounding liquid environment. The temperature at the lower side may be kept lower than 0 °C by introducing ethanol as a cooling medium.

The moving drops were recorded on-line by CCD video camera and VTR during the experiment; and gave the measured drop diameters, drop migration trajectories and velocities. A PIV method was used for flow visualization. The convection in the test cell was visualized by particle tracers. The experimental results showed that the convection in the liquid of the test cell was not strong. Generally, the tracers in the liquid of the test cell for flow visualization may be adsorbed on the interface and varies the original distribution of interfacial tension. However, pure aluminum particles do not act as surfactant of the liquid system, and have no obvious effect on the interfacial tension in the present experiment. Using a long distance microscope and a video camera system, the flow fields inside and outside the drops were observed and recorded.

4 Experimental Results

The migration velocity will be increased during the drop moving upward from the start position to the top of the test cell if the vertical temperature gradient is large enough. The experimental results reveal the tendency of the migration velocity to gradually increase to a steady value if the migration distance is long enough and the vertical temperature gradient uniform. For example, fig. 3 gives the drop velocities depending on the drop diameter during migration for a temperature gradient of 7.5 K/mm. 3 sets of drop diameters, 1.9 mm, 2.5 mm, and 3.1 mm are shown. The broken lines in fig. 3 denote the approximate migration velocities. The experimental migration approaches nearly 97 % of these approximate values. This implies that the measured velocities at the top of the test cell in the present experiment permit nearly to obtain the steady migration velocities, if the density difference, which will be increased during the upward migration of the drop, has only secondary influence. In addition, two independent sets of the

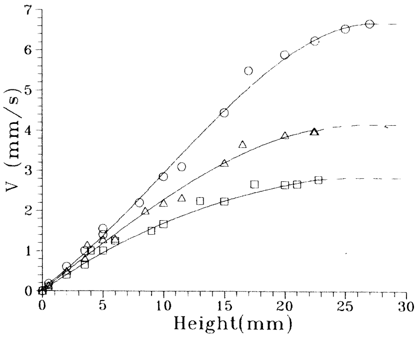

Fig. 3. Typical experimental plots for drop velocities vs drop positions for a temperature gradient of 7.5 K/mm. Drop diameters were 3.1 mm (○), 2.5 mm (△) and 1.9 mm (□), respectively

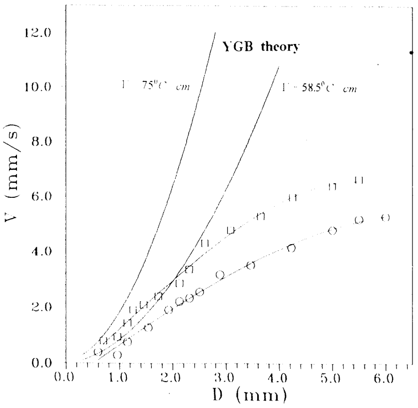

Fig. 4. Experimental results of drop velocities vs drop diameters for vertical temperature gradients of 7.5 K/mm (□) and 5.85 K/mm (○)

Table 2. Comparison of experimental data with YGB linear theory for $dT/dz = 58.5\ K/cm$ and $dT/dz = 75\ K/cm$

$\frac{dT}{dz}$ [K/cm]	diameter D [mm]	experiment V_m [mm/s]	YGB model V_{YGB} [mm/s]	ratio V_{YGB}/V_m	Reynolds number $Re = \frac{V_m D}{\nu}$	Marangoni number $Re \cdot Pr$
1 58.5	0.60	0.031	0.037	1.18	0.09	4.6
	1.15	0.077	0.110	1.42	0.28	18.5
	1.92	0.192	0.274	1.43	1.14	75.4
	2.50	0.257	0.446	1.74	2.00	132.2
	3.46	0.353	0.822	2.33	3.79	250.6
	5.00	0.480	1.660	3.46	7.45	492.5
	6.00	0.530	2.360	4.46	9.88	653.2
2 75.0	0.70	0.080	0.101	1.26	0.22	14.8
	1.15	0.145	0.236	1.63	0.70	46.3
	1.73	0.240	0.474	1.98	1.70	112.4
	2.30	0.338	0.828	2.45	3.10	204.9
	3.08	0.480	1.431	3.00	5.90	390.0
	4.25	0.600	2.745	4.58	10.20	674.3
	5.50	0.665	4.348	6.54	14.60	965.2

migration velocities depending on drop diameters under temperature gradients of 7.5 and 5.85 K/mm were measured and shown in fig. 4. It is seen in fig. 4 that the velocities of both present experiments and the YGB model increase with the drop diameters. The drop migration velocities of the present experiment are smaller than the one of YGB linear model. Table 2 gives a detailed comparison of case 1 ($dT/dz = 58.5$ K/cm) and case 2 ($dT/dz = 75.0$ K/cm) respectively.

The flow field within the drops may be observed by the PIV method to investigate the thermocapillary flow driven by the interfacial tension gradient. A photo of the inner flow pattern in a vertical cross-section of a drop is shown in fig. 5. It shows a cross-section of the drop when the buoyancy and thermocapillary forces are kept in balance. The drop was 9.1 mm in diameter. A small temperature gradient of about 2.4 K/mm was applied. The photo was obtained by using prolonged exposure. A special drop diameter D_* can be obtained for the YGB model by using eq. (1). If the migration velocity is 0 ($V = 0$), the drop diameter is $D_* = 3.4$ mm, which is only 37 % of the experimental drop diameter. This implies that the experimental result for middle Reynolds number has a large dispersion from the YGB model even for drops without migration. One can also note that the flow cells in fig. 5 are nearly symmetric when the drops are suspended in the continuous phase liquid. The 2 cells appear nearly in the middle of the drop in vertical direction. The typical velocity on the interface at the drop equator is about 0.55 mm/s. During the experiment, it also can be observed obviously in real time that when drops migrated upward, the two vortices which appeared in the vertical cross-section moved downward gradually when the drop's velocity was increased, and the upper part of the 2 axial symmetric vortices appeared sepearated.

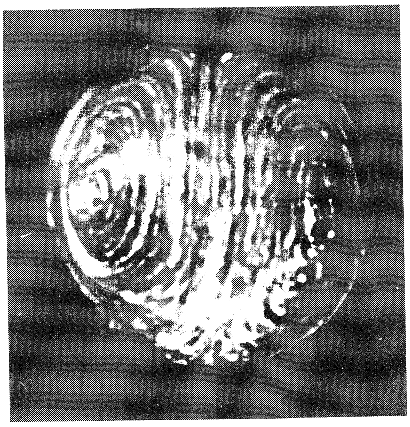

Fig. 5. Flow field inside a drop. Drop diameter is 9.1 mm

5 Discussion

In the present experiment, the features of drop migration of middle Reynolds number and the flow field within the drop were analyzed. The experimental results of drop migration in the case of middle Reynolds number of $O(10^1)$ do not agree with the simplified model given by the YGB theory. The drop migration veolcities of the present experiments are smaller than the one in the YGB model. Non-linear effects should be considered when one investigates the features of drop migration with Reynolds numbers larger than 1. When the Reynolds number is larger, the inertia and convective effects should not be neglected. Furthermore the non-linear effects need to be considered. The effects of buoyancy and thermocapillary convective are coupled in the non-linear case. The experiment in microgravity environment may produce the drop migration driven only by the thermocapillary effect as in [9]. Additionally, more experiments for larger Reynolds numbers should be performed.

The flow pattern within a small drop was obtained by using the PIV method. The quantitaive measurements are difficult due to the movement and the small scale of the drop. The present experiment gave the preliminary results of the velocity field. Further studies are needed.

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