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Fracture criteria for combined cleavage and dislocation emission

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ABSTRACT

A general theory of fracture criteria for mixed dislocation emission and cleavage processes is developed based on Ohr's model. Complicated cases involving mixed-mode loading are considered. Explicit formulae are proposed for the critical condition of crack cleavage propagation after a number of dislocation emissions. The effects of crystal orientation, crack geometry and load phase angle on the apparent critical energy release rates and the total number of the emitted dislocations at the initiation of cleavage are analysed in detail. In order to evaluate the effects of nonlinear interaction between the slip displacement and the normal separation, an analysis of fracture criteria for combined dislocation emission and cleavage is presented on the basis of the Peierls framework. The calculation clearly shows that the nonlinear theory gives slightly high values of the critical apparent energy release rate G_c for the same load phase angle. The total number N of the emitted dislocations at the onset of cleavage given by nonlinear theory is larger than that of linear theory.

§ 1. INTRODUCTION

A general theory of crack propagation due to combined cleavage and dislocation emission is proposed in this paper. The concepts adopted here have been developed by Rice and Thomson (1974), Ohr (1985) and Lin and Thomson (1986). The well known dislocation emission model proposed by Rice and Thomson (1974) gave a quantitative criterion for ductile versus brittle behaviour.

Recently Beltz and Rice (1991), Schoeck (1991), Rice (1992) and Rice, Beltz and Sun (1992), have reanalysed the Rice–Thomson criterion on the basis of the Peierls (1940) framework, in which the fully emitted dislocation is considered as a continuous distribution of infinitesimal continuum dislocations. For the mode II case, Rice (1992) presented an exact solution for the loading as the nucleation instability was developed and identified a solid-state parameter, the unstable stacking energy γ_{us}, which characterizes the resistance to dislocation nucleation.

The brittle cleavage of a crack in a metal is usually accompanied by a considerable number of dislocation emissions. A fracture criterion accounting for the effects of the dislocation emission were proposed by Sinclair and Finnis (1983) with a simple analysis for a pure mode I crack. Their model was constrained to cleavage on one plane and crack branching was ruled out. A general theory for crack propagation in the situation of combined cleavage and dislocation emission was developed by Lin and Thomson (1986). Their model is also constrained to cleavage on one plane and crack branching was not permitted.

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On the basis of numerous observations, Ohr (1985) pointed out that crack propagation was a mixed-mode process in which dislocation emission and cleavage could proceed in the same plane. After emitting a number of dislocations, the crack propagated along the slip plane of these dislocation and stopped after at certain distance. Then a second slip system was activated. After emitting a number of dislocations, the crack propagated along this second slip plane and stopped again at certain distance. This model is consistent with the geometry of zigzag crack propagation. On the basis of this model, we developed a general theory of fracture criteria for mixed dislocation emission and cleavage under mixed loading.

Explicit formulae are proposed for the critical condition of crack cleavage propagation after a number of dislocation emissions. The effects of crystal orientation geometry and load phase angle on the apparent critical energy release rates are analysed in detail.

§ 2. Basic formulae

We consider only the plane strain problem, that is we solve a two-dimensional theory. Suppose that the crack front is contained within one slip plane which is most highly stressed in a crystal. As shown in fig. 1, the slip plane makes an inclined angle θ_0 with respect to the crack plane. Suppose that the crystal is subject to mixed-mode load which includes the stress intensity factors $k_{\rm I}$ and $k_{\rm II}$ at the crack tip with respect to the coordinate system (Oxy).

The in-plane normal stress σ_θ and shear stress $\tau_{r\theta}$ acting on the slip plane, according to the linear elastic theory, can be expressed as

$$\sigma_\theta = \frac{1}{2}\cos\left(\frac{\theta_0}{2}\right)\frac{k_{\rm I}(1+\cos\theta_0) - 3k_{\rm II}\sin\theta_0}{(2\pi r)^{1/2}}, \tag{1}$$

$$\tau_{r\theta} = \frac{1}{2}\cos\left(\frac{\theta_0}{2}\right)\frac{k_{\rm I}\sin\theta_0 + k_{\rm II}(3\cos\theta_0 - 1)}{(2\pi r)^{1/2}}. \tag{2}$$

According to Rice and Thomson (1974), the stress intensity factors contributed by an emitted dislocation along an inclined slip plane are

$$k_{\rm Id} = -\frac{\mu b_{\rm e}}{2(1-\nu)(2\pi r_{\rm c})^{1/2}}\,3\sin\theta_0\cos\left(\frac{\theta_0}{2}\right), \tag{3}$$

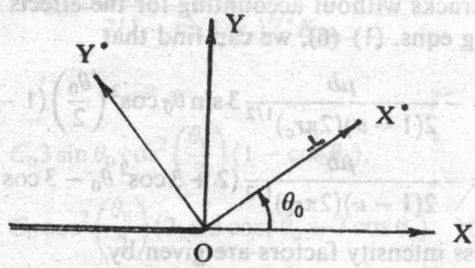
$$k_{\rm IId} = -\frac{\mu b_{\rm e}}{2(1-\nu)(2\pi r_{\rm c})^{1/2}}(3\cos\theta_0 - 1)\cos\left(\frac{\theta_0}{2}\right), \tag{4}$$

where $b_{\rm c}$ is the Burgers vector of an edge dislocation alone the slip direction and $r_{\rm c}$ is the distance from the crack tip to the edge dislocation. It is convenient to introduce the coordinate system (Ox^*y^*) in which the x^* axis is coincident with the slip direction and the y^* axis is perpendicular to the slip plane. With respect to the coordinate system (Ox^*y^*), one can define the stress intensity factors $K_{\rm I}$ and $K_{\rm II}$ as follows:

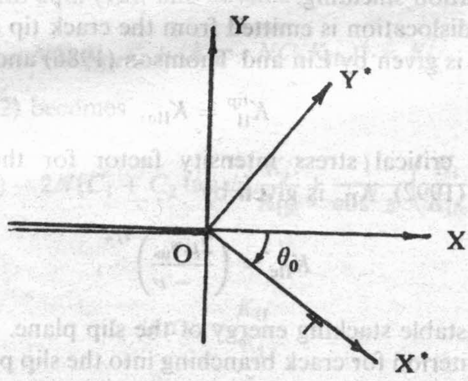
$$K_{\rm I} = \lim_{r\to 0}[(2\pi r)^{1/2}\sigma_\theta], \tag{5}$$

$$K_{\rm II} = \lim_{r\to 0}[(2\pi r)^{1/2}\tau_{r\theta}], \tag{6}$$

Fig. 1



(a)



(b)

Geometry of crack plane and slip plane.

where the σ_θ and $\tau_{r\theta}$ are the stress components acting on the Ox^* plane. Substituting eqns. (1) and (2) into eqns. (5) and (6), one obtains

$$K_{\mathrm{I}} = \frac{1}{2}\cos\left(\frac{\theta_0}{2}\right)[k_{\mathrm{I}}(1+\cos\theta_0) - k_{\mathrm{II}}3\sin\theta_0],$$
$$K_{\mathrm{II}} = \frac{1}{2}\cos\left(\frac{\theta_0}{2}\right)[k_{\mathrm{I}}\sin\theta_0 + k_{\mathrm{II}}(3\cos\theta_0 - 1)]. \tag{7}$$

It is worth noting that eqn. (7) provides only an appropriate approximate expression. As point out by Lin and Thomson (1974), the exact solution for the stress intensity factors at the branched crack tip is complex. The definitions (5) and (6) were introduced by Nuismer (1975) and are widely used in constructing the criteria for mixed-mode cracks without accounting for the effects of dislocation emissions.

Similarly using eqns. (1)–(6), we can find that

$$K_{\mathrm{Id}} = -\frac{\mu b}{2(1-\nu)(2\pi r_c)^{1/2}} 3\sin\theta_0 \cos^2\left(\frac{\theta_0}{2}\right)(1-\cos\theta_0),$$
$$K_{\mathrm{IId}} = -\frac{\mu b}{2(1-\nu)(2\pi r_c)^{1/2}}(2+3\cos^2\theta_0 - 3\cos\theta_0)\cos^2\left(\frac{\theta_0}{2}\right). \tag{8}$$

The local stress intensity factors are given by

$$K_{\mathrm{I}}^{\mathrm{tip}} = K_{\mathrm{I}} + K_{\mathrm{Id}},$$
$$K_{\mathrm{II}}^{\mathrm{tip}} = K_{\mathrm{II}} + K_{\mathrm{IId}}. \tag{9}$$

The physical meaning of eqn. (9) is clear. The local stress intensity factors are less than the applied stress intensity factors. This decrease in local stress intensity factors is caused by dislocation shielding.

When an edge dislocation is emitted from the crack tip along the slip plane, the emission condition is given by Lin and Thomson (1986) and Rice (1992) as follows:

$$K_{\mathrm{II}}^{\mathrm{tip}} = K_{\mathrm{IIe}}, \tag{10}$$

where K_{IIe} is the critical stress intensity factor for the dislocation emission. According to Rice (1992), K_{IIe} is given by

$$K_{\mathrm{IIe}} = \left(\frac{2\mu\gamma_{\mathrm{us}}}{1-\nu}\right)^{1/2}, \tag{11}$$

where γ_{us} is the unstable stacking energy of the slip plane.

The cleavage criterion for crack branching into the slip plane is given by Nuismer (1975) as follows:

$$(K_{\mathrm{I}}^{\mathrm{tip}})^2 + (K_{\mathrm{II}}^{\mathrm{tip}})^2 = K_{\mathrm{Ic}}^2, \tag{12}$$

where K_{Ic} is the fracture toughness for the slip plane. Meanwhile at the initiation of cleavage branching, the local mode II stress intensity factor $K_{\mathrm{II}}^{\mathrm{tip}}$ is given by

$$K_{\mathrm{II}}^{\mathrm{tip}} = \eta K_{\mathrm{IIe}}, \quad 0 \le \eta \le 1. \tag{13}$$

Suppose that the cleavage criterion (12) is met after N edge dislocations are emitted. Then the local stress intensity factors are given by

$$K_{\mathrm{I}}^{\mathrm{tip}} = K_{\mathrm{I}} - \frac{\mu b_e}{2(1-\nu)}\left(\sum_{i=1}^{N}\frac{1}{(2\pi r_i)^{1/2}}\right)3\sin\theta\cos^2\left(\frac{\theta_0}{2}\right)(1-\cos\theta_0),$$
$$K_{\mathrm{II}}^{\mathrm{tip}} = K_{\mathrm{II}} - \frac{\mu b_e}{2(1-\nu)}\left(\sum_{i=1}^{N}\frac{1}{(2\pi r_i)^{1/2}}\right)\cos^2\left(\frac{\theta_0}{2}\right)(2+3\cos^2\theta_0 - 3\cos\theta_0), \tag{14}$$

where r_i $(i = 1, 2, \ldots, N)$ is the distance from the crack tip of the ith emitted dislocation. We introduce several parameters as follows:

$$\frac{1}{(2\pi r_c)^{1/2}} = \frac{1}{N}\sum_{i=1}^{N}\frac{1}{(2\pi r_i)^{1/2}}, \tag{15}$$

$$C_0 = \frac{\mu b_e}{2(1-\nu)(2\pi r_c)^{1/2} K_{IIe}} \tag{16}$$

and

$$\begin{aligned} C_1 &= C_0 3 \sin\theta_0 \cos^2\left(\frac{\theta_0}{2}\right)(1-\cos\theta_0), \\ C_2 &= C_0 \cos^2\left(\frac{\theta_0}{2}\right)(2 + 3\cos^2\theta_0 - 3\cos\theta_0). \end{aligned} \tag{17}$$

Equation (14) can be expressed as follows:

$$\begin{aligned} K_I^{tip} &= K_I - NC_1 K_{IIe}, \\ K_{II}^{tip} &= K_{II} - NC_2 K_{IIe}. \end{aligned} \tag{18}$$

Substituting eqn. (18) into eqn. (12), one obtains

$$(K_I - NC_1 K_{IIe})^2 + (K_{II} - NC_2 K_{IIe})^2 = K_{Ic}^2. \tag{19}$$

Using eqn. (19), eqn. (12) becomes

$$N^2(C_1^2 + C_2^2) - 2N(C_1 + C_2 \tan\psi)\frac{K_I}{K_{IIe}} + \frac{1}{\cos^2\psi}\left(\frac{K_I}{K_{IIe}}\right)^2 = \rho^2 \tag{20}$$

where

$$\tan\psi = \frac{K_{II}}{K_I} \tag{21}$$

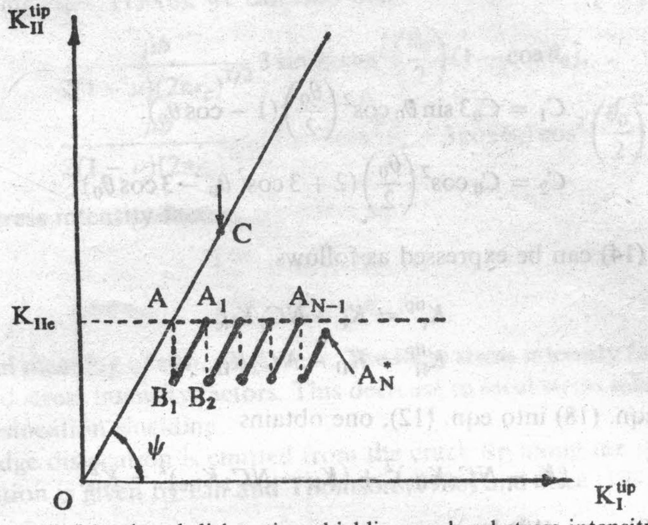
and

$$\rho = \frac{K_{Ic}}{K_{IIe}}. \tag{22}$$

If we know the number N of the emitted dislocations, one can easily obtain the critical value of K_I/K_{IIe} from eqn. (20).

As shown in fig. 2, when the applied load is increased, the stress intensity factors K_I and K_{II} will simultaneously increase along the straight line OC. At point A, the mode II stress intensity factor reaches the critical value K_{IIe}. The first dislocation is fully nucleated at the crack tip, then emitted from the crack tip along the slip plane and finally stopped at a distance r_1. The local stress intensity factors K_I^{tip} and K_{II}^{tip} are decreased from point to point B_1 along the straight line AB_1. The slope of the straight line AB_1 is determined by eqn. (8). As the applied load increases again, the local stress intensity factors K_I^{tip} and K_{II}^{tip} increase along a straight line B_1A_1, which is parallel to the straight line OC. At point A_1, the local mode II stress intensity factor reaches the critical value K_{IIe} again; the second dislocation is fully nucleated at the crack tip, then emitted alone the second slip plane and finally stopped at distance r_2. The local stress intensity factors decrease from point A_1 to point B_1 due to shielding by the second dislocation. As the sequence is repeated, at the critical point A_N^*, the radius OA_N^* is equal to the radius OC after the Nth dislocation is emitted. The fracture criterion (12) is met and the cleavage branching occurs.

Fig. 2



Effects of applied load and dislocation shielding on local stress intensity factors.

Equation (20) can be rewritten as follows:

$$N^2(C_1^2 + C_2^2) - 2N(C_1 \cot\psi + C_2)\frac{K_{II}}{K_{IIe}} + \frac{1}{\sin^2\psi}\left(\frac{K_{II}}{K_{IIe}}\right)^2 = \rho^2. \quad (23)$$

Let us look at straight line $B_N A_N$ which intersects the horizontal line A_1A_2 at point A_N. Obviously the radius OA_N^* is less than the radius OA_N and greater than the radius OA_{N-1}. Let

$$\rho_N^2 = N^2(C_1^2 + C_2^2) - 2N(C_1 \cot\psi + C_2)(1 + NC_2) + \frac{(1 + NC_2)^2}{\sin^2\psi}. \quad (24)$$

Hence we have

$$\rho_{N-1} \leq \rho \leq \rho_N. \quad (25)$$

For a given ρ and ψ, one can easily obtain the number N from eqns. (24) and (25). Substituting N into eqn. (20), one can obtain the critical value of K_I/K_{IIe}. It is given by

$$\frac{K_{II}}{K_{IIe}} = \tan\psi\left(\frac{K_I}{K_{IIe}}\right). \quad (26)$$

The local stress intensity factors are given by eqn. (9). The local load phase angle ψ is determined by following equation:

$$\tan\psi = \frac{\sin\theta_0 + \tan\psi_0(3\cos\theta_0 - 1)}{1 + \cos\theta_0 - \tanh\psi_0 3\sin\theta_0}, \quad (27)$$

where ψ_0 is the load phase angle; $\tan\psi_0 = k_{II}/k_I$. The parameter η is given by

$$\eta = \frac{K_{II}^{tip}}{K_{IIe}}. \tag{28}$$

The apparent critical stress intensity factor K_c is defined as follows:

$$K_c^2 = K_I^2 + K_{II}^2, \tag{29}$$

where K_I and K_{II} are the stress intensity factors at the start of cleavage branching.

The apparent critical release rate G_c is given by

$$G_c = \frac{1-\nu}{2\mu} K_c^2. \tag{30}$$

§ 3. Results

Calculation was carried out with the parameters $\nu = 0{\cdot}3$ and $C_0 = 0{\cdot}01368$.

The average distance r_c in eqn. (15) was taken as $3000b$. This means that r_c is approximately 1 μm.

3.1. $\theta_0 = 0$

In this case, the slip plane is coincident with the crack plane. We have

$$\tan\psi = \tan\psi_0, \tag{31}$$

$$\tan\psi_d = \frac{K_{IId}}{K_{Id}}, \qquad \psi_d = \frac{\pi}{2}. \tag{32}$$

Equation (32) means that K_{Id}, the value contributed by the emitted dislocations, vanishes. Hence the straight lines AB_1 and A_1B_2 are perpendicular to the K_I^{tip} axis. The non-dimensional apparent critical energy release rate G_c/G_{Ic} against load phase angle ψ_0 is shown in fig. 3 for different values of ρ. Figure 4 shows the apparent critical stress intensity factor K_c/K_{Ic} as a function of phase angle ψ_0. The total number N of emitted dislocations at the onset of cleavage is depicted in fig. 5.

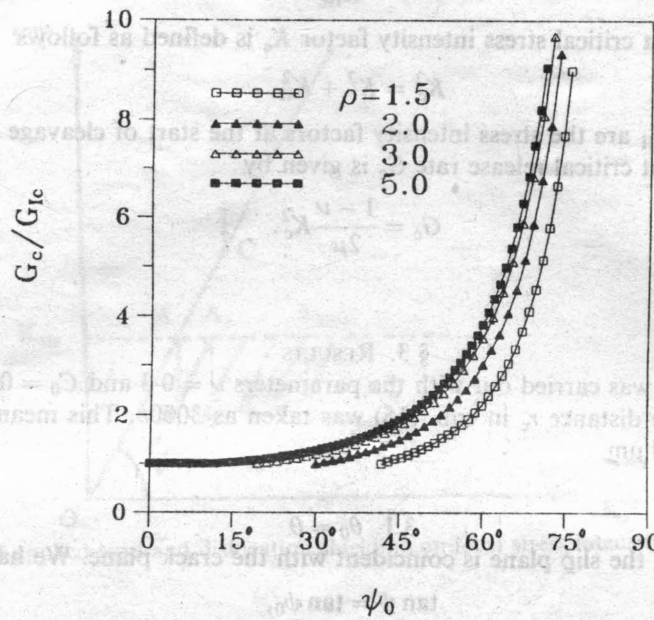
It is clear that, for a given value of ρ, the apparent critical stress intensity factor K_c and the apparent critical energy release rate G_c are significantly increased as the load phase angle ψ_0 increases. The reason for this is that the total number N of emitted dislocations is increased considerably as the load phase angle increases. In other words, when the significant component K_{II} is applied, numerous dislocations will be emitted from the crack tip. The local stress intensity factor K_{II}^{tip} still has a low value. Hence the cleavage fracture occurs when and only when the local stress intensity factor K_I^{tip} reaches a certain value. The major contribution to the cleavage fracture is due to the local stress intensity factor K_I^{tip}.

From eqns. (12) and (13), one can find that

$$\frac{K_{II}^{tip}}{K_I^{tip}} = \frac{\eta}{(\rho^2 - \eta^2)^{1/2}}. \tag{33}$$

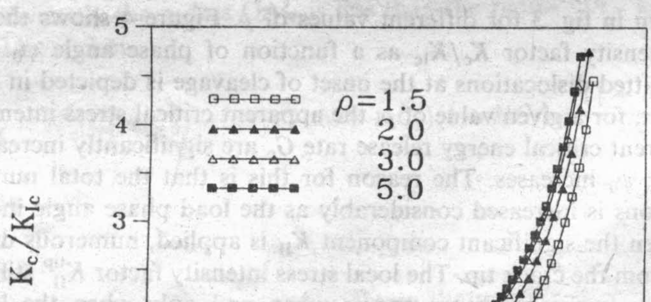
In our calculation, the values of the parameter η are in the interval 0·97–1·0. Hence one can find that the value of K_{II}^{tip}/K_I^{tip} is 0·333 for $\rho = 2$. When $\rho = 1{\cdot}5$, the local shear mode component K_{II} contribution to the local critical energy release rate G_c is approximately equal to 0·45, which corresponds to the case of NaCl listed in the review by Kelly (1966). When $\rho < 1{\cdot}5$, the cleavage criterion (12) may not be suitable

Fig. 3



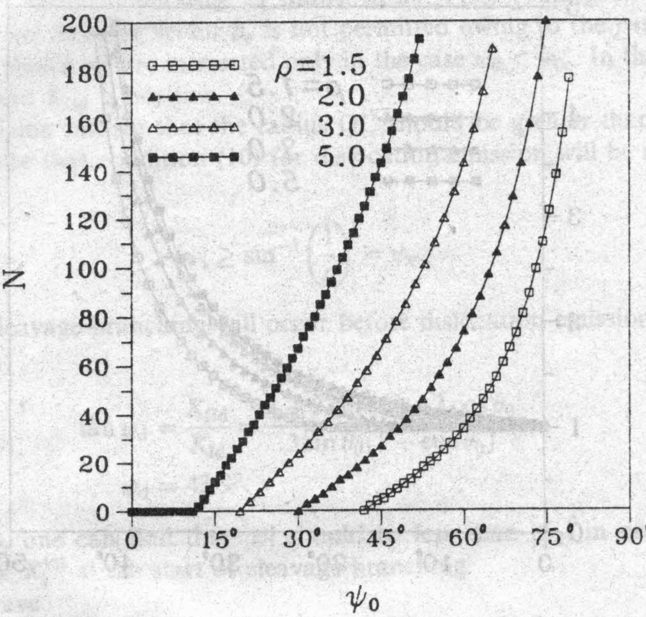
Apparent critical energy release rate G_c/G_{Ic} against load phase angle ψ_0 for $\theta_0 = 0$.

Fig. 4



Apparent critical stress intensity factor K_c/K_{Ic} as the function of phase angle ψ_0 for $\theta_0 = 0$.

Fig. 5



The total number N of emitted dislocations at the onset of cleavage for $\theta_0 = 0$.

and the criterion $K_I = K_{Ic}$ proposed by Lin and Thomson (1986) could be more reasonable.

3.2. $\theta_0 = 30°$

Without loss of generality, we consider only the cases $0 < \psi_0 < \pi/2$. Hence the applied stress intensity factors k_I and k_{II} are always positive. As a consequence, K_{II} is always positive and K_{Id} and K_{IId} are always negative in the case of $\theta_0 = 30°$.

Let

$$\tan\psi_{0c} = \frac{1+\cos\theta_0}{3\sin\theta_0}, \qquad \psi_{0c} = 51{\cdot}20°. \tag{34}$$

According to eqns. (7) and (20), when $\psi \to \psi_{0c}$, $\psi \to \pi/2$.

From eqn. (7), it is clear that, if the load phase angle ψ_0 is larger than ψ_{0c}, the stress intensity factor K_I becomes negative and cleavage branching is not possible. Hence we are interested only in the case $0 < \psi_0 < \psi_{0c}$.

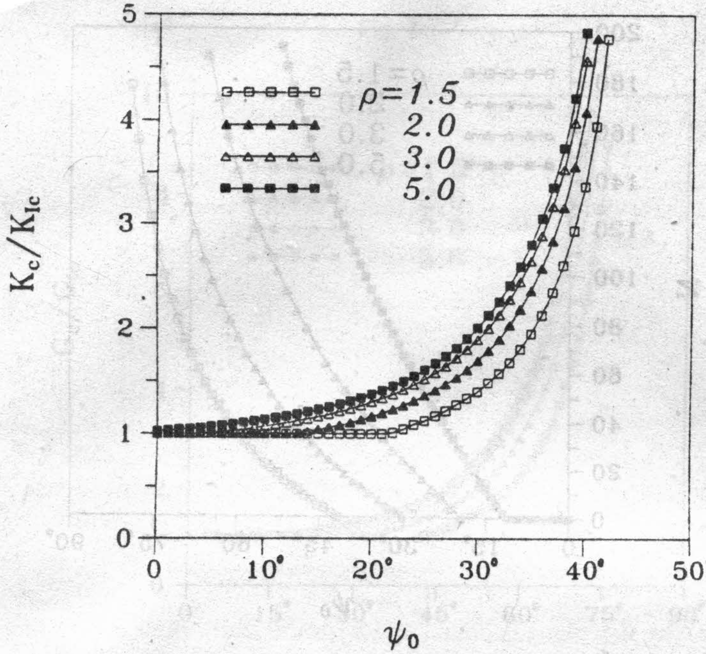
The apparent critical stress intensity factor K_c against the phase angle ψ_0 is plotted in fig. 6. The total number N of emitted dislocations at the initiation of crack branching is shown in fig. 7.

3.3. $\theta_0 = -60°$

For this case, K_I is always positive. Let

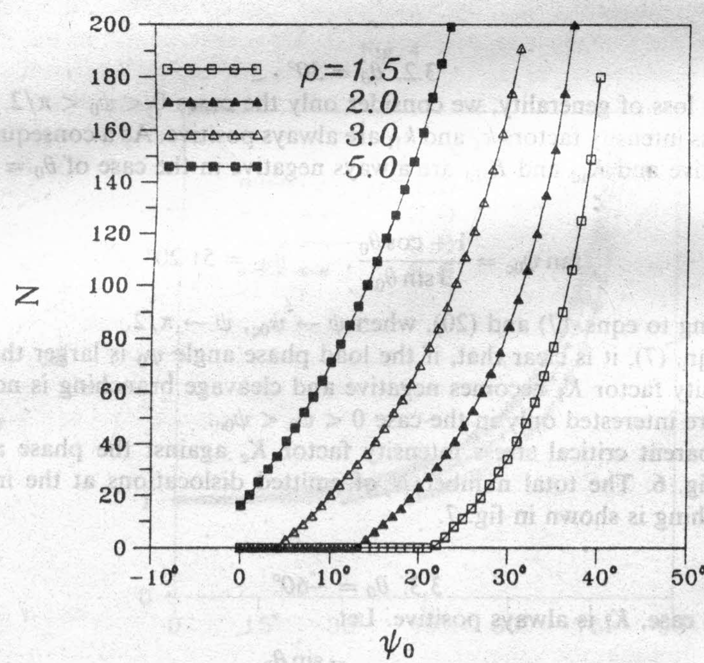
$$\tan\psi_{0c} = \frac{-\sin\theta_0}{3\cos\theta_0 - 1}. \tag{35}$$

Fig. 6



Apparent critical stress intensity factor K_c/K_{Ic} against ψ_0 in the case of $\theta_0 = 30°$.

Fig. 7



The total number N of emitted dislocations at onset of cleavage for $\theta_0 = 30°$.

The ψ_{0c} is equal to 60°. When the phase angle $\psi_0 < \psi_{0c}$, K_{II} is negative; hence the Burgers vector b_e must be opposite as shown in fig. 1 (*b*). From geometrical considerations, positive Burgers vector b_e is not permitted owing to the penetration of the crack faces. Hence we are interested only in the case $\psi_0 < \psi_{0c}$. In this situation, K_{Id} is negative and K_{IId} is positive.

From fig. 8, one can see that the radius OC should be greater than the OA in order to guarantee that condition (10) for dislocation emission will be met first, so that

$$|\psi| \geq \sin^{-1}\left(\frac{1}{\rho}\right) = \psi_e.$$

Otherwise cleavage branching will occur before dislocation emission.

Let

$$\tan\psi_d = \frac{K_{IId}}{K_{Id}} = \frac{2 + 3\sin^2\theta_0 - 3\cos\theta_0}{3\sin\theta_0(1-\cos\theta_0)}, \tag{36}$$

$$\psi_d = 43{\cdot}9°.$$

From fig. 8, one can find that $|\psi|$ should be less than $|\psi_d|$ in order to get a positive value of K_I^{tip} at the start of cleavage branching.

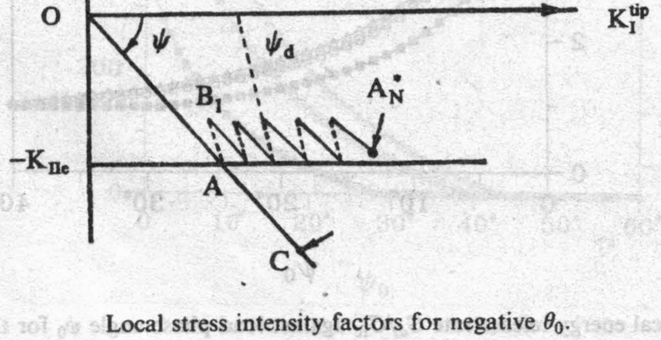
Hence we have

$$|\psi_d| \geq |\psi| \geq \psi_e.$$

Figure 9 shows the relation of ψ against ψ_0. The values of ψ_e are 11·5, 7·18 and 5·74° for the cases when $\rho = 5{\cdot}0, 8{\cdot}0$ and 100 respectively.

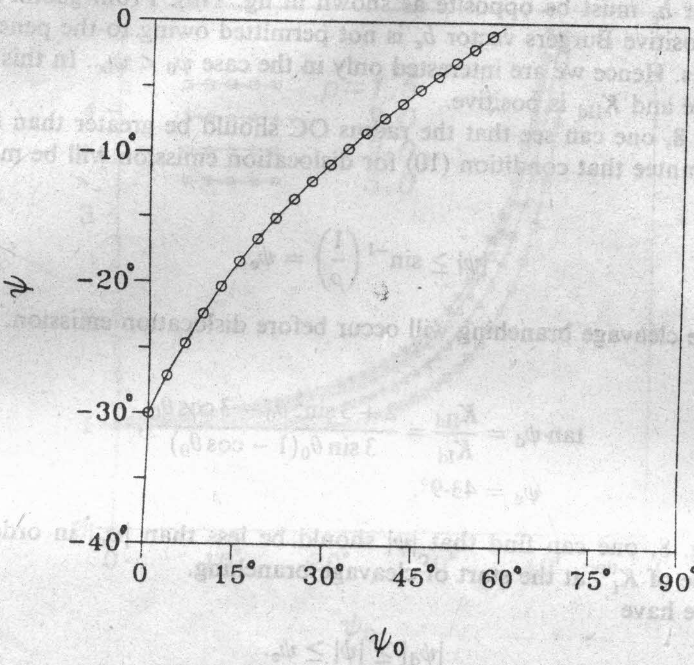
The non-dimensional apparent critical energy release rate G_c/G_{Ic} against load phase angle ψ_0 is shown in fig. 10 for different values of ρ. Figure 11 shows the apparent critical stress intensity factor K_c/K_{Ic} as a function of phase angle ψ_0. The

Fig. 8



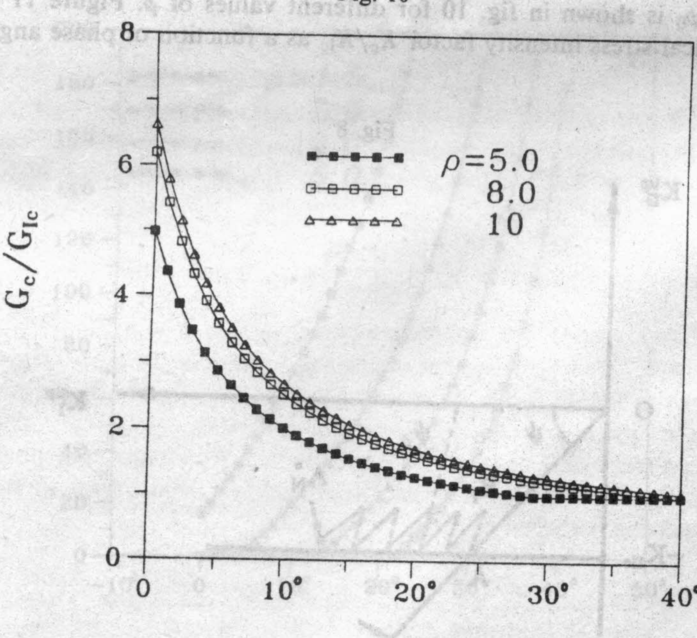
Local stress intensity factors for negative θ_0.

Fig. 9



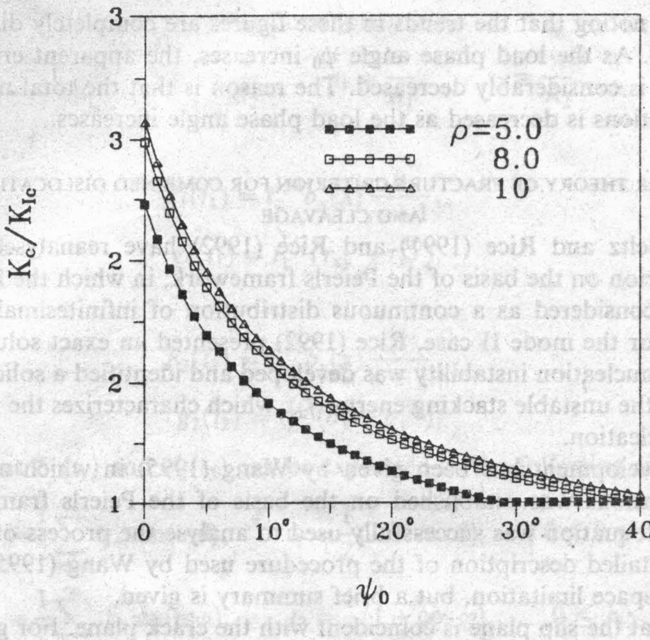
The relation of ψ against ψ_0 for $\theta_0 = -60°$.

Fig. 10



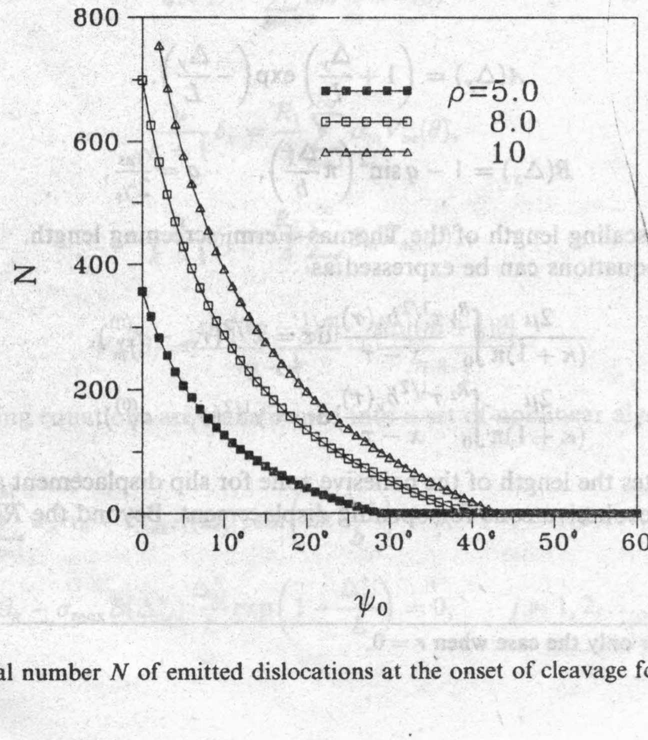
Apparent critical energy release rate G_c/G_{Ic} against load phase angle ψ_0 for the $\theta_0 = -60°$.

Fig. 11



Apparent critical stress intensity factor K_c/K_{Ic} as the function of phase angle ψ_0 for $\theta_0 = -60°$.

Fig. 12



The total number N of emitted dislocations at the onset of cleavage for $\theta_0 = -60°$.

total number N of emitted dislocations at the start of the cleavage is depicted in fig. 12.

It is worth noting that the trends in these figures are completely different from that for $\theta_0 = 0$. As the load phase angle ψ_0 increases, the apparent critical energy release rate G_c is considerably decreased. The reason is that the total number N of emitted dislocations is decreased as the load phase angle increases.

§ 4. Nonlinear theory of fracture criterion for combined dislocation emission and cleavage

Recently Beltz and Rice (1991) and Rice (1992) have reanalysed the Rice–Thomson criterion on the basis of the Peierls framework, in which the fully emitted dislocation is considered as a continuous distribution of infinitesimal continuum dislocations. For the mode II case, Rice (1992) presented an exact solution for the loading as the nucleation instability was developed and identified a solid-state parameter, namely the unstable stacking energy γ_{us}, which characterizes the resistance to dislocation nucleation.

Further development has been given by Wang (1995) in which a new set of governing equations was established on the basis of the Peierls framework. The new governing equation was successfully used to analyse the process of dislocation emission. A detailed description of the procedure used by Wang (1995) is omitted here owing to space limitation, but a brief summary is given.

Suppose that the slip plane is coincident with the crack plane. For general loadings, Beltz and Rice (1991) proposed a generalized constitutive relation†:

$$\begin{aligned} \tau &= \tau_{\max} A(\Delta_y) \sin\left(2\pi \frac{\Delta_x}{b}\right), \\ \sigma &= \sigma_{\max} B(\Delta_x) \frac{\Delta_y}{L} \exp\left(1 - \frac{\Delta_y}{L}\right), \end{aligned} \tag{37}$$

where

$$A(\Delta_y) = \left(1 + \frac{\Delta_y}{L}\right) \exp\left(-\frac{\Delta_y}{L}\right), \tag{38}$$

$$B(\Delta_x) = 1 - q \sin^2\left(\pi \frac{\Delta_x}{b}\right), \qquad q = \frac{\gamma_{us}}{2\gamma_s}, \tag{39}$$

where L is the scaling length of the Thomas–Fermi screening length.

The basic equations can be expressed as

$$\frac{2\mu}{(\kappa+1)\pi} \int_0^{R_1} \frac{\tau^{1/2} b_x(\tau)}{x - \tau} \, d\tau = x^{1/2}(\tau_{xy} - \tau_{xy}^{(0)}), \tag{40 a}$$

$$\frac{2\mu}{(\kappa+1)\pi} \int_0^{R_2} \frac{\tau^{1/2} b_y(\tau)}{x - \tau} \, d\tau = x^{1/2}(\sigma_y - \sigma_y^{(0)}), \tag{40 b}$$

where R_1 denotes the length of the cohesive zone for slip displacement and R_2 is the length of the decohesive zone for opening displacement. Beyond the R_1, there is no

†We consider only the case when $r = 0$.

discontinuity for slip displacement, but beyond R_2, discontinuity of the opening displacement vanishes.

We introduce following non-dimensional quantities:

$$t_1 = \frac{x}{R_1}, \qquad t_2 = \frac{x}{R_2}, \qquad s_1 = \frac{\tau}{R_1}, \qquad s_2 = \frac{\tau}{R_2}.$$

Let

$$F_1(t_1) = t_1^{1/2} b_x(x) \frac{\mu}{\kappa+1}, \qquad g_1(t_1) = t_1^{1/2}(\tau_{xy} - \tau_{xy}^{(0)}), \tag{41}$$

and

$$F_2(t_2) = t_2^{1/2} b_y(x) \frac{\mu}{\kappa+1}, \qquad g_2(t_2) = t_2^{1/2}(\sigma_y - \sigma_y^{(0)}), \tag{42}$$

The functions $F_1(t_1)$ and $F_2(t_2)$ can be expressed as the following sine series:

$$F_1(t_1) = \frac{1}{2}\sum_{m=1}^{\infty} \alpha_m \sin(m\theta), \qquad t_1 = \frac{1}{2}(1+\cos\theta), \qquad 0 \le \theta \le \pi,$$
$$F_2(t_2) = \frac{1}{2}\sum_{m=1}^{\infty} \beta_m \sin(m\varphi), \qquad t_2 = \frac{1}{2}(1+\cos\varphi), \qquad 0 \le \varphi \le \pi, \tag{43}$$

One obtains

$$g_1(t_1) = \sum_{m=1}^{\infty} \alpha_m \cos(m\theta),$$
$$g_2(t_2) = \sum_{m=1}^{\infty} \beta_m \cos(m\varphi), \tag{44}$$

and

$$\frac{\mu}{k+1}\delta_x = \frac{R_1}{4}\sum_{m=1}^{\infty} \alpha_m V_m(\theta), \tag{45}$$

$$\frac{\mu}{k+1}\delta_y = \frac{R_2}{4}\sum_{m=1}^{\infty} \beta_m V_m(\varphi), \tag{46}$$

$$V_m(\theta) = \frac{\sin[(m-\frac{1}{2})\theta]}{m-\frac{1}{2}} - \frac{\sin[(m+\frac{1}{2})\theta]}{m+\frac{1}{2}}. \tag{47}$$

The governing equations are transformed into a set of nonlinear algebraic equations:

$$\sum_{k=1}^{M} a_{ik}\alpha_k - \tau_{\max} A(\Delta_{yi}) \sin\left(2\pi \frac{\Delta_{xi}}{b}\right) = 0, \qquad i = 1, 2, \ldots, M,$$
$$\sum_{k=1}^{N} b_{jk}\beta_k - \sigma_{\max} B(\Delta_{xj}^*) \frac{\Delta_{yj}^*}{L} \exp\left(1 - \frac{\Delta_{yj}^*}{L}\right) = 0, \qquad j = 1, 2, \ldots, N, \tag{48}$$

where

$$a_{ik} = \frac{\cos(k\theta_i) - \cos(k\pi)}{t_{1i}^{1/2}},$$
$$b_{jk} = \frac{\cos(k\varphi_j) - \cos(k\pi)}{t_{2j}^{1/2}}, \tag{49}$$
$$\theta_i = \frac{(i-1)\pi}{M}, \quad \varphi_j = \frac{(j-1)\pi}{N}.$$

Δ_{xi} and Δ_{yi} are the shearing and opening displacements at $x_i = R_i(1+\cos\theta_i)/2$; Δ_{xj}^* and Δ_{yj}^* are the shearing and opening displacements at $x_j = R_2(1+\cos\varphi_j)/2$.

Equation (48) is solved by the Newton–Raphson method. The iterating convergence is guaranteed after five to ten iterations. Most calculations in this paper were carried out with five digits of accuracy for stress fields in the cohesive zone.

The emitted dislocation are located in the interval (R_a, R_b). Suppose that all emitted dislocations are far away from the crack tip. We have $R_b - R_a \ll R_a$. In order to simulate the effects of the N emitted dislocations on the stress fields in the cohesive zone immediately ahead of the crack tip, a single edge dislocation with Burgers vector Nb is used in the present calculation instead of the N discrete emitted dislocations.

The stress field ahead of the crack tip, produced by a single emitted dislocation with Burgers vector Nb is evaluated by

$$\sigma_y = 0,$$
$$\tau_{xy} = \frac{N\mu b}{2(1-\nu)\pi}\left(\frac{x_c}{x}\right)^{1/2}\frac{1}{x-x_c}. \tag{50}$$

The stress field induced by the remote applied load is given by

$$\sigma_y = \frac{K_I}{(2\pi x)^{1/2}},$$
$$\tau_{xy} = \frac{K_{II}}{(2\pi x)^{1/2}}. \tag{51}$$

Hence the $\sigma_y^{(0)}$ and $\tau_{xy}^{(0)}$ in eqn. (4) are

$$\sigma_y^{(0)} = \frac{K_I}{(2\pi x)^{1/2}},$$
$$\tau_{xy}^{(0)} = \frac{K_{II}}{(2\pi x)^{1/2}} + \frac{N\mu b}{2(1-\nu)\pi}\left(\frac{x_c}{x}\right)^{1/2}\frac{1}{x-x_c}. \tag{52}$$

In the cohesive zone, the stress field should be balanced with the cohesive stress field of eqn. (37).

The crack tip normal stress σ_y^{tip}, less than the $0{\cdot}0001\sigma_0$, is taken as the fracture criterion for the combined cleavage and dislocation emission process in the calculation. A typical calculation was carried out with materials parameters $h/b = 1$, $L/b = 0{\cdot}4$, $\tau_0/\mu = 0{\cdot}01$, $\sigma_0/E = 0{\cdot}016$, $\nu = 0{\cdot}3$, $\tau_{max}/\mu = 0{\cdot}159$, $\sigma_{max}/E = 0{\cdot}153$, $M = 180$ and $N = 180$.

According to Rice (1992), we have

$$\gamma_{us} = \frac{\mu b^2}{2\pi^2 h}, \qquad 2\gamma_s = \frac{EL^2}{h}. \tag{53}$$

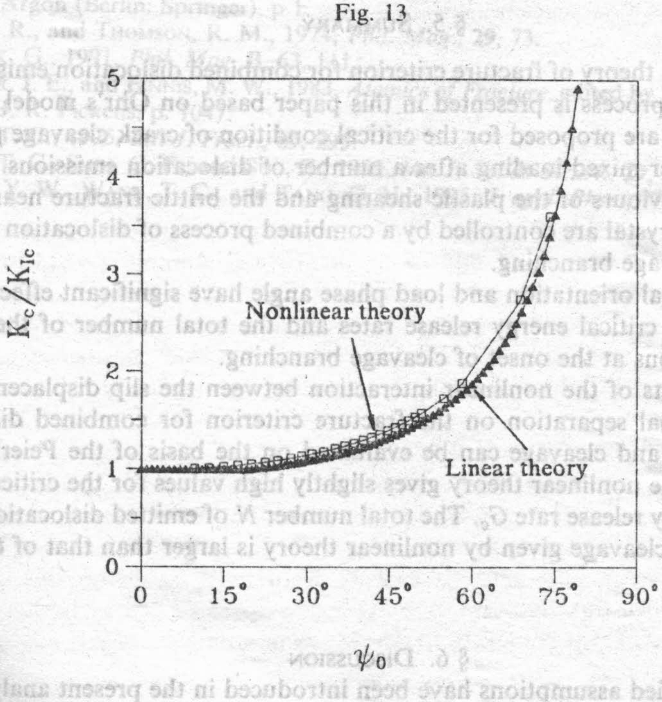
Hence the parameter ρ is given by

$$\rho = \frac{K_{Ic}}{K_{IIe}} = \left(\frac{2\gamma_s}{\gamma_{us}}\right)^{1/2} = 2\pi\frac{L}{b}(1+\nu)^{1/2} = 2{\cdot}866. \tag{54}$$

The results of calculation are depicted in fig. 13 and fig. 14 for a combined loading. For comparison, the results of the linear theory given in § 2 and 3 are also plotted in these figures. One can see that the results of the nonlinear theory agree well with the results of linear theory. The nonlinear theory gives slightly higher values of the critical apparent energy release rate G_c and the critical apparent stress intensity factor K_c for the same load phase angle. On the other hand, the total number N of the emitted dislocations at the incipient cleavage given by nonlinear theory is larger than that of the linear theory.

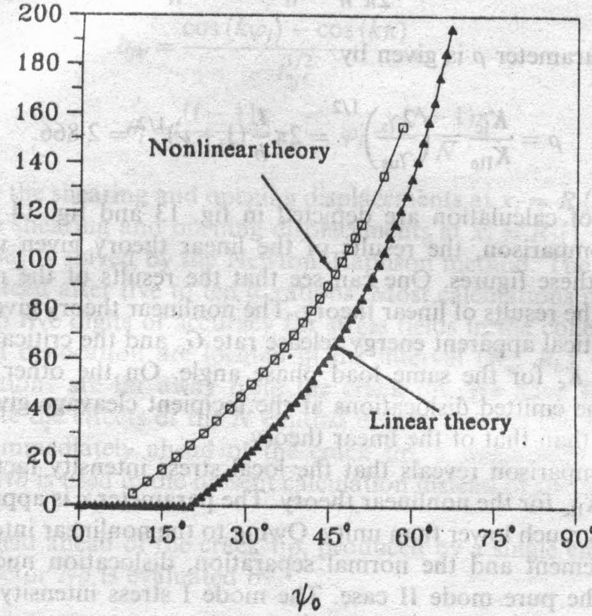
Detailed comparison reveals that the local stress intensity factor K_{II}^{tip} is much lower than the K_{IIe} for the nonlinear theory. The parameter η is approximately equal to 0·33, which is much lower than unity. Owing to the nonlinear interaction between the slip displacement and the normal separation, dislocation nucleation is much easier than in the pure mode II case. The mode I stress intensity factor not only makes a significant contribution to the cleavage but also has an important influence on the dislocation nucleation.

Fig. 13



Apparent critical stress intensity factor K_c/K_{Ic} as the function of phase angle ψ_0 for $\theta_0 = 0$ and $\rho = 2{\cdot}866$.

Fig. 14



The total number N of emitted dislocations at the onset of cleavage for $\theta_0 = 0$ and $\rho = 2{\cdot}866$.

§ 5. Summary

(1) A general theory of fracture criterion for combined dislocation emission and cleavage process is presented in this paper based on Ohr's model. Explicit formulae are proposed for the critical condition of crack cleavage propagation under mixed loading after a number of dislocation emissions.

(2) The behaviours of the plastic shearing and the brittle fracture near a crack tip in a crystal are controlled by a combined process of dislocation emission and cleavage branching.

(3) The crystal orientation and load phase angle have significant effects on the apparent critical energy release rates and the total number of the emitted dislocations at the onset of cleavage branching.

(4) The effects of the nonlinear interaction between the slip displacement and the normal separation on the fracture criterion for combined dislocation emission and cleavage can be evaluated on the basis of the Peierls framework. The nonlinear theory gives slightly high values for the critical apparent energy release rate G_c. The total number N of emitted dislocations at the onset of cleavage given by nonlinear theory is larger than that of the linear theory.

§ 6. Discussion

Many simplified assumptions have been introduced in the present analyses. The dislocation emission actually takes place in three dimensions by a dislocation loop. The present analyses are concerned only with a two-dimensional description. Elastic anisotropy, which is a main feature for a crystal, is neglected in our analyses. The elastic anisotropy has not only a remarkable effect on the stress fields, but also an important effect on the crack growth resistance. The molecular dynamic simulation by Zhang, Wang and Tang (1995) has shown that the elastic anisotropy will reduce or increase the critical stress intensity factor K_{IIe} for dislocation emission from a stressed crack tip by about 30%. The analytical calculation for crack extension and kinking in laminates and bicrystals by Wang, Shih and Suo (1992) has also indicated that the energy release rates will increase or decrease by about 40% owing to elastic anisotropy for the same crack geometries and same loading conditions. In order to justify quantitatively the effects of elastic anisotropy on the fracture criteria, further investigation is needed.

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