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Hopf bifurcation in wakes behind a rotating and translating circular cylinder

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(Received 18 September 1995; accepted 13 March 1996)

A low-dimensional Galerkin method, initiated by Noack and Eckelmann [Physica D **56**, 151 (1992)], for the prediction of the flow field around a stationary two-dimensional circular cylinder in a uniform stream at low Reynolds number is generalized to the case of a rotating and translating cylinder. The Hopf bifurcation describing the transition from steady to time-periodic solution is investigated. A curve indicating the transitional boundary is given in the two-dimensional parameter plane of Reynolds number Re and rotating parameter α. Our results show that rotation may delay the onset of vortex street and decrease the vortex-shedding frequency. © *1996 American Institute of Physics.* [S1070-6631(96)00107-9]

The problem of the flow around a uniformly rotating and translating circular cylinder has been investigated by several researchers due to its engineering importance and academic interest. Badr *et al.*[1] numerically simulated the steady and unsteady flow past a rotating circular cylinder at low Reynolds numbers Re with rotating parameter α, in which Re is based on the cylinder radius R and the incoming velocity U_∞ and $\alpha = R\omega/U_\infty$, where ω represents the angular velocity of the rotating cylinder. Ingham,[2] Ingham and Tang,[3] D'Alessio and Dennis[4] considered numerical solutions of the steady-state N–S equation at subcritical Re. The investigations of the unsteady flow for supercritical Reynolds numbers are relatively fewer than the case of the steady-state flow. Badr *et al.*[5] numerically studied the time-dependent flow past an impulsively rotating and translating circular cylinder started from rest for Re>200, while Coutanceau and Menard[6] gave corresponding experimental results. Chang and Chen[7] investigated the same problem at some higher Re for $0 \leqslant \alpha < 2$, and suggested there are three modes of vortex shedding existing in wakes depending on Re and α.

Meanwhile, the research on the bifurcation structure in an open-flow at low Reynolds numbers is of great interest. Provansal *et al.*,[8] Sreenivasan *et al.*,[9] and Schumm *et al.*[10] experimentally studied the onset of 2-D vortex shedding in the wakes behind a stationary circular cylinder and showed that the transition from the steady to the periodic flow is characterized by a Hopf bifurcation and can be described by the Stuart–Landau equation. Jackson,[11] Zebib[12] and Noack *et al.*[13,14] numerically investigated the onset of vortex shedding in flow past a stationary circular cylinder by applying the linear stability analysis to an autonomous dynamical system.

Following Noack's work,[13] a low-dimensional Galerkin method (LDGM) is generalized to the case of a 2-D uniformly rotating and translating circular cylinder. Although the LDGM cannot compete with grid-based computational techniques for high accurate simulations of the velocity fields or the resolution of far-wake properties, it is confirmed to be an ideal tool for investigations on global stability and chaos-theoretical analysis.[13,14] In the present Galerkin method, the streamfunction is approximated by a finite ex-

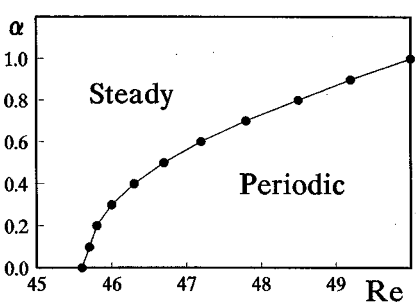

FIG. 1. Transitional curve of Hopf bifurcation in wakes.

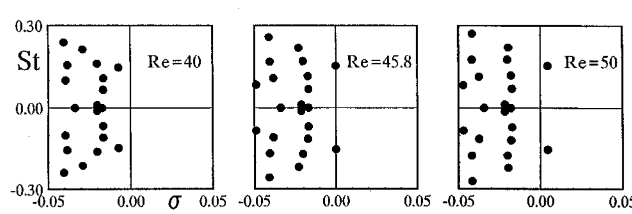

FIG. 2. Eigenvalue spectrum for $\alpha=0.2$. Before the occurrence of the Hopf bifurcation (Re=40) all eigenvalues $\lambda = \pi(\sigma + i\mathrm{St})$ have negative amplification rates σ. After the Hopf bifurcation (Re=50), a complex-conjugate pair of eigenvalues from the spectrum has crossed the imaginary axis.

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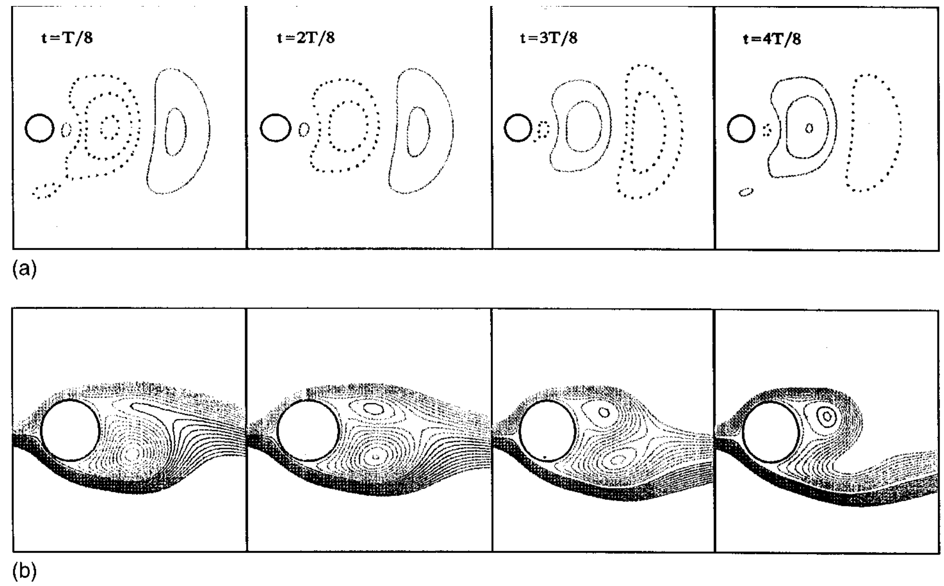

FIG. 3. The critical perturbations for half period (a) the streamlines for the real part of the critical perturbations $a_{ij}^{(p)}$ at Re=46.8 for α=0.5 (the negative streamlines are shown dashed); (b) the corresponding streamlines when the perturbations are superimposed on the steady flow.

pansion (1), which is written as a sum of the basic mode and perturbation modes

$$\psi=\psi^0(r,\theta)+\sum_{i,j} a_{i,j}(t)R_i(r)\Theta_j(\theta)$$
$$i=0,\ldots,K;j=-L,\ldots,L, \quad (1)$$

where the basic mode is

$$\psi^0(r,\theta)=\left(r-\frac{1}{r}\right)(1-e^{-[(r-1)/\delta_{bm}]})\sin\theta+\alpha\ln r \quad (2)$$

which satisfies the no-slip condition on the body surface and approaches to a potential solution at the far field, with $\delta_{bm} = 4/\sqrt{\mathrm{Re}}$.[13] The radical modes $R_i(r)$ and azimuthal modes $\Theta_j(\theta)$ are chosen so that the perturbation modes satisfy all the homogeneous boundary conditions. For a detailed descriptions of the LDGM, the reader is referred to Ref. 13. The evolution equations for the Fourier coefficients a_{ij} in (1) are obtained, which are expressed as a set of nonlinear ordinary differential equations (ODEs)

$$\frac{d}{dt}a_{ij}=c_{ij}+\sum_{kl} l_{ij,kl}a_{kl}+\sum_{klmn} q_{ij,klmn}a_{kl}a_{mn}. \quad (3)$$

Using the fourth-order Runge–Kutta algorithm for these ODEs with $K=6$ and $L=4$, flows around rotating and translating circular cylinder were computed. In the meantime, the Newton–Raphson iteration was utilized to (3) in order to obtain the equilibrium solutions of the ODEs (3) for both subcritical and supercritical Reynolds numbers.

TABLE I. The critical Reynolds numbers vs α with larger numbers of K and L.

α	$K=6$, $L=4$	$K=8$, $L=6$	$K=12$, $L=10$
0.0	45.6	55.4	45.8
0.5	46.7	55.5	45.9
0.8	48.5	56.2	46.3

As we know, the equilibrium of the ODEs correspond to steady solutions (stable or unstable). The linearized ODEs in the vicinity of the equilibrium $a_{ij}^{(s)}$ are given as the following:

$$\frac{d\xi_{ij}}{dt}=\sum_{\substack{k=0,\ldots,K\\ l=-L,\ldots,L}}\left[l_{ij,kl}+\sum_{\substack{m=0,\ldots,K\\ n=-L,\ldots,L}}(q_{ij,klmn}+q_{ij,mnkl})a_{mn}^{(s)}\right]\xi_{ij} \quad (4)$$

in which $\xi_{ij}=a_{ij}-a_{ij}^{(s)}$ is the perturbation of the steady solutions. The stability characteristics of the ODEs are determined by the eigenvalues of the Jacobian matrix in the neighborhood of $a_{ij}^{(s)}$. The eigenvalues λ_{ij} and eigenvectors $a_{ij}^{(p)}$ of this matrix are computed by the QR method. It is well known that the occurrence of a pair of eigenvalues with positive real part implies the global instability of steady solutions and the onset of periodic solutions. The α-dependent critical Reynolds numbers $\mathrm{Re}_{\mathrm{cr}}(\alpha)$ are defined as those Re with an isolated pair of eigenvalues crossing the imaginary axis, and the eigenvectors $a_{ij}^{(p)}$ associated with $\mathrm{Re}_{\mathrm{cr}}(\alpha)$ represent the critical perturbations.

The validity of the code for the present method was checked first for the case of the flow around a stationary

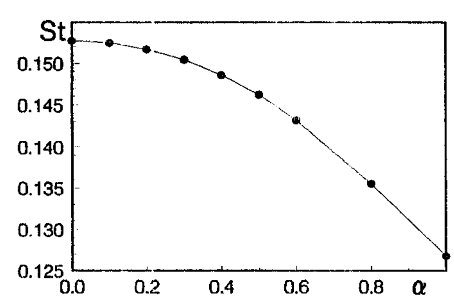

FIG. 4. Strouhal numbers St vs α at Re=60.

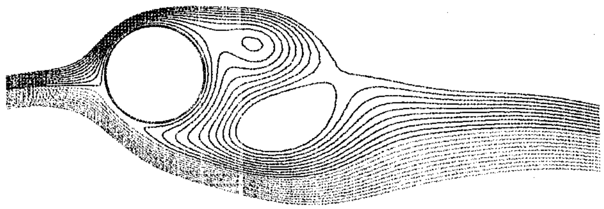

FIG. 5. Steady streamlines for $\alpha=0.8$ at Re=48.

circular cylinder, namely $\alpha=0$. The flow pattern shows that a pair of vortices emerges at the rear of the cylinder for Re$\approx$5. Furthermore, linear stability analysis of the steady solution of the ODEs predicts that the first Hopf bifurcation occurs at $\mathrm{Re}_{cr}(0)=45.6$. These results are in good agreement with previous authors' work.[13,11,12]

Now, the attention is focused on the bifurcation characteristics of the wakes behind a rotating and translating circular cylinder with two parameters Re and α. The stability analyses of wakes indicate that steady flows are stable at low Re $[\mathrm{Re}<\mathrm{Re}_{cr}(0)]$ for all α and steady flows become unstable when Re exceeds $\mathrm{Re}_{cr}(\alpha)$ for any fixed α. The transitional curve describing the Hopf bifurcation from steady to periodic solutions in wakes can be obtained by computing the critical Reynolds numbers for different α (see Fig. 1). It can be seen that $\mathrm{Re}_{cr}(\alpha)$ increases with α, from $\mathrm{Re}_{cr}(0)=45.6$ to $\mathrm{Re}_{cr}(1.0)=50.0$. This implies that the vortex shedding from the cylinder surface may be delayed by the rotating control. A typical example of computed eigenvalue spectra at Re $=40$, 45.8, and 50 for $\alpha=0.2$ is shown in Fig. 2. The streamfunctions of the critical perturbations $a_{ij}^{(p)}$ are displayed in Fig. 3(a) at Re slightly greater than $\mathrm{Re}_{cr}(\alpha)$. When the critical velocity fields are superimposed on steady flows, the time periodic vortex streets are obtained and shown in Fig. 3(b). Larger numbers of Galerkin modes were used to test the robustness of Hopf bifurcations of ODEs (3) for some single value of α, the results with $K=8$ and $L=6$, and with $K=12$ and $L=10$ are shown in Table I.

Another effect of rotation is a slight decrease of vortex shedding frequency, contradicting Badr's assumption that the Strouhal number is independent of the rotation parameter α.[1] Figure 4 represents the variation of Strouhal number of the periodic flow for $0<\alpha<1$ at Re=60. Figure 5 illustrates the steady streamlines for Re=48 and $\alpha=0.8$ (just above the transitional curve in Fig. 1). Figure 6 gives streamlines of the

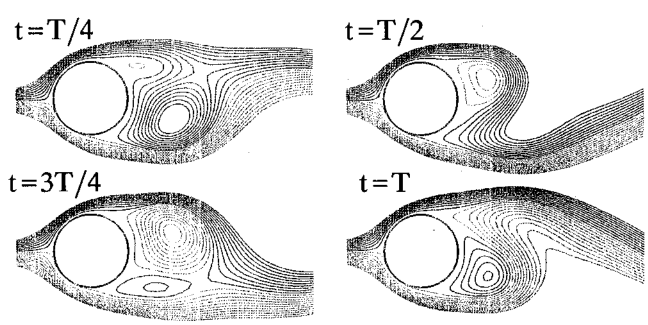

FIG. 6. Instantaneous streamlines within one period T at Re=60 for $\alpha=0.2$.

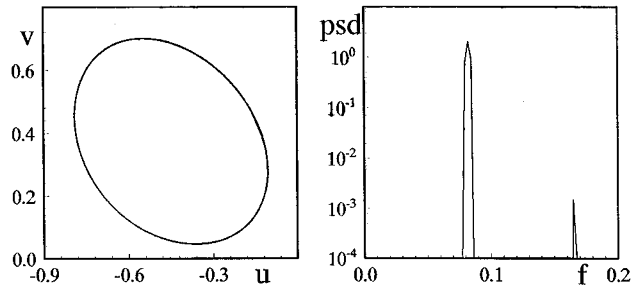

FIG. 7. The velocity portrait (left) and power spectral density vs frequency (right) at Re=60 for $\alpha=0.2$.

time-periodic flow for Re=60 and $\alpha=0.2$, the corresponding phase portrait and power spectral density are given in Fig. 7.

Unfortunately, we have not found any available results, experimental or numerical, for the Hopf bifurcation study of the flow over a rotating and translating circular cylinder which can be compared with the present ones. Our next goal is to examine whether the feature of transition region can be described by the Stuart–Landau equation and to investigate the effects on the 3-D characteristics in wakes behind a 2-D circular cylinder with the rotating control.

ACKNOWLEDGMENTS

We would like to express our gratitude to Dr. B. R. Noack for his continuous help. This work is supported by the National Natural Science Foundation of China.

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