ELSEVIER

0308-0161(95)00023-2

Int. J. Pres. Ves. & Piping **67** (1996) 257-261

0308-0161/96/$15.00

Notch sensitivity of circular rings subjected to impact loading

Ya-Pu Zhao, J. Fang* & T. X. Yu†

LNM, Institute of Mechanics, Chinese Academy of Sciences, Beijing 100080
**Department of Mechanics, Peking University, Beijing 100871, China*
†Dept. of Mech. Eng., The Hong Kong University of Science and Technology, Hong Kong

(Received 1 December 1994; accepted 19 January 1995)

'Notch-sensitive regions' have been observed during a series of experimental investigations into the dynamic plastic behaviour and failure of thin-walled metallic radially notched circular rings with arc-shaped supports subjected to concentrated impact loads. The experimental results show that the exterior notches at some regions have no effect on the deformation of the rings, but do have effect at the remaining regions. The notch-sensitive region is theoretically determined by using the equivalent structures technique; fairly good agreement has been reached between the simple theory and the experimental results. Both dimensional and theoretical analyses prove that whether a plastic hinge formed or not at the notched section does not depend on the mean radius of the ring and the input kinetic energy. It depends on the weak coefficient of the notched section and the angle of the support. Generally speaking, there are mainly three failure modes for a notched circular ring with arc-shaped support under impact loading: Mode I, large inelastic deformation when the notch is outside the sensitive region, in this case the ring deforms as a normal one; Mode II, large inelastic deformation only at some part of the ring and tearing occurred at the notched sections; Mode III, large inelastic deformation and total rupture occurred at the notched sections. It is believed that the present study could assist the understanding of the dynamic behaviour and failure of other kinds of nonstraight components with macroscopic imperfections under impulsive loading.

1 INTRODUCTION

Both static and dynamic behaviour of metallic thin-walled circular rings with different supporting conditions under various kinds of loading patterns have been studied in depth in view of their practical interest.[1-7] The reason why the mechanical behaviour of rings is investigated is two-fold: firstly, ring and ring systems are the most widely used energy absorbing devices, secondly, a clear understanding of the mechanical behaviour of rings will assist in the interpretation of the mechanical behaviour of pipes.

Arc-shaped supports are commonly used to secure various kinds of pressure vessels and piping. Zhao *et al.*[8-10] have recently made both experimental and theoretical investigations into the mechanical behaviour of normal rings (i.e. without notches) with arc-shaped supports subjected to quasi-static or dynamic loads. In Ref. 8, Zhao *et al.* gave a detailed description about their experimental research of circular rings with arc-shaped supports under impact loads; an approximately linear relationship was found between the drop height of the hammer and the residual displacement between the impact point and the bottom of the ring. Another approximately linear relationship was also found between the dimension less impact kinetic energy and the dimensionless permanent deformation of the ring. By using the 'equivalent structure technique', Zhao *et al.*[9] determined the deformation mode and the load carrying capacity of a normal circular ring with arc-shaped support under a symmetric concentrated load.[9] The history of the load/displacement at the loaded point was also presented; it was found that this structure was a kind of unstable one because a softening region exists.[10]

Circular rings, pressure vessels and piping may contain various kinds of cracks as a result of working, environmental and manufacturing reasons, and even in these circumstances they may be under extreme load conditions. A survey of the existing literature of the dynamic study of structures[11] reveals that no previous study has been made on the influence of notches or cracks on the dynamic behaviour of circular rings. The aim of the present paper is to investigate the notch sensitivity as well as the failure mode of the notched circular rings with arc-shaped supports under dynamic loading.

2 EXPERIMENTAL SETUP

The experimental arrangements and the ring material are the same as those described in Refs 8 and 9. The low-velocity impact tests were conducted using a drop-hammer with a modified Cranz–Schardin camera system to record the deformation process of the notched ring. The rings were positioned on arc-shaped supports of the same radius of curvature of the rings. To give a symmetrically concentrated load on the ring, a steel rod with a squared-off blade of width 4 mm was positioned on the upper side of the rings. The material of all the rings was as-received aluminium alloy, designated LD8, which is quite widely used in China. The density of this material is $\rho = 2{\cdot}7 \times 10^3\,\mathrm{kg/m^3}$, and the uniaxial yield stress is $\sigma_0 = 265\,\mathrm{MPa}$. The weight of the drop-hammer is 3·8 kg, and the weight of the rod is much lighter than that of the hammer and therefore, could be neglected. Rings of 100 mm outside diameter, 3 mm radial thickness and 8 mm width were used in the test.

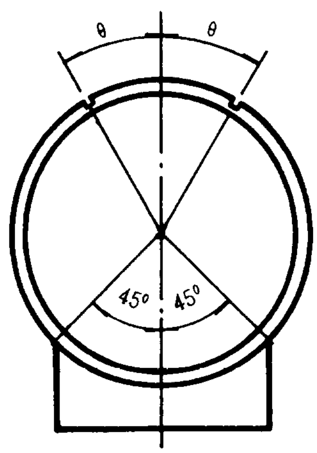

Fig. 1. Initial configuration.

To study the notch sensitivity of the structure under impact loading, two notches were made on the exterior surface of the ring. Because a radial notch can cause the largest strength reduction, the present paper only studied the case that the notch is along the radial direction of the ring. Notch section is 1·2 mm in depth and 0·5 mm in width. The initial configuration of the structure was illustrated in Fig. 1.

3 RESULTS OF EXPERIMENTS

The final shape of tested notched rings was shown in Fig. 2 for different notch angles. Obviously, the experimental results showed that there exist notch-sensitive regions. If the notches were located in these notch-sensitive regions, the notches had a strong influence on the deformation and failure of the rings; in this case a plastic hinge formed at the beginning of the deformation process, then tearing or rupture would happen if the input energy was big enough. On the other hand however if the notices were not located in the notch-sensitive regions, the notches had no influence on the deformation of the rings; in this case the notched ring deformed as a normal one, whose deformation mechanism was a five-hinge mode.[8,9] The method of determination of the notch-sensitive regions will also be given in the next section.

Generally speaking, there are three failure modes for a notched circular ring with arc-shaped support under impact loading, i.e.:

- Mode I, large inelastic deformation when the notch is outside the sensitive region, for example, in Fig. 2(a) and (d);
- Mode II, large inelastic deformation only at some part of the ring and tearing occurs at the notched section, for example, in Fig. 2(b);
- Mode III, large inelastic deformation and total rupture occurs at the notched section, for example, in Fig. 2(c).

The failure process for the case in Fig. 2(b) was shown in Fig. 3, which was recorded by a modified Cranz–Schardin camera system. From

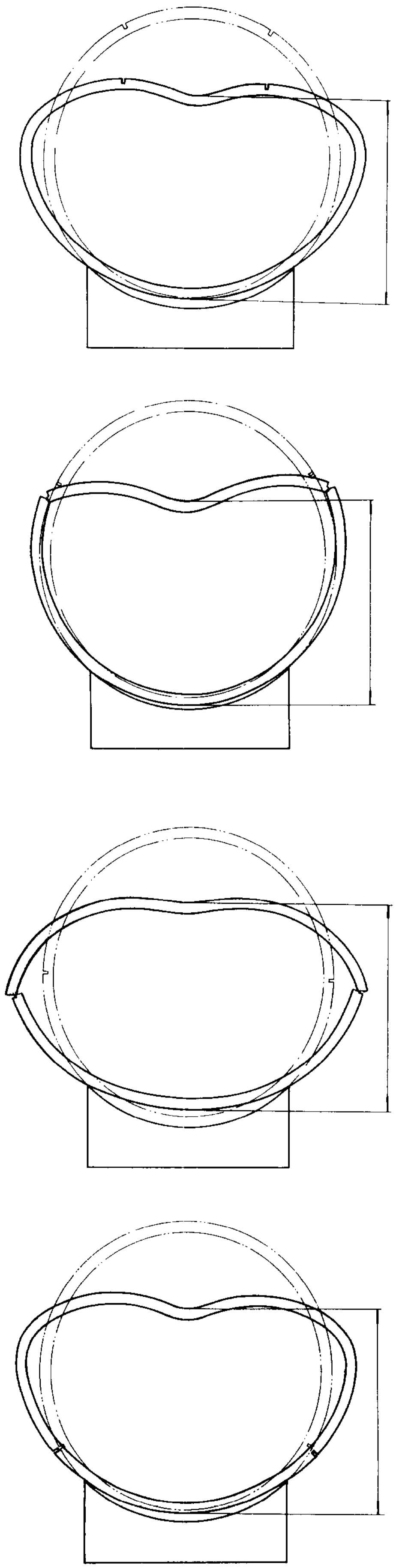

Fig. 2. Typical final shape of notched rings after impact.

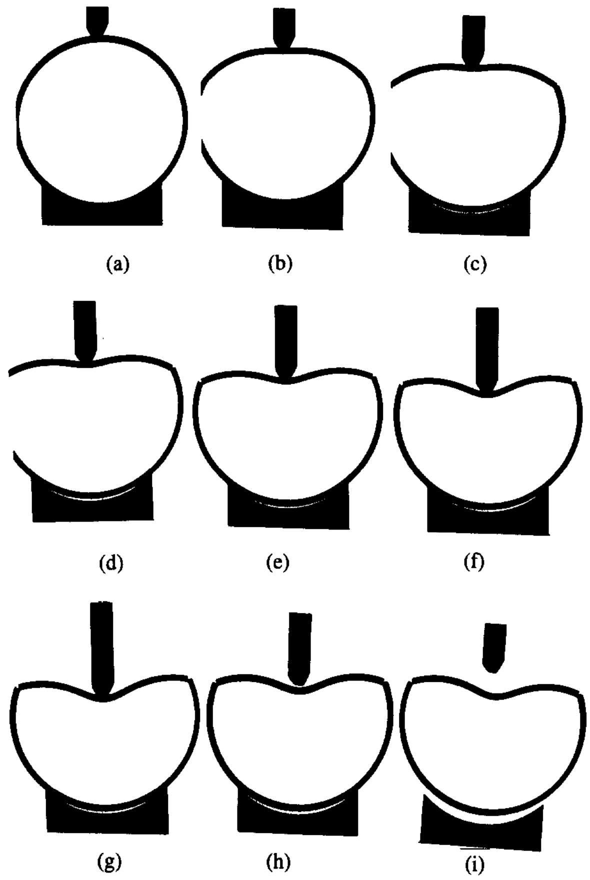

Fig. 3. Failure process of a ring with notch angle of 120°.

the pictures one sees that the deformation and failure process could be divided into three stages. In the first stage, a plastic hinge is formed at the notched section. During the second stage, tearing occurred at the notched sections and the upper part of the ring deformed to its maximum degree. Bounding because of the stored elastic energy belonged to the third stage. The strength at failure[12] of the ring at the upper part was yield-dominant, and the strength at failure of the ring at the notched section was fracture dominant. Thus this was an extremely complex problem.

4 THEORY

4.1 Dimensional analysis

In the case of low-velocity impact, the inertia effect could be neglected in the analysis. Suppose that a plastic hinge formed at the notched section at the beginning of the deformation process of the ring, the half notch angle (as shown in Fig. 1) was denoted by α, the necessary condition could be expressed by the following equation:

$$\alpha_{\text{hinge}} = f(\beta, \gamma, R, E), \tag{1}$$

where α_{hinge} meant that a plastic hinge formed at the notched section; β was the half angle of the arc-shaped support; γ ($0<\gamma\leq 1$) was the weak coefficient of the notched section; R was the mean radius of the ring and E was the input energy. Because α_{hinge}, β, and γ were nondimensional numbers, and because R and E had the dimension of length and energy, respectively, it is therefore quite obvious that α_{hinge} was a function only of β and γ. It was independent of R and E. Thus, equation (1) can be written in the following form:

$$\alpha_{hinge}=f(\beta,\gamma). \qquad (2)$$

4.2 Formula

A typical notched circular ring with arc-shaped support under symmetrically concentrated load is shown in Fig. 4. For simplicity, the two exterior notched rings were symmetrical about the vertical axis passing through the centre of the ring, therefore only half of the ring need be considered in the calculation owing to symmetry. Since the ring is only in contact with the arc-shaped support at the two edge points B and C when the load and deformation is large enough it would be reasonable to assume that the segment BO′C is subjected to two vertical concentrated loads at points B and C.

From the Equivalent Structure Technique[8] we know that the resultant force of the segment ADB of the ring is along the line passing through point K and parallel to line AB, and it is required that HK = KI = Δ. Without loss of generality, we assume that the arc angle of the support is 2β. The moment arm of point H is

$$\Delta=\frac{R}{2}(1-\sin\beta/2) \qquad (3)$$

For normal rings a plastic hinge would be formed always at point H; this point has been proved in a former paper.[9] The moment arm of the notched section N is

$$\Delta^*=R\left[\cos\alpha-\frac{1+\sin\beta/2}{2}\right] \qquad (4)$$

i.e.

$$\frac{\Delta^*}{\Delta}=\frac{2\cos\alpha-(1+\sin\beta/2)}{1-\sin\beta/2} \qquad (5)$$

where α is the angle between line OH and line ON. The condition that a plastic hinge is formed at the notched section is required that

$$\frac{\Delta^*}{\Delta}>\gamma \qquad (6)$$

i.e.

$$\cos\alpha>\frac{1+\gamma+(1+\gamma)\sin\beta/2}{2} \qquad (7)$$

Equation (7) holds for any given β; for the present test $\beta=45°$. The above expression may be rewritten into the following form:

$$\cos\alpha>\frac{\gamma+2{\cdot}24}{3{\cdot}24} \qquad (8)$$

The relationship between α and γ is shown in Fig. 5. From Fig. 5 we know that a plastic hinge is formed at the notched section in region I, and the notch has no influence on the deformation of the ring in region II. In the present test the notch

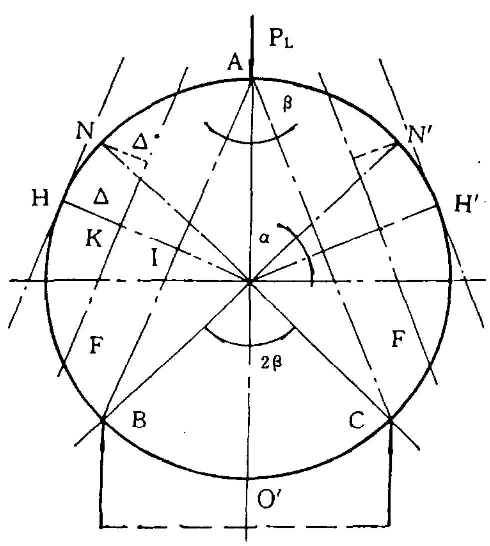

Fig. 4. Illustration of analytical model.

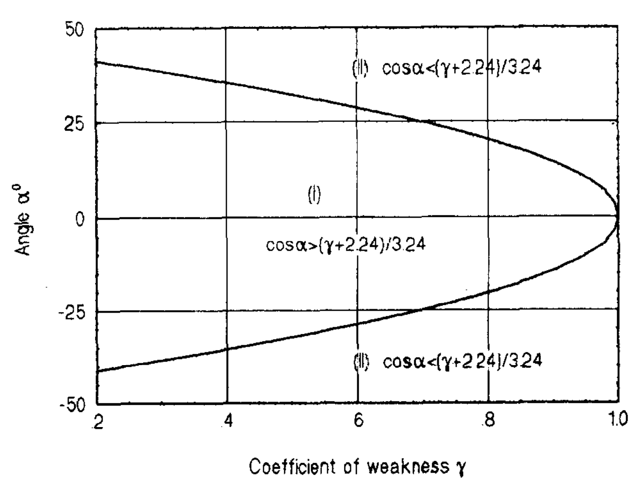

Fig. 5. Relationship between α and γ.

section is 1·2 mm in depth and 0·5 mm in width, and the weakness coefficient of the notched section is then

$$\gamma = (1 - 1{\cdot}2/4)^2 = 0{\cdot}49 \qquad (9)$$

By using eqn (8), we know that a plastic hinge is formed at the notched section if

$$-32{\cdot}6^\circ < \alpha < 32{\cdot}6^\circ \qquad (10)$$

To show the validity of the method presented in this paper, let us take Fig. 2 as a example. For Fig. 2(a) and 2(d), α_1 and α_4 are, respectively, 37·5° and −52·5°, since α_1 and α_4 are not in the range of inequality (11), the two notches have no influence on the deformation of the ring, and it has been shown by Fig. 2(a) and 2(d) that the notched rings deform as normal ones. However, for Fig. 2(b) and 2(c), α_2 α_3 are 7·5° and 22·5°, respectively; as α_2 and α_3 are in the range of inequality (11), it has been shown by Figure 2(b) and 2(c) that the two notches have a strong influence on the deformation of the rings.

5 CONCLUSION

Experimental and theoretical investigations have been made into the mechanical behaviour and failure of notched rings under impact loading. Three major failure modes have been found in the test. Notch-sensitive regions have been found for radially notched rings with arc-shaped supports subjected to impacted or concentrated impact loading. Approximate theoretical analysis has been made to determine the notch-sensitive regions and notch-insensitive regions. It has been shown by the experiments that the approximate theoretical method regarding the influence of notches on the deformation of a ring is valid for low-velocity impact. It should be pointed out the theoretical prediction is also valid for rings with interior radial notches.

It is believed that this test and theoretical analysis would assist in the interpretation of dynamic failure analyses of other kinds of nonstraight components (such as arches, shells, circular beams, *etc.*) with macroscopic imperfection under dynamic loading.

ACKNOWLEDGMENTS

The work reported in this paper is sponsored by the National Natural Science Foundation of China. The authors wish to thank Profs R. Wang and S. R. Reid for their helpful discussion. thanks are also due to Dr A. D. Holmes for his help in the tests.

REFERENCES

1. Hwang, C., *J. Aero. Sci.*, **30** (1953), 819–826.
2. Ghosh, S. K., Johnson, W., Reid, S. R. & Yu, T. X., *Int. J. Mech. Sci.*, **23** (1981), 183–194.
3. Yu, T. X., in *Advance of Plasticity* (in Chinese), Eds. R. Wang, *et al.*, China Railway Press, Beijing, 1988, 387–409.
4. Reddy, T. Y. & Reid, S. R., *Int. J. Mech. Sci.*, **21** (1979), 187–199.
5. Reid, S. R. & Reddy. T. Y., *Int. J. Solids Structures*, **14** (1978), 213–225.
6. Yu, T. X., *Acta Mechanica Sinica*, **1** (1979), 88–91.
7. Zhang, T. G. & Yu, T. X., *Int. J. Impact Engng*, **8** (1989), 43–51.
8. Zhao, Y. P. & Fang, J., Experimental investigation into the dynamic behavior of circular rings with arc-shaped supports, *Int. J. Pres. Ves. & Piping*, **62** (1995), 83–85.
9. Zhao, Y. P., Holmes, A. D. & Fang, J., The load carrying capacities of circular rings with arc-shaped supports, *Int. J. Pres. Ves. & Piping* (in press).
10. Zhao, Y. P., Studies on some problems of the dynamic plastic response and failure of structures under impact loading, Ph.D. Thesis (Chapter 6), Peking University, Beijing (1994).
11. Zhao, Y. P., Yu, T. X. & Fang, J., A brief review on the experimental investigations into the dynamic plastic response of structures, *Advances in Mechanics* (in press, in Chinese).
12. Irwin, G. R. & Paris, P. C., *Fundamental Aspects of Crack Growth and Fracture*, in *Fracture*, Vol. 3, H. Liebowitz (ed.), Academic Press (1971).