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ON THE INTERACTION OF WATER WAVES AND SEABED BY THE POROUS MEDIUM MODEL*

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ABSTRACT: The interaction of water waves and seabed is studied by using Yamamoto's model, which takes into account the deformation of soil skeletal frame, compressibility of pore fluid flow as well as the Coulumb friction. When analyzing the propagation of three kinds of stress waves in seabed, a simplified dispersion relation and a specific damping formula are derived. The problem of seabed stability is further treated analytically based on the Mohr-Coulomb theory. The theory is finally applied to the coastal problems in the Lian-Yun Harbour and compared with observations and measurements in soil-wave tank with satisfactory results.

KEY WORDS: porous medium, wave attenuation, Coulomb friction, seabed stability

I. INTRODUCTION

In coastal engineering design, people always need to know the degree of attenuation of the incident water wave travelling over muddy seafloor and the criteria for instability or liquifaction of seabed to occur. Hence, the problem of interaction between water waves and seabed has long before been paid attention to and carefully investigated by numerous models[1–4].

Initially, marine soils were regarded as monophase media. Evidently, the applicability of these models seems to be very limited owing to the fact that they entirely neglect roles played by pore fluid and percolation effects. In reality, marine soil consists of three phases: solid, liquid and gas. The solid phase forms the frame of soil; the water fills in soil pores; and the gas only occupies a small portion of the pore volume. So, we believe that marine soil is appropriately characterized by porous media. Even though, there are still a few selections. For example, we can assume soil to be a rigid or elastic frame. Since energy dissipation processes due to inelastic deformation in these models are overlooked, they need further improving. Experiments showed that the shear energy dissipation comes from the Coulomb or solid-solid friction, rather than viscous damping or fluid-solid friction. In contrast, the energy loss of acoustic waves in marine sediments is due to both of them. For silt and clay, the former is proved dominant factor within the whole frequency range. In contrast, for coarse sand, the dominant mechanism of damping is the intergranular friction in the low frequency range (0–1 Hz), but the viscous damping dominates in the high frequency range (over 100 Hz)[1,2]. For this reason, we should never leave out dissipation caused by inelastic deformation of the soil frame and viscosity of pore fluid.

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As a matter of fact, compression of pore fluid flow and deformation of skeletal frame take place simultaneously. Considering energy dissipation mentioned above, M.A. Biot[5] proposed a constitutive equation for fluid-filled porous media in 1962. Then, the stresses in porous media under a plane strain condition are given as

$$\begin{aligned} \tau_{xx} &= He - 2\mu e_z - C\zeta \qquad & \tau_{xz} &= \mu\gamma_y \\ \tau_{zz} &= He - 2\mu e_x - C\zeta \qquad & p &= M\zeta - Ce \end{aligned} \tag{1.1}$$

where τ_{ij} denote the stresses; p is the pore fluid pressure; e_i, the strain components; $e = e_x + e_z$, the bulk strain; γ, the shear strain; ζ, the convergence of weighted fluid relative displacement $\boldsymbol{w} = \beta(\boldsymbol{U} - \boldsymbol{u})$, $\boldsymbol{U}$ is the displacement of pore fluid and $\boldsymbol{u}$, the displacement of soil skeletal frame; H, μ, C, M are the Biot's elastic moduli respectively, which can be determined by the bulk modulus of the grain K_r, the shear modulus G, the Poisson's ratio ν, the specific loss of the shear or compression cycle δ, the bulk modulus of pore fluid K_f, the porosity of the soil β, the degree of saturation s and the absolute pore pressure p_0. All the Biot's elastic moduli, expressed in italic letters, are complex constants accounting for slightly inelastic behaviours in marine soil. Especially, the imaginary part of complex shear modulus μ is nothing but the Coulumb damping due to grain-to-grain friction depending on the specific loss δ in a shear or compression cycle.

II. YAMAMOTO'S POROUS MEDIA MODEL[1,2]

Usually, there are two types of stress waves propagating in monophase solid media. Nevertheless, an additional compressive wave of pore fluid adds to them for marine soil by the porous medium model. When ocean waves are travelling over a seabed, pore pressure gradient accumulates in pores, which induces a flow of pore fluid relative to the soil frame. Thus, pore fluid transmits a force to the skeletal frame through fluid viscosity and pressure gradient. Namely, the wave-induced effective stresses cause compressible and shear deformations of soil skeletal frame and compression of pore fluid, which give rise to three types of stress waves in seabed. During this process, ocean wave energy is dissipated by the Coulumb friction and viscosity of pore fluid.

For dynamic problems, people should take into consideration the inertial term in the equilibrium equation system. It turns out the second derivatives of product of weighted density and displacement vectors: $\rho_r(1-\beta)\boldsymbol{u} + \beta\rho_f\boldsymbol{U} = \rho\boldsymbol{u} + \rho_f\boldsymbol{w}$ with respect to time t, ρ_r and ρ_f are densities of grain and fluid, respectively. And

$$\rho = (1-\beta)\rho_r + \beta\rho_f \tag{2.1}$$

is the weighted density of soil. Based on the Biot's constitutive relation (1.1), the dynamic equation systems for marine soil frame can be written as

$$\frac{\partial^2}{\partial t^2}(\rho\boldsymbol{u} + \rho_f\boldsymbol{w}) = \mu\nabla^2\boldsymbol{u} + (H-\mu)\nabla e - C\nabla\zeta \tag{2.2}$$

Likewise, an inertial term, i.e. the second derivaties of $\rho_f\boldsymbol{u} + m\boldsymbol{w}$ with respect to t should be supplemented into the usual Darcy's law as well, where

$$m = (1+\alpha)\rho_f/\beta \tag{2.3}$$

is the reduced mass including added mass effects of soil skeletal frame in terms of added mass coefficient α. In this way, the motion of pore fluid satisfies

$$-\nabla p = \frac{\partial^2}{\partial t^2}(\rho_f \boldsymbol{u} + m\boldsymbol{w}) + \frac{\eta_f}{k_s}\frac{\partial \boldsymbol{w}}{\partial t} \tag{2.4}$$

where η_f is the viscosity of the pore fluid, k_s, the hydraulic coefficient of permeability of soil. Assume that soil frame displacements and pore liquid motion can be represented by the sum of potentials of compressive and shear waves

$$\begin{aligned} \boldsymbol{u} &= \nabla \Phi_1 + \nabla \times \Psi_1 \\ \boldsymbol{w} &= \nabla \Phi_2 + \nabla \times \Psi_2 \end{aligned} \tag{2.5}$$

If the potential Φ_1 and Φ_2 of the sinosoidal forms

$$\begin{aligned} \Phi_1 &= \phi_1(z)\mathrm{exp}[\mathrm{i}(\omega t - kx)] \\ \Phi_2 &= \phi_2(z)\mathrm{exp}[\mathrm{i}(\omega t - kx)] \end{aligned} \tag{2.6}$$

are substituted into Eqs.(2.4), (2.5), two kinds of compressible waves due to soil skeletal frame movement and relative motion of pore fluid are obtained. They are designated as the first and the second compressible waves, respectively. Please note that k, water wave number is a complex constant due to damping. Complex wave numbers and celerities of compressible waves s_1, s_2 and V_1 and V_2 are independent of water wave parameters except frequency ω. Further assume the potential $\boldsymbol{\Psi}_1$ and $\boldsymbol{\Psi}_2$ to be

$$\begin{aligned} \boldsymbol{\Psi}_1 &= \boldsymbol{\psi}_1(z) \ \mathrm{exp} \ [\mathrm{i}(\omega t - kx)] \\ \boldsymbol{\Psi}_2 &= \boldsymbol{\psi}_2(z) \ \mathrm{exp} \ [\mathrm{i}(\omega t - kx)] \end{aligned} \tag{2.7}$$

Then, substitution of them into (2.4), (2.5) leads to only one kind of shear wave since pore fluid can't bear any shear force. Similarily, its complex wave number s_3 and celerity V_3 are independent of water wave parameters except frequency ω.

To obtain stress waves induced by incident water wave, the solutions are generally assumed to be a linear combination of three kinds of stress waves, with coefficients $a_i, b_i, (i = 1, 2, 3)$, the profiles being exponential function of vertical coordinate z. At the same time, they should meet the conditions at the interface between water and seabed, i.e. continuity of normal velocity and pressure and vanishing of shear and effective normal stresses. As a result, three unknown constants and complex water wave number k can be found by solving the four algebraic equation system given by the foregoing boundary conditions. The imaginary parts of all the wave numbers k and s are responsible for damping of water and stress waves. In the long run, the expression of wave number k, usually known as dispersion relationship, turns out to be

$$\mathrm{th}kh = \frac{\omega^2}{kg}\frac{(1 - kgF/\omega^2)}{(1 - \omega^2 F/kg)} \tag{2.8}$$

where

$$F = \frac{\rho_f \omega^2 \{2k^2[\lambda_1(c_1 - c_3) + \lambda_2 E(c_2 - c_3)] - s_3^2[\lambda_1(1 + c_1) + \lambda_2 E(1 + c_2)]\}}{k(2k^2 - s_3^2)[s_1(C + c_1 M) + s_2^2 E(C + c_2 M)]} \tag{2.9}$$

$$E = -\frac{(2k^2 - s_3^2)\{s_1^2[(C - H) + c_1(M - C)] + 2\mu k^2\} - 4\mu k^2 \lambda_1 \lambda_3}{(2k^2 - s_3^2)\{s_2^2[(C - H) + c_2(M - C)] + 2\mu k^2\} - 4\mu k^2 \lambda_2 \lambda_3} \tag{2.10}$$

where c_i are quotients of b_i over $a_i (i = 1, 2, 3)$.

For a rigid impermeable bed, the stiffness coefficients C and M tend to infinity, thus F tends to zero, and (2.8) resumes the familiar dispersion relation for water wave of finite depth, that is to say, the interaction of water wave and seabed can be neglected.

For a Coulomb-damped poroelastic bed, the complex wave number k can be determined by iteration methods. To simplify this process, we make an approximation to Eq.(2.8). Assuming that damping is small, we have $k = k_r(1 + \mathrm{i}D/2)$, where the specific damping $D = 2k_i/k_r = o(1)$ stands for the decay rate of water wave energy. Since sand bed is rather rigid, $H, C, M \gg \mu$ and F is also small. Moreover, the celerity of water wave is much smaller than that for stress waves in the seabed $V_\omega/V_1, V_\omega/V_3$ (or $s_1/k, s_2/k \ll 1$). Then the characteristic roots are $\lambda_1, \lambda_3 \approx 1, \lambda_2 = k[1 - (s_2/k)^2]^{1/2}$. According to above assumptions, the iteration formula for k_r and D with simplified E and F can be reduced to

$$\begin{aligned} &\mathrm{th}k_r h = \omega^2[1 - \mathrm{sech}^2 k_r h \mathrm{Re}F]/k_r g \\ &D = -4\mathrm{Im}F/(2k_r h + \mathrm{sh}2k_r h) \end{aligned} \tag{2.11}$$

where

$$F = \frac{\rho_f \omega^2 (c_1 - c_3)}{[s_1^2(C + c_1 M) + s_2^2 E(C + c_2 M)]}$$

$$E = -\frac{s_1^2[(C - H) + c_1(M - C)]}{s_2^2[(C - H) + c_2(M - C)]}$$

where $k^{(i)} = k_r^{(i)}(1 + \mathrm{i}D^{(i)}/2)$, if initial value $k_r^{(0)}$ is given as the solution of $\mathrm{th}k_r h = \omega^2/k_r h$, $D^{(0)} = 0$, then the complex wave number can be ultimately found out. With wave number k and combination coefficients given, we are able to derive displacements of marine soil and relative motion of pore fluid. By using the Biot's constitutive relation, the stress expressions are immediately yielded. Then, the vertical distributions for the ratios of pressure of pore fluid and shear stress to the bottom pressure look like

$$\frac{P}{P_B} = \frac{(C + c_1 M)s_1^2 a_1 \exp(\lambda_1 z) + (C + c_2 M)s_2^2 a_2 \exp(\lambda_2 z)}{(C + c_1 M)s_1^2 a_1 + (C + c_2 M)s_2^2 a_2}$$

and

$$\frac{\tau_{xz}}{P_B} = \frac{-2\mathrm{i}\mu k \lambda_1 a_1 \exp(\lambda_1 z) - 2\mathrm{i}\nu k \lambda_2 a_2 \exp(\lambda_2 z) - \mu(2k^2 - s_3^2)\exp(\lambda_3 z)}{(C + c_1 M)s_1^2 a_1 + (C + c_2 M)s_2^2 a_2}$$

which enable us further to examine the stability of seabed.

III. APPLICATIONS AND DISCUSSION

We have applied the foregoing theoretical results to a few coastal problems of the Lian-Yun Harbour. To begin with, we tabulate the physical parameters of marine soil in the Lian-Yun Harbour in Table 1[1,2,6]. Since measured data exhibit some scattering, an average of them measured in situ at twelve points along the interface of marine water and soil at depth 8.6m is chosen as representative parameters for the Lian-Yun Harbour's soil. We also list Yamamoto's parameters in the same table for comparison.

We have plotted the specific damping of water wave versus wave number according to (2.11). Just as shown in Fig.1, the larger the nondimensional water-wave number is, the smaller the wave damping or vice versa. The reason responsible for that is: seabed in shallower water exerts apparent influences on the attenuation of water wave over it. On the

Table 1

Soil Parameters

	ρ_r [kg/m^3]	ρ_f [kg/m^3]	G [N/m^2]	K_r [N/m^2]	K_f [N/m^2]	ν	β_f	δ	α	s
Lian-Yun	2.75	1.03	6.1E5	4.15E6	1.92E6	0.49	0.59	0.05	0.25	1.0
Yamamoto	2.65	1.03	1.0E5	3.60E6	1.92E6	0.45	0.40	0.05	0.25	1.0

contrary, bottom in deeper water merely exerts minor influences on it. Figure 1(a) displays the relation between the specific damping and the shear modulus. Specific damping decreases with the decrease of shear modulus as $k_r h \geq 1.5$ and increases with it as $k_r h \leq 1.5$. Figure 1(b)(c) provide the relations of specific damping with poroe water viscosity and soil permeability, respectively. Obviously, the phenomenon can be easily imagined intuitively from physical considerations, that is to say, smaller viscosity and larger permeability mean smaller damping.

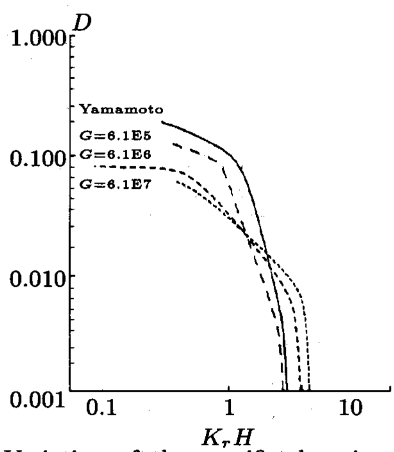


Fig.1(a) Variation of the specific damping with the wave number for the different shear moduli of soil

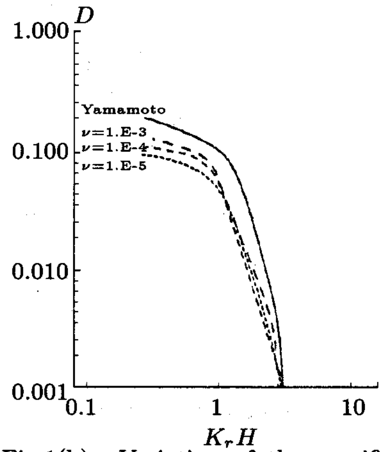


Fig.1(b) Variation of the specific damping with the wave number for the different viscosities of pore fluid

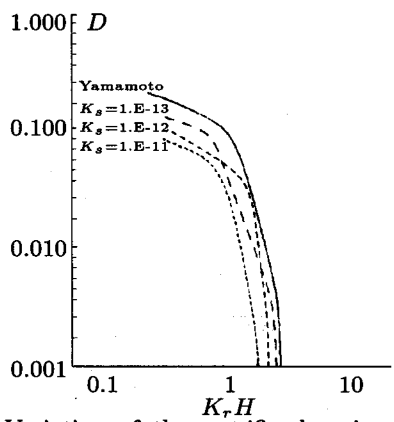


Fig.1(c) Variation of the specific damping with the wave number for the different hydraulic coefficients permeability

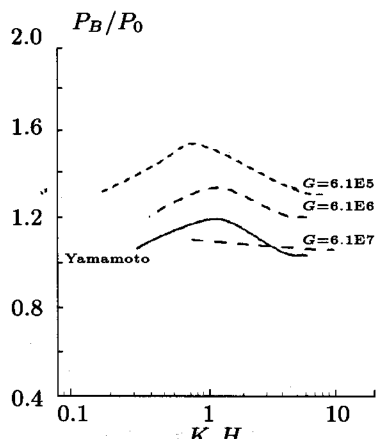


Fig.2 Variation of the bottom pressure with $k_r H$ over the soft clay bed

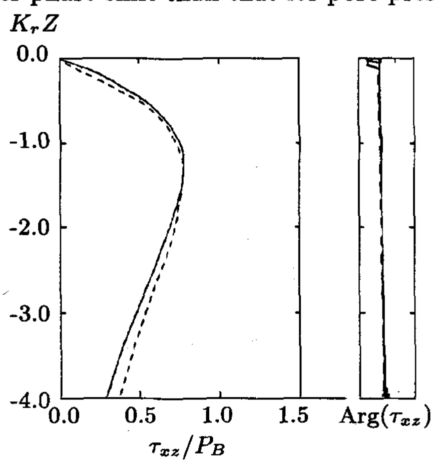
The comparison of theory with experiments performed at the Tsinghua University[6] have been made as well. The wave flume is equipped with wooden wedge wave maker at one end to produce various water waves we need and with wave absorber in preventing incident waves from reflecting at another end. Table 2[7] lists the theoretical and experimental values of k_i. We can see from them that there is no evident difference between them qualitatively, namely, in the magnitude of order. What accounts for these errors is that the effects of nonlinearity tend to increase damping, whereas thinner seabed tend to decrease it. Hence, cases 4 and 6 show better results.

Table 2

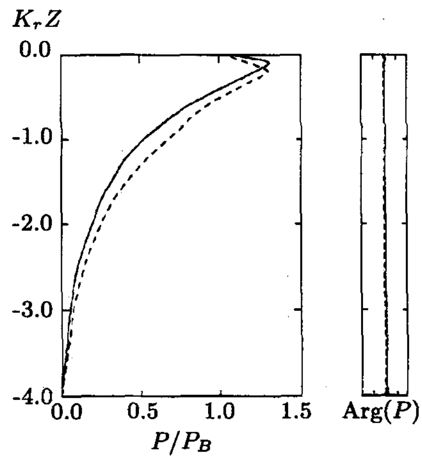
Theoretical and Experimental Values of k_i

Case	h/[m]	ω/[1/s]	$k_r h$	D	k_r/[1/m]	k_i/[1/m] Theory	k_i/[1/m] Measured
1	0.125	1.0	0.775	0.07	6.2	0.217	0.455
2	0.205	1.0	1.055	0.04	5.1	0.102	0.221
3	0.205	1.25	0.798	0.07	3.9	0.136	0.194
4	0.205	1.5	0.645	0.085	3.1	0.131	0.130
5	0.285	1.0	1.323	0.02	4.6	0.046	0.0598
6	0.285	1.25	0.977	0.045	3.4	0.0765	0.0757

Furthermore, the amplitude distribution of pore water pressure and shear stresses in seabed are calculated. Since λ_i are complex, these quantities often decay exponentially with a phase shift. The pore pressure, normal and shear stressses are shown in Figs.2, 3, 4, respectively, from which we can see that the pore water pressure increases from 1 at the interface to a maximum due to phase shift and then decreases exponentially. However, the shear stress increases from zero at the interface. We also notice that the shear stresses have a larger phase shift than that for pore pressure.



(a) Distribution of shear stresses in seabed



(b) Distributions of pore pressure in seabed

Fig.3 — Yamamoto ···· L-Y Harbor

Although we know that the rate of external loading has apparent influences on the strength of soils, for instance, the strength of soil under slowly varying loading is larger

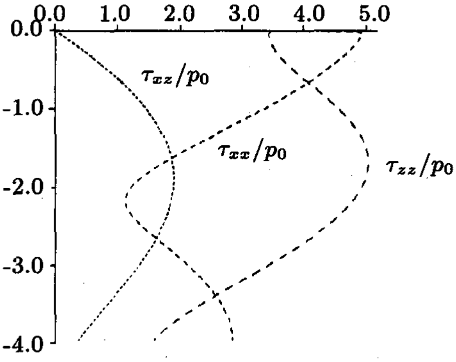


Fig.4 Normal and shear stresses variation versus depth

than that under fast loading; periodic loading reduces the shear strength of soils etc., the dynamic theory of strength for the failure of soil base is not well understood yet[7]. Hence, the stability analysis in the present paper remains to use the static theory of strength for soils. The soil breaks down when the shear stress in soil exceeds the shear strength and leads to the occurrence of land slide. The Mohr-Coulomb theory of strength can be used:

$$\tau_f = c + \sigma \text{tg}\phi$$

in which c, ϕ are the soil strength parameters, i.e. cohesion and friction angle depending on the soil compositions and the types of loading. σ is the effective normal stress. If external forcing is fast without drainage, we call it a fast shear process; if the soil is subjected to a fast shear force, it is referred to as unconsolidated-undrained test (UUT). On the other hand, the soil is subjected to a slow shear force, it is referred to as consolidated-drained test (CDT). The parameters in the present study are: c = 40.0 N/m^2, ϕ = 1.05° (UUT); c = 121.0 N/m^2, ϕ = 16.3° (CDT). We can see that the friction angle ϕ for UUT is considerably small so that the line of shear stress strength looks almost horizontal. With the distribution of shear stress and strength theory given, it is not difficult to judge when the marine soil will slide and break down under the action of incident ocean waves over seabed.

Now we are able to conduct stability analysis. According to the soil data of the Lian-Yun Harbor given in above section, Fig.5 and Fig.6 are plots exhibiting shear stresses

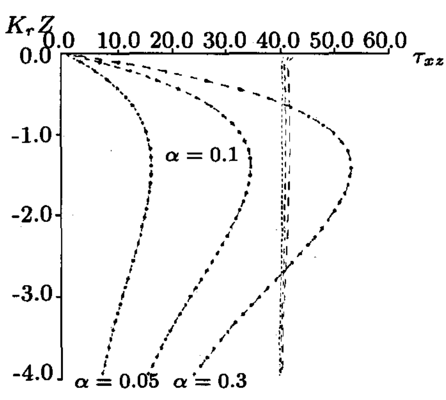


Fig.5 Shear stresses variation versus depth (UUT)

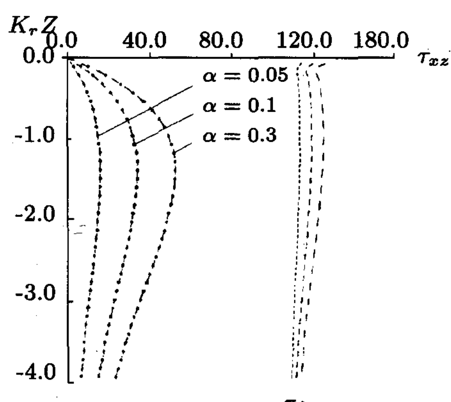


Fig.6 Shear stresses variation versus depth (CDT)

varying with water wave amplitudes chosen as 0.05, 0.1, 0.3, respectively. Besides, the shear strengths are plotted simultaneously in the same figures for the sake of comparison. Then, we can conclude that under CDT condition the seabed is completely stable. In contrast, the soil stability under UUT condition seems to depend on the amplitude of incidence water wave. In the present example, if the amplitude is 0.3, the marine soil is unstable; while the amplitude is reduced to 0.1 or below, the marine soil becomes stable.

In conclusion, we must emphasize that the porous medium model suggested in the present paper, has some limitations owing to the Biot's linear constitutive relation. So the model cannot be applied to large deformation or even soil moving conditions. Moreover, the linearized water wave theory in the present study imposes some restrictions on wave amplitude and water depth as well. Consequently, it is necessary for us to further study nonlinear interaction of sea wave and muddy bed in the future. Since the soil shear strength is a function of cyclic shear-strain and cyclic numbers, the stability criteria for seabed is related to the waves-induced cyclic loading and its period. The resolution of these challenging problems in ocean engineering is a task of top priority of the moment[8].

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REFERENCES

[1] Yamamoto T. On the response of a Coulomb-damped poroelastic bed to water waves. *Marine Geotechnology*, 1983, 5(2): 93–130

[2] Yamamoto T. Nonlinear mechanics of wave interaction with sediment beds. *Appl Ocean Res*, 1982, 4(2): 99–106

[3] Rahman MS. Instability and movements of oceanfloor sediments: A review. In: Chung Jin S, Natvig Birger, Das Braja M eds. the Proceedings of the Fourth (1994) International Offshore and Polar Engineering Conference, 1994. 589–599

[4] Shimizu M. A simplified one-dimensional theory for the wave-induced liquefaction of sandy seabeds. In: Chung Jin S, Natvig Birger, Das Braja M eds. the Proceedings of the Fourth (1994) International Offshore and Polar Engineering Conference, 1994. 600–606

[5] Biot MA. Mechanics of deformation and acoustic propagation in porous media. *Jour Appl Phys*, 1962, 33(4): 1254–1264

[6] Huhe Aode et al. Wave attenuation over muddy seabed in the Lian-yun-gang port experiment, observation and theoretical analysis. *The Ocean Engineering*, 1994, 12(2): 68–77

[7] Gu Xiaoyun. On the analytic methods of stability of submarine slopes. *Advances in Mechanics*, 1989, 19(1): 50–59 (in Chinese)

[8] Prakash S. On dynamic soil properties for marine environment. In: Chung Jin S, Natvig Birger, Das Braja M eds. Proceedings of the Fourth (1994) International Offshore and Polar Engineering Conference, 1994. 404–412