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EXPERIMENTAL AND THEORETICAL STUDY ON NUMERICAL DENSITY EVOLUTION OF SHORT FATIGUE CRACKS*

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ABSTRACT: Fatigue testing was performed using a kind of triangular shaped specimen to obtain the characteristics of numerical density evolution for short cracks at the primary stage of fatigue damage. The material concerned is a structural alloy steel. The experimental results show that the numerical density of short cracks reaches the maximum value when crack length is slightly less than the average grain diameter, indicating grain boundary is the main barrier for short crack extension. Based on the experimental observations and related theory, the expressions for growth velocity and nucleation rate of short cracks have been proposed. With the solution to phase space conservation equation, the theoretical results of numerical density evolution for short cracks were obtained, which were in agreement with our experimental measurements.

KEY WORDS: short fatigue cracks, crack numerical density, fatigue damage evolution

I. INTRODUCTION

Engineering components such as various parts of aeroplane and automobile structure are inevitably subjected to cyclic loading during routine service. Fatigue damage of these components usually originates from the short crack whose dimension is comparable to the size of the microstructural unit, e.g., the grain diameter. The process of fatigue failure may include different regimes of crack initiation, short crack growth, long crack propagation and final fracture, of which the short crack regime takes up a large portion of total fatigue life. The evaluation of fatigue failure therefore relies on the understanding of the short crack behaviour and the damage evolution characteristics in this regime.

The behaviour of initiation and propagation for short fatigue cracks in structural steels and aluminium alloys has been reported in the literature[1−3], where the observed phenomena have been interpreted to a certain extent. However, in most cases, the monitoring and the analysis of the behaviour were focused only on a few single cracks. In practice, the development of short crack damage is mostly a collective evolution process of a large number of short cracks.

To account for this, the application of statistic physics to the study of short crack damage becomes an attractive approach. Bai et al.[4,5] proposed a method to analyze the

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evolution of micro-damage, in which the equilibrium evolution equation was established based on the balance of damage numerical density within the concerned phase space. For the damage type with simple shape like short crack, crack length a may be used to denote its dimension. By introducing the distribution function of numerical density for micro-damage, $n(a,t)$, then at time t, the number of crack with size between a and $a+\mathrm{d}a$ within a unit volume is $n(a,t)\mathrm{d}a$. At primary stage of damage, the interaction between cracks is negligible and the process of initiation and growth for a short crack predominantly depends on the local environment where it originates. We define the distribution of such short cracks as ideal-crack-system, which can be expressed by the following equilibrium equation[4,5]

$$\frac{\partial n}{\partial t}+\frac{\partial(A\cdot n)}{\partial a}=n_N \tag{1}$$

where, A is the crack growth rate and n_N the specific crack nucleation rate. Equation (1) is established based on the consideration that the total number of short cracks is determined by both the crack nucleation and the crack growth. Thus, the evolution of n is dependent on the average crack growth rate A and the specific crack nucleation rate n_N. In the following, we attempt to construct the equations for A and n_N from the experimental observations so as to analyze the evolution characteristics of short crack damage.

II. EXPERIMENTAL DETERMINATION OF NUMERICAL DENSITY EVOLUTION FOR SHORT CRACKS

1. Test Material

A structural alloy steel (30CrMnSi) was used as the test material, whose main chemical compositions are (wt%): C: 0.30, Si: 1.0, Mn: 0.90, Cr: 0.90, and Fe balance. The material was annealed from 860°C and possesses the microstructure of ferrite and pearlite. The average ferrite grain size is 17.9μm. The results of uniaxial tensile testing show that the Young's modulus E: 206 GPa, yield strength σ_y: 380 MPa, elongation δ: 23%, and area reduction ψ: 74%.

2. Fatigue Testing

A kind of triangular specimen[6,7] shown in Fig.1 was used in fatigue tests at room temperature. The wider end of the specimen was clamped in a support and the other end was connected to a vibration testing machine through a pliable bar to permit displacement

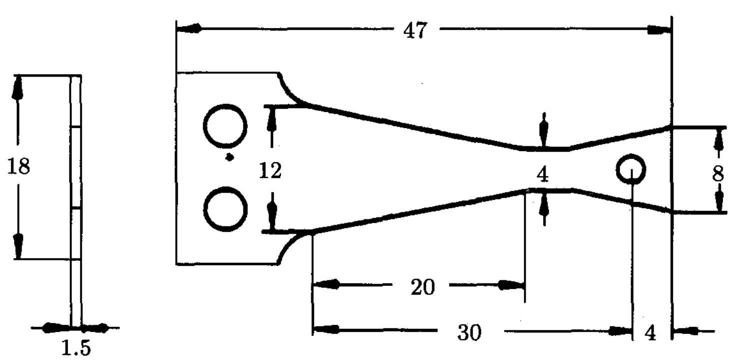

Fig.1 Triangular specimen (in mm)

in the longitudinal direction of the specimen. When the specimen is subjected to an end flexural loading, the surface tensile stress σ within the isosceles triangular part is constant. A large area for monitoring the characteristics of short cracks is therefore available. From both calibration and calculation[7], the relation between bending deflection y at the end and the surface tensile stress σ for this specimen is

$$\sigma = 1.04\frac{Eby}{L^2}$$

where E is Young's modulus, b the thickness of specimen, and L the height of triangular section of the specimen. Fatigue testing was periodically interrupted at pre-set intervals. During each interval, the specimen was removed from the test rig and examined under an optical microscope. After examination, the specimen was put back to the machine and the same fatigue loading was again applied. The testing control parameters were as follows: frequency $f = 8 - 10$Hz, stress ratio $R = -1$, maximum fatigue stress $\sigma_{\max} = 1.0 - 1.5\sigma_y$ and maximum fatigue cycles $N_{\max} = 10^6$.

3. Results

After a few thousand cycles of fatigue loading, slip steps were observed on the surface of polished but unetched specimen, some of which consequently evolved into surface short cracks. Evidently, short cracks were initiated from heavily developed persistent slip bands with the surface appearance of so called "extrusion" and "intrusion". From the observation on etched specimens, it was seen that short cracks predominantly formed within ferrite grain domains. The number of short cracks gradually increased with fatigue cycles whereas the

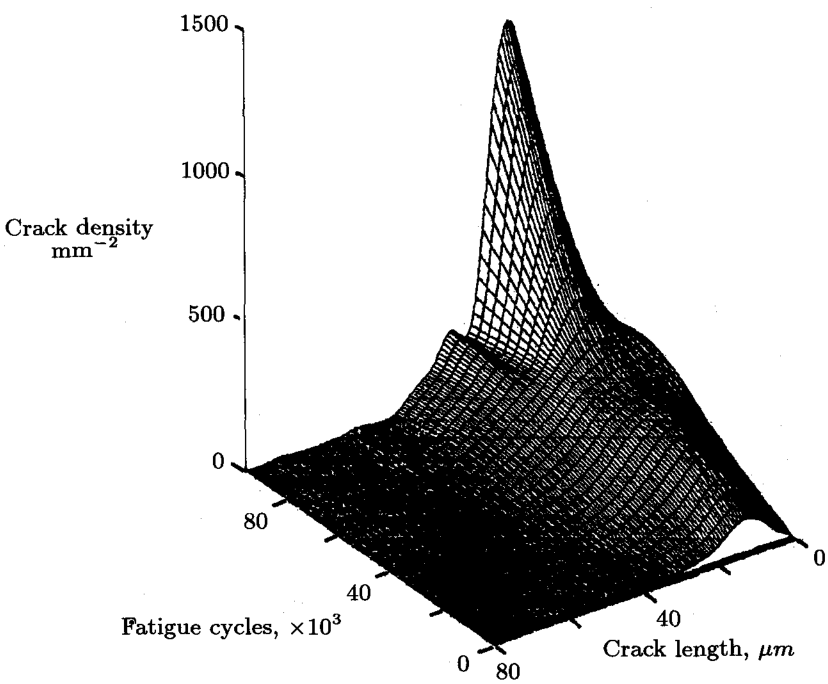

Fig.2 Experimental results of crack density as a function of crack length and fatigue cycles

length of most short cracks that terminated at grain boundaries was kept almost unchanged. The number and the size of short cracks were measured with a quantitative image analyzer from the photographs of specimen surface taken at each test interval. Figure 2 shows the cumulative results of crack numerical density as a function of crack length and fatigue cycles. It is seen that crack length gradually converges to the size between 10 and 17 μm, i.e. the size slightly less than the average ferrite grain diameter. This implies that, when growing short cracks approach to grain boundaries, the resistance against the growth abruptly increases. As a result, the number of the short cracks with the length close to grain diameter becomes increasingly large, leading to the emergence of a large peak located at the vicinity of average grain size (Fig.2).

In addition to the dominant phenomenon, Fig.2 also presents a small peak appeared approximately at the crack size of 30μm. The reason for this small peak remains for further investigations and arguments. Here it is regarded as the fact that short cracks may terminate at the second grain boundary when such a crack grows across the first grain boundary or links with an adjacent crack.

III. THEORY AND BASIC EQUATIONS FOR SHORT CRACK NUMERICAL DENSITY CALCULATION

The development of damage is a cumulative process that consists of contributions from each single crack. In the light of this consideration, we define D_1 the one-dimensional damage of short cracks as

$$D_1 = \gamma \int_0^\infty n(a,t) \cdot a \cdot \mathrm{d}a \tag{2}$$

where $n(a,t)$ is the distribution function of short crack numerical density with the dimension of (length^{-2}), a is crack length and γ is a dimensionless parameter. Generally, the effective stress σ may increase with the development of damage, such that

$$\sigma = \frac{\sigma_0}{1 - D_1} \tag{3}$$

where σ_0 is the stress when $D_1 = 0$ and depends on the applied loading. Equations (1)–(3) are basic equations for the concerned problem.

For the sake of consistent analysis and calculation, the following normalized functions are defined

$$\overline{\sigma} = \frac{\sigma}{\sigma_0} \qquad \overline{t} = \frac{tA^*}{d}$$

$$\overline{a} = \frac{a}{d} \qquad \overline{A} = \frac{A}{A^*}$$

$$\overline{n}_N = \frac{n_N}{n_N^*} \qquad \overline{n} = \frac{n}{n^*} \qquad \overline{D}_1 = D_1$$

where d is the average ferrite grain diameter, A^* the characteristic crack growth rate, n_N^* the characteristic crack nucleation rate, and n^* the characteristic crack numerical density. Consequently, the basic equations are rewritten as

$$\frac{\partial \overline{n}}{\partial \overline{t}} + \frac{\partial(\overline{A} \cdot \overline{n})}{\partial \overline{a}} = N_g \cdot \overline{n}_N \tag{4}$$

$$\overline{D}_1 = \overline{\gamma}\int_0^\infty \overline{n}\cdot\overline{a}\cdot d\overline{a} \tag{5}$$

$$\overline{\sigma} = \frac{1}{1-\overline{D}_1} \tag{6}$$

with $\overline{\gamma}$ and N_g being dimensionless parameters as

$$\overline{\gamma} = \gamma\cdot n^*\cdot d^2 \qquad \text{and} \qquad N_g = \frac{d\cdot n_N^*}{A^*\cdot n^*}$$

N_g can be regarded as the ratio of crack nucleation quantity to total crack number within a given period of time. Moreover, the boundary and initial conditions can be shown as

$$\overline{n} = 0 \quad \text{at} \quad \overline{a} = 0 \tag{7}$$

and

$$\overline{n} = \overline{D}_1 = 0 \qquad \overline{\sigma} = 1 \qquad \text{when } \overline{t} = 0 \tag{8}$$

At this stage, crack growth rate A, and specific crack nucleation rate n_N are still needed for obtaining the solution to the problem. From the experimental observations, it is evident that short crack growth rate is inversely proportional to the crack length at the primary stage, which can practically be ascribed to the resistant effect of grain boundary, and such effect becomes less when a short crack extends across the first grain boundary. Besides, crack growth rate is related to the cyclic excursion and material intrinsic propensity. Taking into account the above considerations, we propose that

$$\overline{A} = \begin{cases} (1-\overline{a})(m\overline{\sigma}-1) & \text{for } \overline{a} \le 1 \\ \overline{a}\cdot(m\overline{\sigma}-1) & \text{for } \overline{a} > 1 \end{cases} \tag{9}$$

where $m = \sigma_0/\sigma_1^*$ with σ_1^* being the critical stress for crack extension. In the construction of the expression for crack nucleation rate, it is considered that the formation of a new crack is associated with energy activation process and with the cumulation of local plastic deformation and damage, which were previously considered by Curran et al.[8] and Bai et al. [9]. For the short crack damage situation, we assume

$$\overline{n}_N = \overline{a}^2\cdot(m\overline{\sigma}-1)\exp(-\overline{a}) \tag{10}$$

where $m = \sigma_0/\sigma_2^*$ with σ_2^* being the critical stress for crack nucleation.

IV. CALCULATION RESULTS AND DISCUSSION

Figure 3 is the calculation result of damage development, which shows that the damage cumulates gradually with time. In terms of $\dot{\overline{D}}$ vs $\overline{D}$ for the same result, one notices that the damage rate linearly increases with damage level (Fig.4). From Fig.4, we may write

$$\frac{\mathrm{d}\overline{D}_1}{\mathrm{d}\overline{t}} = \kappa\overline{D}_1 + \omega \tag{11}$$

with κ and ω being constants. Consequently, the damage evolution process can be described as

$$\overline{D}_1 = \frac{\omega}{\kappa}[\exp(\kappa\overline{t}) - 1] \tag{12}$$

Generally, κ and ω depend on test material and loading condition, and can be determined from relevant experiments. For the calculation result of Fig.4, one readily obtains $\kappa = 1.06$ and $\omega = 0.012$ from linear regression.

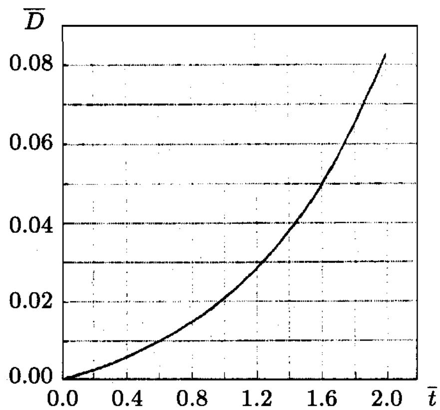

Fig.3 Damage development vs time with $N_g = 1.0$ and $m = 2.0$

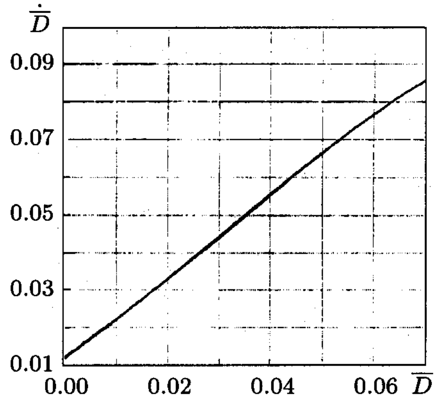

Fig.4 Relationship between $\dot{\overline{D}}$ and $\overline{D}$ with $N_g = 1.0$ and $m = 2.0$.

Figures 5 and 6 demonstrate the variation of short crack numerical density with crack length and loading cycles. In order to relate the calculation results to the experimental data, we convert the normalized parameters into physical variables

$$\begin{aligned} a = d &= 17.9\mu\text{m} \quad &&\text{for} \quad \bar{a} = 1 \\ n &= 1000\text{mm}^{-2} \quad &&\text{for} \quad \bar{n} = 1 \\ t &= 13,600\text{cycles} \quad &&\text{for} \quad \bar{t} = 1 \end{aligned}$$

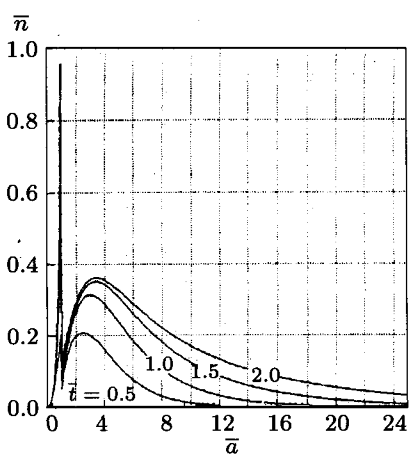

Fig.5 Crack density vs crack length with $N_g = 1.0$ and $m = 2.0$.

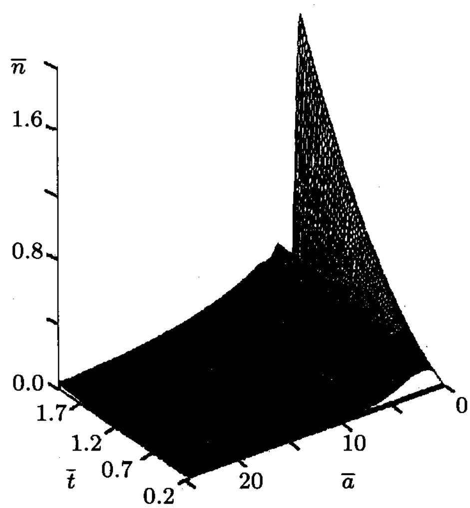

Fig.6 3-D construction of crack density evolution with $N_g = 1.0$, $m = 2.0$

For the t conversion ($\bar{t} = tA^*/d$), we take $A^* = 10^{-6}$mm/cycle. Referring to above correlations, one can discuss the physical implications of the calculation results. It is observed from Figs.5 and 6 that the number of short cracks increases with time, i.e. fatigue cycles, and it reaches the maximum value when normalized crack length approaches to the unity of the coordinate, i.e. at the location of average grain diameter. This reflects the obstacle effect of grain boundaries and the deceleration of short crack growth at the vicinity of the barriers. In addition to the large extreme value, there also exists a short peak on the crack density histogram located in relation to crack size ranging from 30 to 40μm. Comparing the appearance of these two peaks and the general outlook of the three-dimensional construction of Fig.6 with the experimental results of Fig.2, one finds an obvious similarity. This suggests that the present model and the considerations for the expressions of $\overline{A}$ and $\overline{n}_N$ are physically applicable. Note that the above results (Figs.3–6) were obtained by setting $N_g = 1.0$ and $m = 2.0$. In the following, the influences of N_g and m on the short crack damage evolution are respectively discussed.

First, the variation of N_g is examined and Figs.7–9 show the results for $N_g = 0.5$. The decrease in the value of N_g is associated with the reduction in numbers of nucleated short cracks. It is observed that the damage level reduces nearly to one half by comparing Fig.7 with Fig.3, and that the numerical density of short cracks accordingly drops upon referring Fig.9 to Fig.6. Despite of these changes, the damage rate and the damage extent are still linearly related (Fig.8). From the linear regression of the same formula of Eq.(11), we have κ=1.03 and $\omega = 0.006$. Note that the slop κ is almost independent of N_g, whereas the value of ω is considerably lowered, corresponding to the reduction in damage extent.

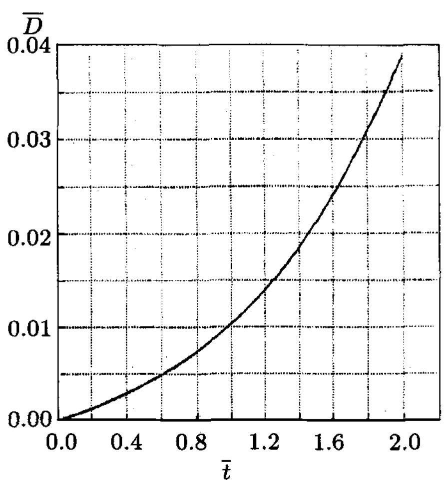

Fig.7 Damage development vs time with $N_g = 0.5$ and $m = 2.0$

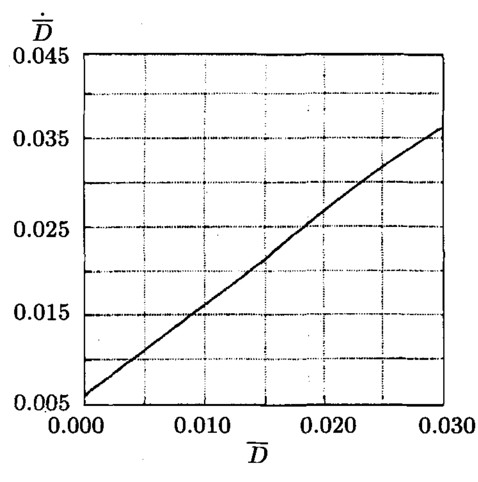

Fig.8 Relationship between $\dot{\overline{D}}$ and $\overline{D}$ with $N_g = 0.5$ and $m = 2.0$

Secondly, the change in the value of m is discussed, with the demonstration of Figs.10–12 for m=1.5. The value of m is the ratio of applied stress to the critical stress for short crack propagation. For the case of m=1.5, linear correlation again exists between the damage rate and the damage amount (Fig.12). By linear regression, we obtain: κ=0.56 and ω=0.006. Evidently, when reducing the applied stress, both values of κ and ω decreases markedly, implicating the drops in the damage rate and the damage level of short cracks (Figs.10,11).

In the results of numerical calculation, the maximum value of short crack density occurs when crack length is close to the size of the average ferrite grain diameter, whereas in the experimental data, the peak is located at the position slightly less than the average grain diameter. This discrepancy may be attributed to the constructed expression for short crack growth rate, in which the rate linearly decreases when a growing crack approaches to grain boundary. In practice, however, the resistant effect of grain boundary to crack growth varies nonlinearly with the distance between crack tip and the boundary. A further study will be carried out to examine both the nonlinear effect of grain boundary on short crack growth and the interesting phenomenon of the second peak in the crack numerical density distribution.

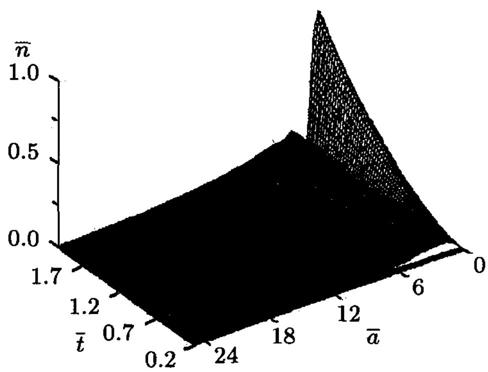

Fig.9 3-D construction of crack density evolution with $N_g = 0.5$, $m = 2.0$

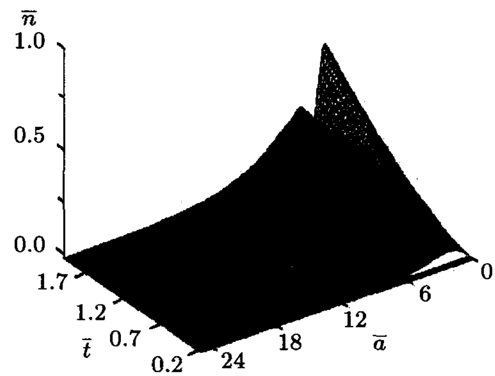

Fig.10 3-D construction of crack density evolution with $N_g = 1.0$, $m = 1.5$

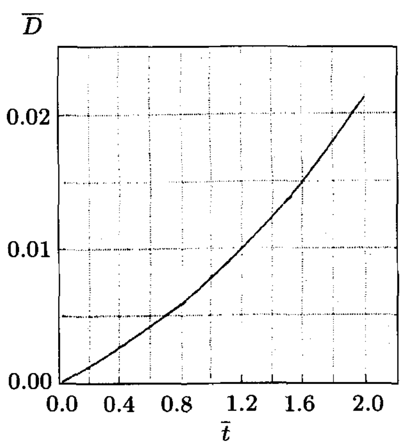

Fig.11 Damage development vs time with $N_g = 1.0$, $m = 1.5$

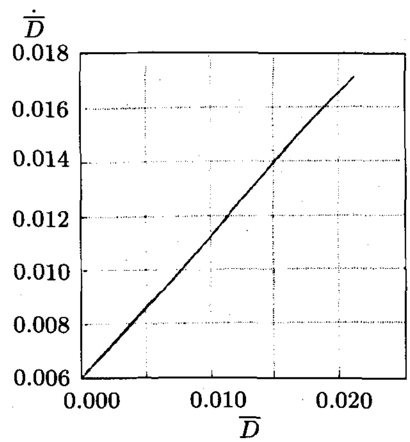

Fig.12 Relationship between $\dot{\bar{D}}$ and $\bar{D}$ with $N_g = 1.0$, $m = 1.5$

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