JOURNAL DE PHYSIQUE IV
Colloque C8, supplément au Journal de Physique III, Volume 4, septembre 1994

Influence of inertial and thermal effects on the dynamic growth of voids in porous ductile materials

J. Zheng, Y.L. Bai and Z.P. Wang

Institute of Mechanics, Chinese Academy of Science, China

ABSTRACT The influence of inertial, thermal and rate – sensitive effects on the void growth at high strain rate in a thermal – viscoplastic solid is investigated by means of a theoretical model presented in the present paper. Numerical analysis of the model suggests that inertial, thermal and rate – sensitive effects are three major factors which greatly influence the behavior of void growth in the high strain rate case. Comparison of the mathematical model proposed in the present work and Johnson's model shows that if the temperature – dependence is considered, material viscosity η can take the experimentally measured values.

1. INTRODUCTION

In ductile materials, spallation is a consequence of the nucleation, growth, and coalescence of voids. Meyers and Aimone[1] indicated that the dynamic growth of a void presents some additional complications: (a) The heat generated by plastic deformation cannot dissipate itself due to the high rate of deformation. (b) The inertial effect associated with the displacement of the material adjoining the void walls becomes an important consideration. (c) Wave interactions have a bearing on the final configuration. The spall process is found to be temperature sensitive. The purpose in the present paper is to develop a mathematical model of ductile void growth under high rate loading, which can represent the influence of the heating effect due to high speed deformation, the inertial effect, the material strain rate – dependence, the action of the applied external deviatoric stress, and the change of the surface energy of a void. In order to simplify theoretical analysis, we assume that the matrix material is incompressible during void growth. We also assume that the void remains spherical all the time. These assumptions lead to a great simplification of the theoretical analysis, so that we obtain the exact relations for the void growth.

2. DYNAMIC GROWTH OF A VOID

The problem analysed in this paper is shown diagrammatically in Fig. 1. We assume that the porous material consists of a suspension of pores in a matrix of homogeneous isotropic solid ductile material which is subjected to an external stress Σ_{ij}, and that the porous material is statistically homogeneous and isotropic so that it can be effectively modeled by homogeneous isotropic solid material. With these assumptions, we can study the void growth by considering a spheral hole of the matrix material of inner radius a and outer radius b. Distention α is defined as

$$\alpha = \frac{b^3}{b^3 - a^3} \tag{1}$$

We investigate the response of this hollow sphere to time – dependent external stress and zero internal pressure, and attempted to obtain the relation between an exteral stress Σ_{ij} and the distention $\alpha(t)$. It is expected that these relations will adequately describe the void growth for the effective homogeneous material.

Take the matrix material and void as a system, the work done by the external applied stress Σ_{ij} is equal to the change of the system energy:

$$W = \triangle E_k + \triangle E_s + \triangle E_i \tag{2}$$

where$\triangle E_k$, $\triangle E_s$ and $\triangle E_i$ denote the changes of the kinetic energy, the surface energy of a void, and the internal energy in the system respectively. W denotes the work done by the external stress Σ_{ij}. We neglect the initial elastic and elastic – plastic phases of the process, and begin to consider fully plastic deformation in the solid around the void[2]. A general spherically symmetric motion gives the following solutions

$$r^3 = r_0^3 - B(t), \qquad B(t) = a_0^3 \frac{\alpha_0 - \alpha}{\alpha - 1} \tag{3}$$

$$\frac{B(t)}{a_0^3} = \frac{\alpha_0 - \alpha}{\alpha - 1} \qquad \frac{B(t)}{b_0^3} = \frac{\alpha_0 - \alpha}{\alpha} \tag{4}$$

where r and r_0 denote the Eulerian and Lagrangian spherical polar coordinates, respectively. The subscript 0 used in this work denotes the initial value.

Consider $\triangle E_k$, $\triangle E_s$, $\triangle E_i$ and W respectively. $\triangle E_k$ is given by

$$\triangle E_k = E_k(\alpha) - E_k(\alpha_0) \tag{5}$$

with

$$E_k(\alpha) = \left[\frac{4\pi a_0^3}{9(\alpha_0 - 1)}\right] \frac{a_0^2 \rho_s}{2(\alpha_0 - 1)} \left(\frac{\alpha_0 - 1}{\alpha - 1}\right)^{1/3} \left[1 - \left[\frac{\alpha - 1}{\alpha}\right]^{1/3}\right] \dot{\alpha}^2 \tag{6}$$

where ρ_s is the density of the matrix material. We can also obtain the expression of $\triangle E_s$

$$\triangle E_s = E_s(\alpha) - E_s(\alpha_0) \tag{7}$$

with

$$E_s(\alpha) = \left[\frac{4\pi a_0^3}{9(\alpha_0 - 1)}\right] \frac{9(\alpha_0 - 1)^{1/3}}{a_0} (\alpha - 1)^{2/3} \tag{8}$$

where γ is the surface energy expended per unit area during the hole expansion.

To study the thermal effect on the dynamic growth of voids, a simple form of linear thermal softening is chosen in the rate – dependent constitutive relation in the matrix material[3]

$$\sigma_e = (Y_0 + \eta\dot{\varepsilon}_e^p)\left(1 - \frac{T}{T_m}\right) \qquad (T \leqslant T_m) \tag{9}$$

where T is temperature, T_m is the melting temperature of the matrix materials. σ_e and $\dot{\varepsilon}_e^p$ denote the equivalent stress and the equivalent plastic strain rate in the matrix material. Y_0 is the yield strength of the matrix material, η is the material viscosity. Since the condition considered here is a high strain rate, the heat generated by plastic deformation cannot dissipate itself, then the temperature approximately satisfies[4]

$$\frac{\partial T}{\partial t} = \frac{\kappa}{\rho_s C_v} \sigma_e \frac{\partial \varepsilon_e^p}{\partial t} \tag{10}$$

where κ is a constant which is taken as 0.9[5]. C_v is the heat capacity of the matrix material.

Since we assume a plastic deformation process with spherical symmetry, the equivalent plastic strain ε_e^p is given by Johnson and Mellor[6]

$$\varepsilon_e^p = 2ln\frac{r}{r_0} \tag{11}$$

The change of internal energy in the system is

$$\triangle E_i = \frac{1}{\rho_s}\int_a^b \left[\int_0^{\varepsilon_e^p} \sigma_e d\varepsilon^p\right] 4\pi\rho_s r^2 dr \tag{12}$$

With the help of Eqs. (3) – (4) and (11), Eq. (12) is given in the following form

$$\triangle E_i = \left[\frac{4\pi a_0^3}{9(\alpha_0 - 1)}\right][F_3(\alpha) + F_4(\alpha)\dot{\alpha}] \tag{13}$$

with

$$F_3(\alpha) = 2Y_0\left(1 - \frac{T_0}{T_m}\right)(\alpha - \alpha_0)\ln\frac{\alpha}{\alpha - 1} \tag{14}$$

$$F_4(\alpha) = \frac{2}{3}\left(1 - \frac{T_0}{T_m}\right)\left(1 + \frac{2}{3}\frac{Y_0\kappa}{T_m\rho_s C_v}\right)\eta\ln\frac{\alpha}{\alpha - 1} \tag{15}$$

Functions $F_3(\alpha)$ and $F_4(\alpha)$ denote the influence of the yield stress of the matrix material, and the material viscosity on the increment of internal energy. The work W done by the external stress is as follows

$$W = \frac{3}{4}\pi\frac{a_0^3}{\alpha_0 - 1}\int_{\alpha_0}^{\alpha}\left[\frac{2}{3}\Sigma_e(\alpha') - P(\alpha')\right]d\alpha' \tag{16}$$

where Σ_e and P denote the macroscopic equivalent and hydrostatic stresses. We assume Σ_e and P to be the functions of distention α, that is, $\Sigma_e = \Sigma_e(\alpha)$, $P = P(\alpha)$. This assumption was also adopted by Johnson[2] and Curran et al.[7]

Substitution of the expressions of $\triangle E_k$, $\triangle E_s$, ΔE_i and W into Eq.(2) gives

$$\dot{\alpha} = \frac{1}{2F_1(\alpha)}\left\{-F_4(\alpha) + \sqrt{[F_4(\alpha)]^2 - 4F_1(\alpha)F_5(\alpha)}\right\} \tag{17}$$

with

$$F_1 = \frac{\rho_s a_0^2}{2(\alpha_0 - 1)}\left(\frac{\alpha_0 - 1}{\alpha - 1}\right)^{1/3}\left[1 - \left(\frac{\alpha - 1}{\alpha}\right)^{1/3}\right] \tag{18}$$

$$F_5(\alpha) = F_2(\alpha) + F_3(\alpha) - 3\int_{\alpha_0}^{\alpha}\left[\frac{2}{3}\Sigma_e(\alpha') - P(\alpha')\right]d\alpha' - F_1(\alpha_0)\dot{\alpha}_0^2 - F_2(\alpha_0) \tag{19}$$

$$F_2(\alpha) = \frac{9(\alpha_0 - 1)^{1/3}\gamma}{a_0}(\alpha - 1)^{2/3} \tag{20}$$

Equation (17) is the relationship from which we obtain the rate – dependent response of the void growth with temperature – dependence under dynamic loading.

3. NUMERICAL ANALYSIS

A copper – like material, with $\rho = 8.92\text{g/cm}^3$, $\gamma = 9\times10^{-5}\text{GPa/cm}^2$, $T_m = 1356\text{K}$, $C_v = 385\text{J/kg.K}$, $a_0 = 0.0005\text{cm}$, $\alpha_0 = 1.0003$, is selected for numerical analysis of the model. We must state that the initial temperature T_0 is composed of two parts, namely, $T_0 = T_{01} + T_{02}$. T_{01} is the environment temperature (or preheating temperautre), and T_{02} is the temperature generated by high speed plastic deformation in the matrix material before the voids grow.

To simplify the analysis, the material is assumed to be subjected to a linearly external stress

$$\Sigma(\alpha) = \Sigma_e(\alpha) - P(\alpha) = \Sigma_0 + G(\alpha - \alpha_0) \tag{21}$$

where $\Sigma_0 = \Sigma(\alpha_0)$ and G is a constant. The material viscosity η is, in general, considered to be proportional to $1/\sqrt{\dot{\varepsilon}}$ [3,8], where $\dot{\varepsilon}$ is the strain rate. We may estimate that for $\dot{\varepsilon} \sim 10^3 s^{-1}$, $\eta \sim 1.0$GPa. μs, and for $\dot{\varepsilon} \sim 10^5 s_{-1}$, $\eta \sim 0.1$GPa. μs. A comparison of the relative magnitudes of the internal energy E_i (viscoplastic dissipation) and the void surface energy to the total work $E_{total} = E_k + E_s + E_i$ for different values of material viscosity η is illustrated in Fig. 2. The surface energy of a void is significant at initial stage of the void growth, but decreases quickly with increase of distention α. The inertial effect, as illustrated in Fig. 2, becomes dominant in high stain rate case (small material viscosity), with initial temperature $T_0 = 393K$. However, the viscoplastic effect mainly dominates the void growth in low strain rate case.

A comparison of the model with temperature – dependence proposed in this paper (Eq. (17)) and Johnson's model[2] is shown in Fig. 3 with initial temperature $T_0 = 373(K)$. Johnson's model is given by

$$\dot{\alpha} = -\frac{(\alpha_0 - 1)^{2/3}\alpha(\alpha - 1)^{1/3}}{\eta}\left(P + \frac{2}{3}\frac{Y_0}{\alpha}ln\frac{\alpha}{\alpha - 1}\right) \tag{22}$$

in which the inertial effect is neglected. The strain rate in the process of spallation, in general, is in the range of $10^3 s^{-1} \sim 10^5 s^{-1}$. We suppose that the strain rate is about $10^4 s^{-1}$, therefore $\eta = 0.5$GPa. μs is taken. Calculations in Fig. 3 show that in Johnson's model, $\eta = 0.002$GPa. μs must be taken, then, the rate of distention α is approximatively equal to the result computed by our model. Johnson himself also noticed this problem. He found that the "viscosity" of metals is usually found to be on the order of anywhere from 0.1 to 1.0 GPa. μs for conditions of homogeneous compressive deformation. But, in his numerical simulation of spallation in copper, he must take $\eta = 0.001$GPa. μs. Seaman et al.[9] also found that a value of η(~0.0075GPa. μs) slightly lower than expected was required for the NAG model to reproduce these data on copper, but did not speculate on possible sources of this low value. Numerical analysis of Johnson's model and ours suggests that the thermal effect generated by high speed deformation greatly influence the void growth, which must be considered in high strain rate case.

4. CONCLUSIONS

In the present work, a relation of dynamic ductile growth of voids, in which the thermal effect, the inertial effect, the material rate sensitivity, the effect of void surface energy as well as the action of deviatoric stress are considered, is presented. Theoretical analysis of the mathematical model presented in this paper suggests that the thermal effect and the inertial effect greatly influence on the behavior of the void growth at high strain rate. It also shows that the inertial effect is no longer significant (for $\dot{\varepsilon} < 10^3 s^{-1}$). In this case, the viscoplastic effect becomes dominant.

References

[1] Meyers M. A. and Aimone C. T., *Prog. Mater. Sci.* **28** (1983) 1–96.
[2] Johnson J. N., *J. Appl. Phys.* **52** (1981) 2812–2825.
[3] Cortes R., *Int. J. Solids Structures* **29** (1992) 1339–1350.
[4] Tvergaard V. and Needleman A., *Int. J. Fracture* **59** (1993) 53–67.

[5] Taylor G. J. and Quinney H., *Proc. R. Soc. London* **143A** (1934) 307 – 326.
[6] Johnson W. and Mellor P. B., *Engineering Plasticity* (Van Nostrand Reinhold. London) (1973).
[7] Curran D. R., Shockey D. A. and Seaman L., *J. Appl. Phys.* **44** (1973) 4025 – 4036.
[8] Chhabildas L. C. and Asay J. R., *J. Appl. Phys.* **50** (1979) 2749 – 2756.
[9] Seaman L., Barbee T. W. Jr. and Curran D. R., *Stanford Res. Inst. Tech. Report No. AFWL – TR – 71 – 156*, Dec. 1971 (unpublished).

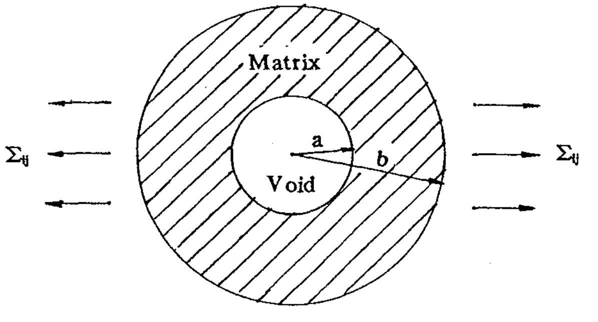

Fig. 1 A spherical element of material of radius b containing a single void of radius a.

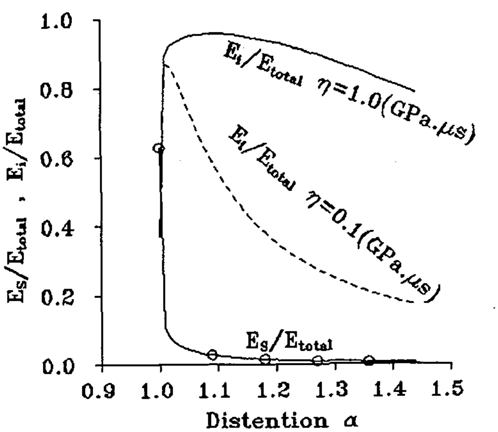

Fig. 2 Ratios of viscoplastic work (E_i) and surface energy of a void (E_s) to total work E_{total} from the void growth model with temperature – dependence for material viscosity $\eta = 0.1$ and 1.0 GPa·μs with initial temperature $T_0 = 373$K.

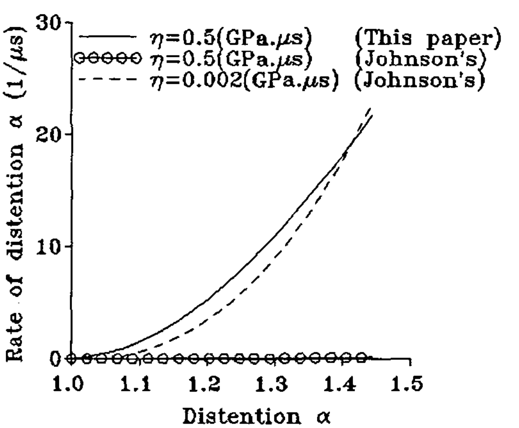

Fig. 3 A comparison of the rate of increase of distention $\dot{\alpha}$ vs. distention α from temperature dependent model (Eq. (17)) with Johnson's model (Eq. (22)) with initial temperature $T_0 = 373$K.