Theoretical Interpretation on Ion Heating Experiments in Reverse Field Pinch Devices

QIU Xiaoming (X. M. Qiu), WANG Aike
Southwestern Institute of Physics, P. O. Box 432, Chengdu 610041
XUE Minglun (M.L.Xue)
Institute of Mechanics, Academia Sinica, Beijing 100080

(Received 13 September 1993)

A new set of equations for the energies of the mean magnetic field and the mean plasma velocity is derived taking the dynamo effects into account, by which the anomalous phenomenon, $T_i > T_e$, observed in some reversed field pinches (RFP's) is successfully explained.

PACS: 52. 50. Gj, 52. 25. Gj, 52. 55. Ez

In the "ohmically" heated reversed field pinch (RFP), it is often observed that the loop voltage increases anomalously and the ion temperature T_i can exceed the electron temperature T_e, namely, $T_i > T_e$.[1-4] Since the only external heating is ohmic, the only classical ion energy input is from collisional equipartition with the electrons. This process will always cause $T_i < T_e$, by the second law of thermodynamics. Hence, another important ion heating mechanism must be present. Several specific scenarios have been proposed for revealing the ion heating mechanism (see, e. g., Refs. 5–7). The common view in Refs. 5 and 6 is that the dynamo fluctuations present in the RFP heat ions via classical collisional viscous damping. As pointed out in Ref. 7, however, because of small ion viscosity coefficient, such heating is so weak that it is not adequate to explain the high ion temperatures observed in the RFP's. Mattor *et al.*,[7] according to the dual cascade and the tearing mode theory, suggested that even if the viscosity of plasma is zero, the turbulent magnetic energy can be transferred directly to the ions at the scale where $\omega \approx \omega_{ci}$ (ion cyclotron frequency). In this letter, first of all, we point out that because in the RFP the mean kinematic motion of plasma, $\boldsymbol{U}$, is physically coupled with the turbulent motion-modes as well as the mean magnetic motion $\boldsymbol{B}$ so that the originally small $\boldsymbol{U}$ is most likely amplified by the turbulent motion or/and the mean magnetic motion, the viewpoint that in the RFP the mean kinematic motion of plasma is not very important (see, e. g., Ref. 8) is hardly justifiable. Secondly, from the mean magnetic induction equation, the mean motion equation, and the mean parts of Ohm's and Faraday's laws, without assuming specific instability mode, we utilize the turbulent dynamo model[8] to derive a set of equations for the energies of the mean magnetic field and the mean plasma velocity. In the two energy-equations χ and κ, reflecting the turbulent dynamo effects, are the turbulent counterparts of the classical collisional viscosity ν and resistivity η, respectively. Therefore, we call χ and κ the anomalous viscosity and resistivity.

In the following, we use the ensemble mean—, to divide a magneto-hydrodynamics (MHD) quantity into the mean and fluctuating parts:

$$\boldsymbol{u} = \boldsymbol{U} + \delta \boldsymbol{u}, \quad \boldsymbol{U} = \bar{\boldsymbol{u}} \quad , \tag{1}$$

$$\boldsymbol{b} = \boldsymbol{B} + \delta\boldsymbol{b}, \quad \boldsymbol{B} = \bar{\boldsymbol{b}} \quad , \tag{2}$$

$$p = P + \delta p, \quad P = \bar{p} \quad , \tag{3}$$

Then, the mean magnetic induction equation, the mean motion equation, and the mean parts of Ohm's and Faraday's laws can be written as

$$\partial \boldsymbol{B}/\partial t = \nabla \times (\boldsymbol{U} \times \boldsymbol{B}) + \nabla \times \boldsymbol{F}_{\mathrm{emf}} + \lambda \Delta \boldsymbol{B} \quad , \tag{4}$$

$$\partial \boldsymbol{U}/\partial t = -\boldsymbol{U} \cdot \nabla \boldsymbol{U} + \mu \boldsymbol{J} \times \boldsymbol{B} + \nabla \cdot \boldsymbol{R}_v + \nu \Delta \boldsymbol{U} - (1/2)\overline{(\nabla \delta \boldsymbol{b})^2} - \nabla P \quad , \tag{5}$$

$$\lambda \nabla \times \boldsymbol{B} = \boldsymbol{E} + \boldsymbol{U} \times \boldsymbol{B} + \boldsymbol{F}_{\mathrm{emf}} \quad , \tag{6}$$

$$\mu \boldsymbol{J} = \nabla \times \boldsymbol{B} \tag{7}$$

with

$$\nabla \cdot \boldsymbol{B} = 0 \text{ and } \nabla \cdot \boldsymbol{U} = 0 \quad . \tag{8}$$

Here, $\boldsymbol{B}$ is the mean magnetic field measured in Alfvén velocity units; it is defined as the original magnetic field divided by $(\rho\mu)^{1/2}$ (ρ is the uniform plasma density and μ is the magnetic permeability). $\boldsymbol{U}$ is the mean plasma velocity, P is the mean pressure divided by ρ, $\boldsymbol{J}$ and $\boldsymbol{E}$ are the mean current density and electric field, respectively, λ is the magnetic diffusivity defined by $(\sigma\mu)^{-1}$ (σ is the electric conductivity), and ν is the kinematic plasma viscosity, $\boldsymbol{F}_{\mathrm{emf}}$ is the turbulent electromotive force, $\overline{\delta\boldsymbol{u} \times \delta\boldsymbol{b}}$, and $\boldsymbol{R}_v$ is the MHD stress defined as[8]

$$\boldsymbol{R}_v = -\overline{(\delta\boldsymbol{u}\delta\boldsymbol{u} - \delta\boldsymbol{b}\delta\boldsymbol{b})}$$

and for which we make use of the result[8] from a two-scale direct interaction approximation (TSDIA)

$$\boldsymbol{R}_v = \chi(\nabla \boldsymbol{U} + \nabla \boldsymbol{U}^+) \tag{9}$$

($\nabla \boldsymbol{U}^+$ is the transposed of tensor $\nabla \boldsymbol{U}$), where

$$\chi = C_v(K^2/\varepsilon) \quad , \tag{10}$$

$$K = (1/2)[\overline{(\delta\boldsymbol{b})^2} + \overline{(\delta\boldsymbol{u})^2}] \quad , \tag{11}$$

$$\varepsilon = \lambda\overline{\left(\frac{\partial \delta b^a}{\partial x^b}\right)\left(\frac{\partial \delta b^a}{\partial x^b}\right)} + \nu\overline{\left(\frac{\partial \delta u^a}{\partial x^b}\right)\left(\frac{\partial \delta u^a}{\partial x^b}\right)} \tag{12}$$

(a, b=1, 2, 3, the summation convention is adopted for the repeated superscripts) with

$$C_v \approx 0.1 \quad ,$$

χ is the turbulent viscosity. Here, the turbulent viscosity present in the earth's magnetic dynamo is introduced into the RFP. Concerning the turbulent electromotive force $\boldsymbol{F}_{\mathrm{emf}}$, several theoretical models have been put forward.[8-12] Here, we take the turbulent dynamo model in Ref. 8; namely, such a dynamo model gives

$$\boldsymbol{F}_{\text{emf}} = a_H \boldsymbol{B} - \beta \nabla \times \boldsymbol{B} \quad , \tag{13}$$

where

$$a_H = C_H (K/\varepsilon) h \quad ,$$
$$\beta = C_\beta (K^2/\varepsilon) \quad ,$$
$$h = \overline{\delta \boldsymbol{b} \cdot (\nabla \times \delta \boldsymbol{b})} - \overline{\delta \boldsymbol{u} \cdot (\nabla \times \delta \boldsymbol{u})}$$

with

$$C_\beta \approx 0.2 \quad .$$

Above, C_v, C_H and C_β and model constants, β is the turbulent magnetic diffusivity. Because the RFP magnetic configuration is near minimum energy state to which the plasma spontaneously relaxes,[14] in Eq. (13) we omitted the term reflecting the inhomogeneity of fluctuations.

From Eqs. (4)–(8) and (13), we have

$$\frac{\partial \langle (1/2) \boldsymbol{B}^2 \rangle}{\partial t} = I V_{\text{L}} - \Gamma + \Gamma_1 - \Gamma_2 \quad , \tag{14}$$

$$\frac{\partial \langle (1/2) \boldsymbol{U}^2 \rangle}{\partial t} = \Gamma - \Gamma_3 + \langle \nabla \cdot \boldsymbol{D} \rangle \quad . \tag{15}$$

where

$$\Gamma = \langle (\nabla \times \boldsymbol{B}) \times \boldsymbol{B} \cdot \boldsymbol{U} \rangle \quad ,$$
$$\Gamma_1 = \langle a_H \boldsymbol{B} \cdot (\nabla \times \boldsymbol{B}) \rangle \quad ,$$
$$\Gamma_2 = \langle \mu (\kappa + \eta) \boldsymbol{J}^2 \rangle \text{ with } \kappa \equiv \mu \beta \text{ and } \eta \equiv \mu \lambda \quad ,$$
$$\Gamma_3 = \langle (\chi + \nu) \nabla \boldsymbol{U} : \nabla \boldsymbol{U} \rangle \quad ,$$
$$\boldsymbol{D} = \boldsymbol{R}_v \cdot \boldsymbol{U} + (1/2) \nu \nabla \boldsymbol{U}^2 - (1/2) \boldsymbol{U}^2 \boldsymbol{U} - P \boldsymbol{U} - (1/2) \overline{(\delta \boldsymbol{b})^2 \boldsymbol{U}} \quad .$$

Here, V_{L} is the toroidal loop voltage, I is the toroidal current, and $\langle \ \rangle$ represents a volume integral. Equations (14) and (15) exactly are the equations for the above-mentioned energies of the mean magnetic field and the mean plasma velocity. This set of equations is an important new result obtained in the present letter; it is applicable for analyzing lots of nonlinear phenomena occurred in the RFP's. We are now mainly interested in explaining the anomalous phenomenon, $T_{\text{i}} > T_{\text{e}}$. The physical meaning of each term in Eqs. (14) and (15) is easily clarified. At this point we would like to go into $\Gamma, \Gamma_1, \Gamma_2$ and Γ_3 in particular. Γ expresses the mean magnetic energy transfered to the mean kinetic energy. Γ_1 is the fluctuating field energy cascaded back to the mean magnetic field.[7] Γ_2 is, because of the presence of classical collisional resistivity η, the resistive dissipation which consists of the turbulent (κ) and the Spitzer's (η). Analogously, Γ_3 is the viscous dissipation which includes the anomalous (χ) and the classical (ν). As well known, the resistive dissipation mainly makes the electrons heat, whereas the viscous one largely renders the ions hot. We employ T_{e}^{η} and T_{e}^{κ} to denote the electron temperatures associated with η- and κ-dissipations, respectively. Similarly, we have T_{i}^{ν} and T_{i}^{χ}. Invoking dimensional arguments from MHD turbulence theory,[13] we easily estimated that $\kappa \sim \eta$ and $\chi \gg \nu$ in dynamo problem in the RFP's, showing that $T_e^\kappa \sim T_e^\eta$ and $T_i^\chi \gg T_i^\nu$. This, because T_i^ν is not much less than T_e^η, implies that

$$T_i(\equiv T_i^\chi + T_i^\nu) > T_e(\equiv T_e^\kappa + T_e^\eta) \quad .$$

We are discussing other applications of the set of Eqs. (14) and (15). The results will be published in coming papers.

REFERENCES

[1] E. Scime, S. Hokin, N. Mattor and C. Watts, Phys. Rev. Lett. 68 (1992) 2165.

[2] S. Ortolanir, in *Proceedings of International Conference on Plasma Physics*, Kiev, 1987, edited by A. G. Sitenko, (World Siencific Publishing, Singapore, 1987) Vol. 2, p. 802.

[3] T. Fujita, K. Saito, J. Matusi, Y. Kamada, H. Morimoto, Z. Yoshida and N. Lnoue, Nucl. Fusion, 31 (1991) 3.

[4] A. Fujisawa, H. Ji, K. Yamagishi, S. Shinobara, H. Toyama and K. Miyamoto, Nucl. Fusion, 31 (1991) 1443.

[5] C. G. Gimblett, Europhys. Lett. 11 (1990) 541.

[6] A. Lazaros, Plasma Phys. Controlled Fusion, 31 (1989) 1995.

[7] N. Mattor, P. W. Terry and S. C. Prager, Comments Plasma Phys. Controlled Fusion, 15 (1992) 65.

[8] A. Yoshizawa and F. Hamba, Phys. Fluids, 31 (1988) 2276.

[9] C. G. Gimblett and M. L. Watkins, in *Proceedings of the 7th European Conference on Controlled Fusion and Plasma Physics*, Lausanne, 1975 (European Physical Society, Geneva, 1975) Vol. 1, p. 103.

[10] R. K. Keinigs, Phys. Fluids, 26 (1983) 2558.

[11] D. Montgomery and H. Chen, Plasma Phys. Controlled Fusion, 26 (1984) 1199.

[12] D. Montgomery and H. Chen, Plasma Phys. Controlled Fusion, 29 (1987) 205.

[13] R. H. Kraichnan, Phys. Fluids, 8 (1965) 1385.

[14] J. B. Taylor, Phys. Rev. Lett. 33 (1974) 1139.