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IMPACT RESPONSE OF A FINITE INTERFACE CRACK WITH ADHESIVE TIPS

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Of considerable importance in structural analysis is the transient response of a flaw to a time dependent stress field. A number of papers on the area of dynamic crack analysis have been reviewed in [1]. The impact response of a finite crack in plane extension has been considered in a paper by Sih and Embley [2]. Impact response of the interface crack is, however, a very complex problem due to the strong discontinuity of the material constants. The general elasticity solutions, that have been worked out for such a crack, involve oscillatory singularities which lead to wrinkling of the crack and overlapping of the materials [3]. These unreasonable phenomena have bothered scholars and research workers for a long time. In 1977, Comninou [4] proposed a frictionless contact model and solved the relevant static problem. The size of the contact region that is worked out according to the model is, however, too small to be acceptable to the assumption of continuum mechanics. In 1978, J.D. Achenbach [5] proposed another model which assumed that the crack faces are in adhesive contact near the tips. He solved the static problem and obtained some excellent results. The scattering of steady elastic waves by this kind of crack has been considered in a paper by Zhou Zhengong [6], and Rice [7] has further considered static contacting problems of the interface crack.

This paper presents the impact response of an interface crack. It is assumed that two dissimilar half planes are bonded together perfectly except in a small region occupied by an interface crack with adhesive contact tips. The elastic constants of the two half planes are ρ_0, μ_0, λ_0 and ρ_1, μ_1, λ_1, respectively, as shown in Fig. 1.

The governing equations are

$$\nabla^2\phi_j^0 - \frac{1}{C_{Lj}^2}\frac{\partial^2\phi_j^0}{\partial t^2} = 0 \tag{1}$$

$$\nabla^2\psi_j^0 - \frac{1}{C_{Tj}^2}\frac{\partial^2\psi_j^0}{\partial t^2} = 0 \tag{2}$$

where

$C_{Lj}^2 = (\lambda_j + 2\mu_j)/\rho_j, C_{TJ}^2 = \mu_j/\rho_j (j = 0,1)$ and ψ_j^0, ψ_j^0

are potential functions.

According to what is described above, the boundary and continuous conditions can be written as:

$$|x| \le a, \quad y = 0, \quad \sigma_y^1 = \sigma_y^0 = -\sigma_0 H(t), \quad \sigma_{xy}^1 = \sigma_{xy}^0 \tag{3}$$

$$a \le |x| \le 1, \quad y = 0, \quad \sigma_y^1 = \sigma_y^0 = T(t)\left(\frac{|x| - a}{1 - a}\right)^2 H(t) - \sigma_0 H(t) \tag{4}$$

$$\frac{\partial}{\partial x}(u_x^1 - u_x^0) = -A(t)(1 - x^2)^{\frac{1}{2}} \tag{5}$$

$$|x| > 1, \quad y = 0, \quad u_y^1 = u_y^0, \quad u_x^1 = u_x^0, \quad \sigma_y^1 = \sigma_y^0, \quad \sigma_{xy}^1 = \sigma_{xy}^0 \tag{6}$$

where a, A and T are to be determined, H(t) is step function.

To solve the problem, Fourier transform and Laplace transform are used to reduce the problem to a set of singular integral equations

$$\begin{aligned} &h_1\phi_1(x,t) + h_2\int_{-1}^{1}\frac{\phi_2(u,t)}{u - x}du + \int_0^t\{\int_{-1}^{1}\phi_1(u,\tau)\bar{n}_1(u,x,t-\tau)du \\ &+ \int_{-1}^{1}\phi_2(u,\tau)\bar{n}_2(u,x,t-\tau)du\}d\tau = P(x,t) \quad (|x| \le 1) \end{aligned} \tag{7}$$

$$\phi_1(x,t) = m_1(t)(1 - x^2)^{\frac{1}{2}} \qquad (a \le |x| \le 1) \tag{8}$$

$$\begin{aligned} &h_3\phi_2(x,t) + h_4\int_{-1}^{1}\frac{\phi_1(u,t)}{u - x}du + \int_0^t\{\int_{-1}^{1}\phi_1(u,\tau)\bar{n}_3(u,x,t-\tau)du \\ &+ \int_{-1}^{1}\phi_2(u,\tau)\bar{n}_4(u,x,t-\tau)du\}d\tau = 0 \quad (|x| \le a) \end{aligned} \tag{9}$$

where

$$h_1 = 2\pi d_i i,\quad h_2 = 2id_2,\quad m_1 = -Ai,\quad h_3 = -2\pi i d_1 \text{ and } h_4 = 2id_2$$

$$\rho(x,t) = 2\pi\{T\left(\frac{|x|-a}{1-a}\right)^2 [H(|x|-a)-H(|x|-1)]H(t)-\sigma_0 H(t)$$

d_1 and d_2 are two constants. ϕ_1 and ϕ_2 are the dislocation density functions.

The singular integral equations given above are solved numerically by making use of Chebyshev polynomial expansion of the dislocation density function. Using the orthogonality conditions of Chebyshev polynomials, a set of algebraic equations are obtained. These equations are linear for coefficients of Chebyshev polynomials, the shear dislocation density coefficients A, and the adhesive stress coefficients T, but nonlinear for the unknown length a.

In numerical solution to the above equations, the inversion of the Fourier and the Laplace transforms as in the following form is very complex and difficult.

$$\bar{n}_1(u,x,t) = L^{-1}\int_{-\infty}^{\infty}(D_1(s,p)-id_1)e^{is(u-x)}ds.$$

It can be changed into the following form by using the uniform function $D_1(s,p)$, the transforms $s=p\eta$ and $t = \eta|u-x|$

$$\bar{n}_1(u,x,t) = \frac{i}{|x-u|}[g'_t(x,u,t)+g(x,u,0)\delta(t)]$$

where

$$g(x,u,t) = \theta_1^+\left(\frac{it}{|x-u|}\right) - \theta_1^-\left(\frac{it}{|x-u|}\right),\quad \theta_1(\eta) = D_1(\eta,1)-id_1$$

θ^+_1 and θ^-_1 represent the expression of θ_1 in the first and the fourth quadrant respectively, the other inversion of the Fourier and the Laplace transforms can be obtained similarly.

By solving the relevant algebraic equations, the coefficient of the stress and the length a of the adherent region of the crack faces can be obtained simultaneously. From the numerical solution, the following conclusion can be drawn.

a: When the wave arrives at a point, the normal stress of this point increases to a value at once, and then it decreases. As time goes on the normal stress of this point appears at several peak values. However, the back peak value is lower than the front one.

b: When the inmpact load acts on the crack face, the adhesive region decreases to about 5 percent of the length of the crack at once, and then it increases rapidly. Then it becomes smooth.

As an example, numerical results for a special material combination is given here. The material constants are as follows:

$$\lambda_1 = 98 \times 10^9 (N/m^2), \quad \mu_1 = 77 \times 10^9 (N/m^2), \quad \rho_1 = 7.7 \times 10^3 (kg/m^3)$$

$$\lambda_0 = 41.4 \times 10^9 (N/m^2), \quad \mu_0 = 41.4 \times 10^9 (N/m^2), \quad \rho_1 = 7.1 \times 10^3 (kg/m^3)$$

The results of stress in the interface cracks tips and the length of adherent region of the crack face are shown in Figs. 2 and Fig. 3, respectively.

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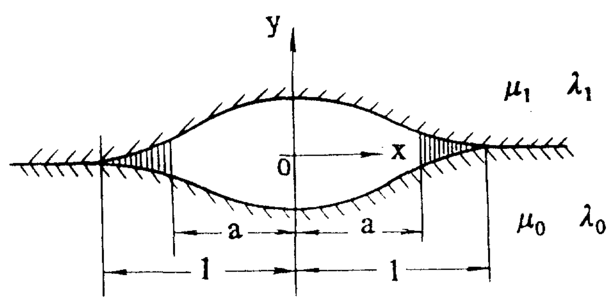

Figure1.

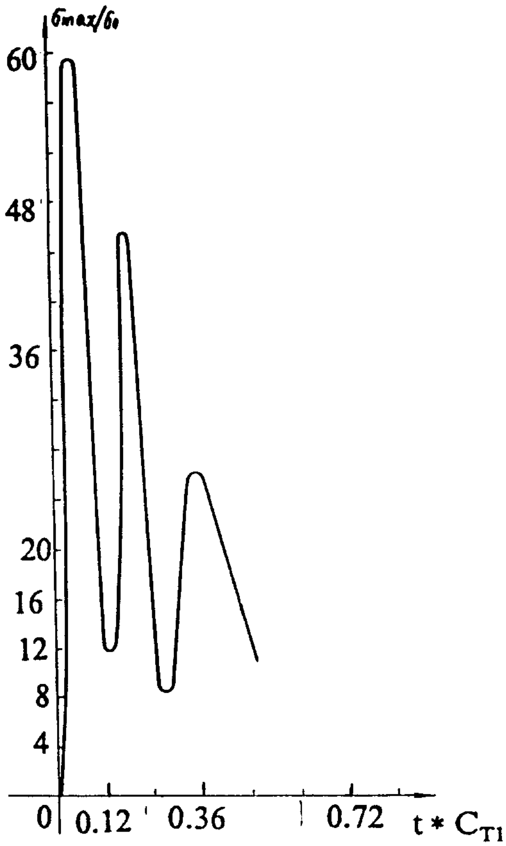

Figure 2.

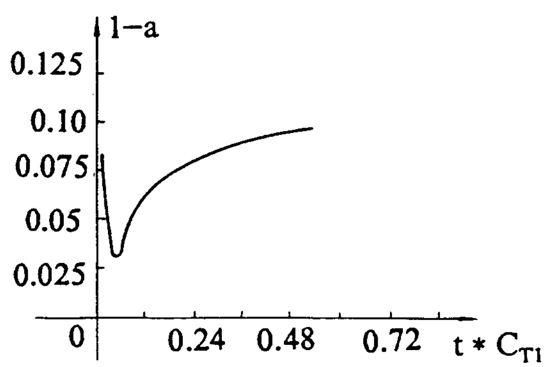

Figure 3.