Ablation of hydrogen isotopic spherical pellet due to the impact of energetic ions of the respective isotopes

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The ablation rate of a hydrogen isotopic spherical pellet G_{is} due to the impact of energetic ions of the respective isotopes and its scaling law are obtained using the transsonic neutral-shielding model, where subscript s might refer to either hydrogen or deuterium. Numerical results show that if $E_{0s}/E_{0e}^2 \geqslant 1.5$, $G_{is}/G_{es} \geqslant 20\%$, where E_{0s} and E_{0e} are the energy of undisturbed ion and electron, respectively, and G_{es} is the ablation rate of a pellet due to the impact of electrons. Hence, under the NBI heating, the effect of the impact of energetic ions on the pellet ablation should be taken into consideration. This result also gives an explanation of the observed enhancement of pellet ablation during NBIH.

I. INTRODUCTION

The injection of a hydrogen-isotope solid-target pellet into the fusion reactor is commonly recognized as one method of adding reactor fuel.[1] In addition, the target pellet injection can also be considered as a diagnostic tool to measure the various parameters of the plasma, to determine the transport coefficients for the main elements and impurities in the plasma,[2] and to improve the macroscopic stability of the plasma.[3] Therefore, scientists in many countries have in recent years performed many extensive theoretical and experimental studies of the ablation of the target pellets in the plasma and have tried to determine the speed of injected target needed for the experiments.

When a target pellet is injected into the plasma, its surface is bombarded by neutrons, alpha particles, ions, and electrons, etc. This causes surface heating and ablation. The ablated material forms a dense layer surrounding the target pellet. The shielded target can also be bombarded by high energy particles. In the past, it has been recognized that the ablation of the target comes mainly from the electrons in the plasma.[1] However, experiments in recent years have shown that in many annular fusion reactor setups, when neutral particles were used for target heating, the ablations of the target pellets were increased. The same experiments also revealed that the energy exhibited by the plasma was far higher than the high-energy ions or the electrons. The results are shown in Table I. The reason for this occurrence could be due to the

TABLE I. The ion and electron energy data in some fusion experimental setups.[4]

Name of setup	PDX	TFTR
neutral-particle, MW	1.1	33.5
high-energy ion energy, keV $D_2 \to D^+$	50	120
electron energy at center of plasma, keV.	2.2	8.5

bombardment, which strengthened the ablation of the target pellets. Therefore, the effects of the target bombardment by high energy ions on the ablation of the target pellets need to be studied further.

Since the deuterium-deuterium reaction is one of the more interesting practical fusion reactions besides performing the calculations on the hydrogen target pellet ablation coefficient under bombardment by high energy ions and determining the laws for the target ablation, this paper also calculates the ablation for deuterium target pellets and makes comparisons with the ablation by bombardment with electrons.

II. EQUATIONS AND BOUNDARY CONDITIONS

From Ref. 5 it is known that the dissociation and the ionization of the ablation gas have very little effect on the ablation rate of a target pellet. Therefore, we use the transsonic neutral gas shielding model of Ref. 5. The ablation gas current can then be described using the gas state equations and the conservation equation as follows:

$$p=\frac{\rho}{m}T, \tag{1}$$

$$\frac{G}{4\pi}\frac{\mathrm{d}}{\mathrm{d}r}(r^2\rho u)=0, \tag{2}$$

$$\rho u\frac{\mathrm{d}u}{\mathrm{d}r}+\frac{\mathrm{d}p}{\mathrm{d}r}=0, \tag{3}$$

$$\rho u\frac{\mathrm{d}}{\mathrm{d}r}\left[\frac{\gamma T}{(\gamma-1)m}+\frac{u^2}{2}\right]=Q, \tag{4}$$

where p, ρ, T, and u are the pressure, density, temperature, and velocity of the ablation gas, respectively; γ and m are the specific heat ratio and molecular mass of the ablation gas, respectively; r is the radial distance from the origin, which is taken to be the center of the target pellet. The target pellet ablation rate is $G=4\pi r^2\rho u=$ constant. Q is the volume energy source, which is supplied by the injected high-energy ions; that is

$$Q=\frac{\mathrm{d}q}{\mathrm{d}r}, \tag{5}$$

q in the equation denotes the energy current of the injected ions.

The energy current q of the ions is related to its energy by the following equation

$$q=\frac{n_i v_i}{4}E. \tag{6}$$

Here we also have

$$v_i=\left(\frac{8T_i}{\pi m_i}\right)^{1/2}, \tag{7}$$

$$E=2T_i, \tag{8}$$

where n_i, v_i, m_i, and T_i are the number density, velocity, mass, and temperature of the ions, respectively. The subscript "i" denotes ion.

When the ions interact with the ablation gas current, the equations for E and q are respectively given below:

$$\frac{\mathrm{d}E}{\mathrm{d}r}=\frac{\rho L_m(E)}{m\langle\cos\theta\rangle}, \tag{9}$$

$$\frac{\mathrm{d}q}{\mathrm{d}r}=\frac{\rho}{m}\Lambda(E)q, \tag{10}$$

where $\langle\cos\theta\rangle$ is the average value of the cosine of the angle between the injected ion beam direction and the magnetic field direction. When the ion beam is isotropic, we have

$$\langle\cos\theta\rangle \approx \frac{1}{2}. \tag{11}$$

The ion-energy current-attenuation total cross section $\Lambda(E)$ is determined by the energy loss function $L_m(E)$ and the total elastic backscattering cross section $\sigma(E)$ of the ions

$$\Lambda(E) = \frac{2L_m(E)}{E} + \sigma(E). \tag{12}$$

In this way, Eqs. (1)–(5) and Eqs. (9) and (10) combined to form a set of ablation equations that can be used to describe the ablation of hydrogen isotope spherical target pellets under the bombardment by the high energy ions. In this paper, while the energy is given in eV units, all other quantities are expressed in international units.

In order to solve the above set of equations, it is necessary to know $L_m(E)$ and $\sigma(E)$. Generally speaking, these functions depend on the elements. Therefore, the ablation equations are expected to be different because the forms of Eqs. (9) and (10) are different. However, from the Appendix, it is known that when the hydrogen or deuterium gas molecules collide with the corresponding ions, $\sigma_s(E)$ can be taken to be zero and their attenuation functions are, given below as

$$L_{mH}(E) = \begin{cases} 0.9424\times10^{-20}E^{0.5} \text{ eV}\cdot\text{m}^2, & (E\geqslant 500\text{eV}) \\ 2.107\times10^{-19} \text{ eV}\cdot\text{m}^2, & (E<500\text{eV}) \end{cases} \tag{13}$$

$$L_{mD}(E) = \begin{cases} 0.1187\times10^{-19}E^{0.5} \text{ eV}\cdot\text{m}^2, & (E\geqslant 500\text{eV}) \\ 2.6548\times10^{-19} \text{ eV}\cdot\text{m}^2. & (E<500\text{eV}) \end{cases} \tag{14}$$

respectively.

It can be seen that as far as hydrogen and deuterium are concerned, $L_{ms}(E)$ and $\Lambda_s(E)$ differ only by a constant factor, that is

$$\frac{L_{mD}(E)}{L_{mH}(E)} = \frac{\Lambda_D(E)}{\Lambda_H(E)} = 1.26. \tag{15}$$

In this way, their ablations can also be described by the same set of equations. In the discussion to follow, the subscript s can be either H or D, which denotes hydrogen or deuterium, respectively.

Since the energy of the injected ions is very high, the majority of the energy of the ions is deposited within the distance of the acoustic radius $r=r_*$. Therefore, it can be assumed that the energy of the hydrogen isotope in this location is $E_{*s}\geqslant 500$ eV. The subscript in this paper denotes the ablation gas current and the transsonic parameter of the ablation gas current. After the calculations and the normalization of the set of equations to obtain dimensionless quantities for the corresponding quantities at $r=r_*$, we next obtain the dimensionless set of equations that can be used to describe the hydrogen-isotope spherical-target pellet ablation

$$\frac{d\bar{\rho}_s}{dr} = \frac{2\bar{\rho}_s^3\bar{r}^4\bar{T}_s}{1-\bar{\rho}_s^2\bar{r}^4\bar{T}_s}\left(\frac{\bar{q}_s\bar{\Lambda}_s\bar{\rho}_s\bar{r}^2}{\bar{T}} - \frac{1}{\bar{r}}\right) - \frac{2\bar{\rho}_s}{\bar{r}}, \tag{16}$$

$$\frac{d\bar{T}_s}{d\bar{r}} = 2\bar{q}_s\bar{\Lambda}_s\bar{\rho}_s\bar{r}^2 + \frac{(\gamma-1)}{\bar{\rho}_s^2\bar{r}^4}\left(\frac{2}{\bar{r}} + \frac{1}{\bar{\rho}_s}\frac{d\bar{\rho}_s}{dr}\right), \tag{17}$$

$$\frac{d\bar{q}_s}{d\bar{r}} = \lambda_*\bar{q}_s\bar{\Lambda}_s\bar{\rho}_s, \tag{18}$$

$$\bar{E}_s = \bar{q}_s, \tag{19}$$

$$\bar{\Lambda}_s = \begin{cases} \bar{q}_s^{-0.5}, & (E_s \geqslant 500\text{eV}) \\ 0.2236 \times 10^2 \bar{q}_s^{-1} E_{*s}^{-0.5}. & (E_s < 500\text{eV}) \end{cases} \quad (20)$$

The " – " symbol above letters denotes dimensionless quantities. For simplicity, unless it is absolutely necessary, the subscript "s" will be dropped from here on.

The dimensionless parameter λ_* is

$$\lambda_* = \frac{2}{\gamma - 1} \frac{\rho_* u_*^3}{q_*}, \quad (21)$$

or

$$\lambda_* = \frac{\rho_* \Lambda_* r_*}{m}. \quad (21a)$$

At the target-pellet surface $\bar{r} = \bar{r}_p$, energy-conservation requirement leads to

$$q_v = \rho_v u_v h_v \left(1 + \frac{\gamma - 1}{2} M_v^2 + \frac{\varepsilon}{h_v} \right), \quad (22)$$

where M and h are the Mach number and enthalpy of the ablation gas, respectively; the subscript "v" denotes the value of the parameter for the ablation gas current and ions at the target-pellet surface.

The surface process during the target-pellet ablation lacks experimental observation; however, based on the temperature and density relation of the hydrogen isotopes, the surface temperature is at least equal to the corresponding critical temperature. Therefore, we let

$$T_{vH} = 33\text{K}, \quad T_{vD} = 38\text{K}. \quad (23)$$

Hence, the corresponding energies ε_s needed for sublimation are:

$$\varepsilon_H = 0.475 \times 10^6 \text{J/kg}, \quad (24a)$$
$$\varepsilon_D = 0.760 \times 10^6 \text{J/kg}. \quad (24b)$$

Most experiments have shown that the ablation gas current can effectively shield the target pellets and as a result, we have

$$\bar{q}_v \to 0, \quad M_v^2 \approx 0. \quad (25)$$

Considering Eqs. (24a), (24b), and (25), the dimensionless equation obtained from Eq. (22) is

$$\frac{\bar{T}_v}{\bar{q}_v} = \frac{\bar{r}_p^2}{\lambda_*}. \quad (26)$$

We further analyze the ablation-gas current energy equation and it is not difficult to obtain

$$\frac{\bar{T}_v}{\bar{q}_v} < 1. \quad (27)$$

Finally, the boundary conditions at the target pellet surface at $\bar{r} = \bar{r}_p$ can be written

$$\bar{q}_v \to 0, \quad \frac{\bar{T}_v}{\bar{q}_v} < 1. \quad (28)$$

Downstream from the ablation gas current, that is, at the location where the injected ions have not been disturbed, the current motion tends toward a subsonic state.[6] Here the boundary conditions for $\bar{r} \to \infty$, can be written as

$$\bar{q} \to \hat{q}, \quad \bar{E} \to \hat{E}, \quad (29)$$

$$\hat{M}=\left(\frac{5}{\gamma}\right)^{1/2},$$

$$\hat{T}=\frac{\gamma}{5}\left(\frac{30\hat{\Lambda}\bar{q}}{7\gamma-5}\right)^{2/3}\bar{r}^{2/3},$$

$$\hat{\rho}=\left(\frac{30\hat{\Lambda}\bar{q}}{7\gamma-5}\right)^{-2/3}\hat{r}^{-7/3},$$

where the "∧" above some symbols denotes the downstream subsonic state value of the ablation gas current and ions.

III. EQUATIONS AND THE ABLATION LAWS

From the preceding sections it is obvious that the target-pellet ablation problem does not have a fixed boundary characteristic value λ_* solution. A trial solution is commonly needed to obtain a solution. Figure 1 shows the distribution of the dimensionless parameters as functions of $\bar{r}$ for the hydrogen-isotope target pellet ablation gas current when the unperturbed energy of the ions is $E_0 = 194.81$ keV. The values of λ_* and $\bar{r}_p$ are

$$\lambda_*=0.9460, \quad \bar{r}_p=0.6366. \tag{30}$$

Since the attenuation function $L_m(E)$ for the hydrogen isotope is not a smooth function, the value λ_* changes as E_0 changes. The calculated results are shown in Table II.

The determination of the target-pellet ablation law is one of the main goals in the study of target-pellet ablation. Based on the definition of ablation rate, we have

$$G_{is}\propto(\bar{r}_p^2\bar{q})^{-2/3}\cdot\lambda_*(E_0)\cdot(r_p^4 n_{i0})^{1/3}\cdot E_0^{5/6}\cdot m_{is}^{1/2}. \tag{31}$$

It can be seen from Table II that when $E_0 > 30$ keV, we can assume

$$(\bar{r}_p^2\bar{q})^{-2/3}\cdot\lambda_*(E_0)=1.28=\text{const.}, \tag{32}$$

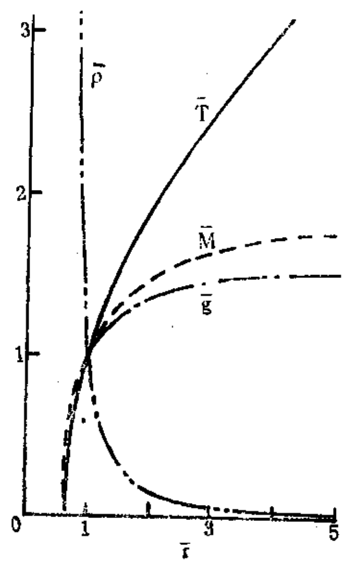

FIG. 1. The distribution of the dimensionless quantities for ablation gas current and ions as functions of $\bar{r}$.

TABLE II. The calculated results for different E_0.

E_0,keV	31.10	77.88	155.80	194.81	233.78
λ_*	0.9405	0.9445	0.9455	0.9460	0.9460
$\bar{r}_p$	0.6380	0.6370	0.6367	0.6366	0.6366
$\hat{q}=\hat{E}$	1.5550	1.5575	1.5580	1.5585	1.5585
$(\bar{r}^2_p\hat{q})^{-2/3}\lambda_*$	1.2761	1.2828	1.2847	1.2854	1.2854

We then obtain the ablation law

$$G_{is} = \frac{1.5523\times 10^4}{C_r^{2/3}}(r_p^4 n_{i0})^{1/3}\cdot E_0^{5/6}\cdot m_{is}^{1/2}. \tag{33}$$

The error in Eq. (33) is less than 1%. For different elements, $L_{ms}(E)$ are different. Therefore, for $H_2 \to H^+$, $C_H = 1$ and for $D_2 \to D^+$, $C_D = 1.26$.

Finally, we obtain

$$G_{iH}=0.8978\times 10^{-9}(r_p^4 n_{i0})^{1/3}\ E_{0H}^{0.833}, \tag{34}$$

$$G_{iD}=1.0884\times 10^{-9}(r_p^4 n_{i0})^{1/3}\ E_{0D}^{0.833}. \tag{35}$$

If r_p, n_{i0}, and E_{0s} are all the same, then

$$G_{iD}/G_{iH}=1.212. \tag{36}$$

From Ref. 6, we know that in the temperature range of fusion-reaction interest, the various parameters of ablation for the hydrogen-isotope spherical-target pellets under plasma ion-electron bombardment are

$\bar{r}_p=0.62-0.64,$

$\lambda_*=0.925-0.98,$

$\hat{q}_e=1.56,$

$\Lambda_{0e}=0.4453\times 10^{-22}\ E_{0e}^{-1.735}.$

From here we obtain the ablation ratio of the hydrogen-isotope target pellets for the corresponding ion and electron bombardments as follows:

$$\frac{G_{iH}}{G_{eH}}= 0.325\left(\frac{n_{i0H}}{n_{0e}}\right)^{1/3}\cdot\left(\frac{E_{0H}}{E_{0e}^2}\right)^{0.833}, \tag{37}$$

$$\frac{G_{iD}}{G_{eD}}= 0.250\left(\frac{n_{i0D}}{n_{0e}}\right)^{1/3}\cdot\left(\frac{E_{0D}}{E_{0e}^2}\right)^{0.833}. \tag{38}$$

The subscript "e" denotes the quantity related to the case when the target pellet is under the bombardment by the electrons.

If neutral particles are used for bombardment and heating, the ratio of the density of the high-energy ions to that of the electrons are

$$\frac{n_{i0s}}{n_{0e}}=10^{-1}. \tag{39}$$

Under this kind of number density, the energy ratio, Eqs. (37) and (38) can be used to obtain the relation between the energy ratio coefficient

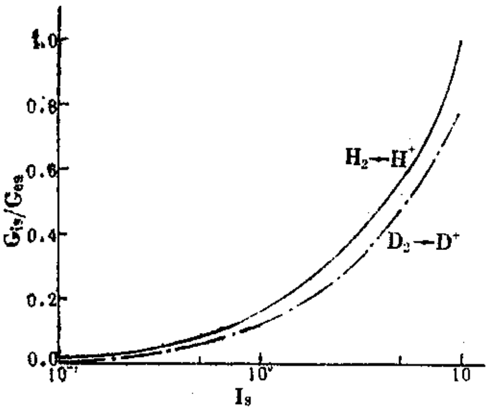

FIG. 2. G_{is}/G_{es} as a function of the energy ratio coefficient I_s.

$$I_s = \frac{E_{0s}}{E_{0e}^2}$$

and

$$G_{is}/G_{es},$$

as shown in Fig. 2.

4. CONCLUSIONS

1. It can be seen from Eqs. (37) and (38) that the influence of E_{0e} on G_{es} is greater than the influence of E_{0s} on G_{is}. The reason is that the change of Λ_e as a function of E_e is much greater than the change of Λ_{is} as a function of E_s;

2. The ablation rate of a deuterium-target pellet is greater than that of a hydrogen target pellet; that is

$$G_{iD} > G_{iH}.$$

However, the relative target-pellet ablation rates for electron bombardment are just the opposite; that is

$$\left(\frac{G_{iD}}{G_{eD}}\right) \Big/ \left(\frac{G_{iH}}{G_{eH}}\right) = 0.77.$$

3. It can be seen from Fig. 2 that when $I_s \geqslant 1.5$, $G_{is}/G_{es} \geqslant 20\%$. When there is neutral-particle bombardment, it is commonly true that $I_s \geqslant 1.5$. Therefore, it is necessary to consider the effect of the high-energy ion bombardment on the target pellet ablation.

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5. APPENDIX

The energy attenuation function $L_m(E)$ and the total elastic backscattering cross section $\sigma(E)$ when a high-energy hydrogen-isotope ion collides with the corresponding gas molecule

When a high-energy hydrogen isotope ion collides with the corresponding gas molecule $L_m(E)$ and $\sigma(E)$ can be obtained in general from the charge-carrying particles and the $L_a(E)$ and $\sigma_a(E)$ of the corresponding gaseous atoms by multiplying them with the number of atoms contained in the molecule.[7] When the ion with a velocity v_i collides with a stationery atom, we have

$$L_s(E)=L_e(E)+L_n(E), \tag{A.1}$$

where $L_e(E)$ is the electron-velocity moderation loss function, $L_a(E)$ is the nuclear-velocity moderation loss function. They can be determined from the reduced mass γ_m, the Lindhard parameter ε_L, the shielding radius a_L, and the velocity moderation loss function $S_x(E)$. That is

$$L_x(E) = \frac{\pi a^2_L \gamma_m}{\left(\frac{\varepsilon_L}{E}\right)} \cdot S_x(E), \tag{A.2}$$

The subscript $x=e$ or n. γ_m is determined below:

$$\gamma_m = \frac{4m_i m_a}{(m_i+m_a)^2}, \tag{A.3}$$

where m_j is the ion mass; m_a is the atom mass. When the energy of the charge-carrying particle reaches 10–100 eV, the Lindhard parameter becomes

$$\varepsilon_L = \frac{32.53\times10^3 m_a E}{Z_i Z_a (m_i+m_a)(Z_i^{2/3}+Z_a^{2/3})^{1/2}}, \tag{A.4}$$

$$a_L = 0.8853 a_0 (Z_i^{2/3}+Z_a^{2/3})^{-1/2}. \tag{A.5}$$

The simplified electron-velocity moderation loss function is

$$S_e(E)=K_L \varepsilon_L^{1/2}, \tag{A.6}$$

and the simplified nuclear-velocity moderation loss function is

$$S_n(E)=\frac{\frac{1}{2}\ln(1+\varepsilon_L)}{\varepsilon_L+0.10718\varepsilon_L^{0.37544}}, \tag{A.7}$$

since

$$K_L = \frac{0.0793 Z_i^{2/3} Z_a^{1/2}(m_i+m_a)^{3/2}}{(Z_i^{2/3}+Z_a^{2/3})^{3/4} m_i^{3/2} m_a^{1/2}}, \tag{A.8}$$

where Z_i and Z_a are the charge number and atomic number, respectively; and a_0 is the Bohr radius, $a_0 = 5.2918\times10^{-11}$m.

In order to determine the total elastic backscattering cross section $\sigma(E)$, we want to use the differential elastic-scattering cross section derived by Lindhard *et al.*[8]

$$d\sigma = \pi a^2_L \frac{dt}{2t^{3/2}} f(t^{1/2}), \tag{A.9}$$

where $t=\varepsilon_L^2\ (T_{re}/T_m)$. The maximum energy transition T_m and the minimum energy transition T_{re} correspond to

$$T_m=\gamma_m E, \tag{A.10}$$

$$T_{re} = \frac{2m_i E}{m_i+m_a}. \tag{A.11}$$

The function $f(t^{1/2})$ can be obtained from Ref. 8.

Since Eq. (A.9) is suitable for both single and multiple scatterings, it can be integrated from T_{re} to T_m to obtain $\sigma(E)$.

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