Journal of Reinforced Plastics and Composites
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Journal of Reinforced Plastics and Composites 1992 11: 1187
DOI: 10.1177/073168449201101009

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ABSTRACT: Creep behavior of [±45°]$_s$ composite material is characterized by using uniaxial creep and recovery tests. The well-known Schapery nonlinear viscoelastic constitutive relation was modified to make it suitable for characterizing the creep behavior of this material. Then, using this modified Schapery constitutive equation, by which the viscoplastic and creep damage can be taken into consideration, the creep behavior of [±45°]$_s$ glass fiber reinforced epoxy laminate was studied. The constitutive parameters of the material were determined experimentally, and the procedure and method of determination of the material parameters are proved to be valid.

KEY WORDS: creep, viscoelasticity, viscoplasticity, fiber reinforced plastic, constitutive relation.

1. INTRODUCTION

THE PRESENT PAPER presents theoretical and experimental investigations on creep behavior of glass fiber reinforced polymer matrix composites (PMC) with [±45°]$_s$ lay-up construction. In the previous works [1,2], the well-known Schapery nonlinear viscoelastic constitutive relation was used to characterize the time-dependent deformation of PMC. The experimental results of Reference [2] indicate that the Schapery equation is inadequate to describe the creep behavior of PMC; in particular, for viscoplastic behavior with creep damage, which is always coupled with creep strain of the material. In the first part of the present paper, the Schapery equation was modified to make it adequate for describing creep deformation in which the viscoplasticity and damage cannot be ignored. Then in the second part of the paper, the creep behavior of [±45°]$_s$ glass fiber reinforced epoxy laminate was studied experimentally. Creep-recovery experiments under different loads and of different creep durations were conducted. The constitutive parameters were determined. The predicted curves of the modified equation were compared with the experimental ones, which demonstrates that the method and procedure of determining the constitutive parameters are effective.

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2. MODIFIED SCHAPERY EQUATION

The well-known Schapery nonlinear viscoelastic constitutive relation is given as [1]:

$$\epsilon(t) = A_0 g_0 \sigma + g_1 \int_0^t \Delta A(\psi - \psi') \frac{\partial(g_2\sigma)}{\partial\tau} d\tau \tag{1}$$

where A_0 and ΔA are instantaneous elastic compliance and creep compliance respectively, and the reduced time ψ is defined as:

$$\psi = \int_0^t \frac{dt'}{a_\sigma} \qquad \psi' = \int_0^\tau \frac{dt'}{a_\sigma} \tag{2}$$

where g_0, g_1, g_2 and a_σ are stress-dependent material parameters. When the input stress σ is low, $g_0 = g_1 = g_2 = a_\sigma = 1$, then Equation (1) reduces to the equation of Boltzmann superposition principle. We see that in Equations (1) and (2) there is one function of time and four functions of stress which have to be evaluated experimentally. In the previous paper [2], the constitutive theory (1) was used to characterize creep behavior of random short fiber composite–chopped strand mat (CSM) glass fiber reinforced epoxy composite. The results showed that the creep damage played a significant role in the creep behavior. That is difficult to be described by Equation (1) without modification. In the experimental study of this paper, we also found the need to modify Equation (1) for characterizing creep of $[\pm 45°]_s$ glass fiber epoxy laminate.

Re-examining the procedure through which R. A. Schapery [3] derived Equation (1) and combining the experimental results we obtained, it seems reasonable [3,4] that the modified Schapery equation was proposed as follows:

$$\epsilon(t) = A_0 g_0 \sigma + \int_0^t \Delta A(\psi - \psi') \frac{\partial(g_2\sigma)}{\partial\tau} d\tau + B\sigma^s t^m \tag{3}$$

In general, total creep strain can be divided into three components: elastic, viscoelastic and viscoplastic strains (see Figure 1). The three terms in Equation (3) are associated with the three components respectively.

By comparison between Equations (1) and (3), we see two differences in them. Firstly, an additional term, i.e., the third term, is introduced into Equation (3). This term represents viscoplastic deformation, where B, s and m are material constants. Secondly, for the second term of Equation (3), which is for the viscoelastic deformation, the parametric function g_1 (σ) appearing in Equation (1) has been discarded. The reason for so doing can be given as follows: since $g_1(\sigma)$ is a function of stress, according to Equation (1), when the input stress changes at time t_1 from $\sigma_1(t_1)$ to $\sigma_2(t_1)$, the output viscoelastic strain will also change

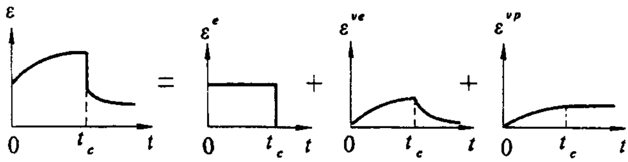

Figure 1. *Dividing creep-recovery strain into three components.*

abruptly at time t_1, namely $\epsilon^{ve}(t_1^+) \neq \epsilon^{ve}(t_1^-)$. Since viscoelastic properties can be modelled with the so-called standard solid model—i.e., the three element model, that is not reasonable. Omitting $g_1(\sigma)$ from Equation (1) this problem is overcome.

For many materials, $\Delta A(\psi)$ can be approximated by the power function

$$\Delta A(\psi) = C\psi^n \tag{4}$$

where C and n are independent of stress level and time. Thus, there are six parameters, A_0, C, n, m, B and s, and three parametric functions, $g_0(\sigma)$, $g_2(\sigma)$ and $a_\sigma(\sigma)$, in Equation (3), which need to be determined experimentally.

If the effect of creep damage is taken into consideration, the nominal stress has to be replaced by real stress $\tilde{\sigma}$, which is related to damage parameter D by the following equation:

$$\tilde{\sigma} = \frac{\sigma}{1 - D} \tag{5}$$

During creep test, the damage grows according to the Kachanov-Rabotnov's damage evolution law:

$$\dot{D} = \left(\frac{\sigma}{R}\right)^r (1 - D)^{-k} \tag{6}$$

where D can be defined in many ways, here we let $D = 1 - (\tilde{E}/E)$. In Equation (6), r, k and R are also material constants.

3. EXPERIMENTAL DETERMINATION OF MATERIAL PARAMETERS

The creep behavior of $[\pm 45°]_s$ glass fiber reinforced epoxy laminate was studied experimentally, and its constitutive parameters were determined experimentally. Creep-recovery tests were carried out by using Instron 1195. The dimension of the specimen is 150 × 25 × 2 mm, which is similar to the dimension of the standard specimen specified by ASTM D3039-76 and by Chinese National Testing Standard, GB3354-82. The tests were conducted under room temperature. Four different loads were used in the tests. The creep and recovery durations, together with the dimensions of the specimens, are listed in Table 1. The strain-time curves are recorded as shown in Figure 2.

Table 1. Specimen, load and duration of creep and recovery.

Spec.	Width (mm)	Thickness (mm)	Load (kg)	Stress (MPa)	Creep (min)	Recovery (min)
1	25.172	2.165	200	35.965	100	100
2	25.202	2.199	200	35.367	50	100
3	25.248	2.172	300	53.611	100	100
4	25.255	2.171	300	53.620	50	100
5	25.187	2.221	400	70.075	100	100
6	25.150	2.202	400	70.784	50	100
7	25.253	2.182	500	88.926	100	100
8	25.283	2.172	500	89.913	50	100

The strain curves can be divided into three curves, i.e., elastic, viscoelastic and viscoplastic ones as illustrated in Figure 1. The instantaneous elastic strain at time of loading is

$$\epsilon^e(0) = A_0 g_0 \sigma \tag{7}$$

The elastic strain at the time of unloading is

$$\epsilon^e(t_c) = \frac{A_0 g_0 \sigma}{1 - D(\sigma,t_c)} \tag{8}$$

where t_c is unloading time. Creep strain in the period of creep is given as:

$$\epsilon^c = C g_2 \sigma \frac{t^n}{a_\sigma^n} + B\sigma^s t^m \tag{9}$$

The strain in the recovery period is

$$\epsilon^r = \epsilon^{vp}(\sigma,t_c) + [\epsilon(t_c^+) - \epsilon^{vp}(\sigma,t_c)][(1 + a_\sigma\lambda)^n - (a_\sigma\lambda)^n] \tag{10}$$

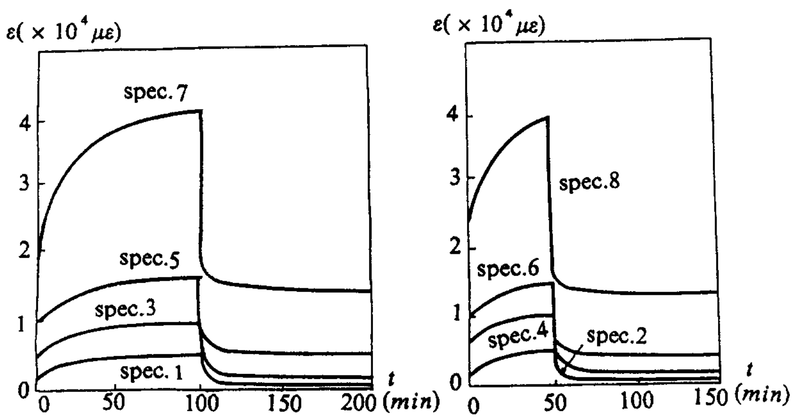

Figure 2. Experimental creep-recovery strain curve.

where

$$\lambda = \frac{t - t_c}{t_c} \qquad \epsilon^{vp}(\sigma,t_c) = B\sigma^s t_c^m \tag{11}$$

Using the curves in Figure 2 and Equations (7) through (11), the constitutive parameters were determined through the following procedures.

1. Under the lowest stress, e.g., for specimens No. 1 and No. 2, g_0 was assumed to be equal to 1. Making use of Equation (7), A_0 can be obtained.

$$A_0 = \frac{\epsilon^e(0)}{\sigma} \tag{7a}$$

Then combining Equation (8), we obtained

$$D(\sigma_{min},t_c) = 1 - \frac{\epsilon(0)}{\epsilon(t_c^-) - \epsilon(t_c^+)} \tag{12}$$

2. Under the lowest stress, let $a_\sigma = 1$, Equation (10) becomes

$$\epsilon^r = \epsilon^{vp}(\sigma,t_c) + [\epsilon(t_c^+) - \epsilon^{vp}(\sigma,t_c)][(1 + \lambda)^n - \lambda^n] \tag{13}$$

Selecting several points on the curve in Figure 2 and employing the least square method, the values of n and $\epsilon^{vp}(\sigma,t_c)$ were acquired.

3. According to the elastic strains under other higher stresses, $g_0(\sigma_i)$ and $D(\sigma_i,t_c)$ can be evaluated as

$$g_0(\sigma_i) = \frac{\epsilon^e(0)}{A_0\sigma_i} \tag{14}$$

$$D(\sigma_i,t_c) = 1 - \frac{\epsilon^e(0)}{\epsilon(t_c^-) - \epsilon(t_c^+)} \tag{15}$$

4. Using the creep-recovery curves of other stresses and Equation (10), the least square method gives the value of $a_\sigma(\sigma_i)$ and $\epsilon^{vp}(\sigma_i,t_c)$.
5. Having obtained $\epsilon^{vp}(\sigma_i,t_c)$ through procedures 2 and 4, the constants B, s and m can be determined as follows. Formulating residual difference function

$$f(\ln B,s,m) = \ln[\epsilon^{vp}(\sigma,t_c)] - \ln B - s\ln\sigma - m\ln t_c \tag{16}$$

and selecting strain values from the tested curves, ln B, s and m can be calculated by the least square method.

6. By using the strain curve of lowest stress, where $g_2 = a_\sigma = 1$, the constant

C was determined from Equation (9). Formulating residual difference function

$$f(c) = \epsilon - \epsilon^e - C\sigma_{\min}t^n - B\sigma_{\min}^s t^m \quad (17)$$

the least square method produces the following formula, from which the value of C can be solved.

$$C = \frac{\sum_{i=1}^{k} (\epsilon_i - \epsilon_i^e - B\sigma_{\min}^s t_i^m)\sigma_{\min} t_i^n}{\sum_{i=1}^{k} (\sigma_{\min} t_i^n)^2} \quad (18)$$

In the same way, using strain curves under other higher stresses, the formulas of $g_2(\sigma_i)$ were obtained:

$$g_2(\sigma_j)C = \frac{\sum_{i=1}^{k} \left[(\epsilon_i - \epsilon_i^e - B\sigma_j^s t_i^m) \left(\frac{\sigma_j t_i^n}{a_\sigma^n} \right) \right]}{\sum_{i=1}^{k} \left(\frac{\sigma_j t_i^n}{a_\sigma^n} \right)^2} \quad (19)$$

After the above six procedures were performed, all the constitutive parameters were determined, which are listed in Table 2.

By using the parameters given in Table 2, we can plot a fitted curve as shown in Figure 3. The coincidence between the experimental and the fitted curves is satisfactory.

According to the values of Table 2, $g_0(\sigma_i)$, $g_2(\sigma_i)$ and $a_\sigma(\sigma_i)$ can be fitted with linear functions, which are akin to those in Reference [5]. The results are shown in Table 3.

Table 2. Fitted parameters based on experiments.

Param. \ Spec. No.	1	2	3	4	5	6	7	8
g_0	1.00	1.00	1.23	1.20	1.29	1.29	1.97	1.93
g_2	1.00	1.00	1.13	1.60	1.86	1.60	7.15	6.23
a_σ	1.00	1.00	1.33	0.92	1.74	1.67	3.17	1.76
$D(t_c)$	.036	.065	.014	.055	.011	.008	0.082	0.089
Viscoela.	A_0 = 102.5			C = 7.22		n = 0.243		
Viscopla.	B = 1.67 × 10^{-4}			s = 3.76		m = 0.279		

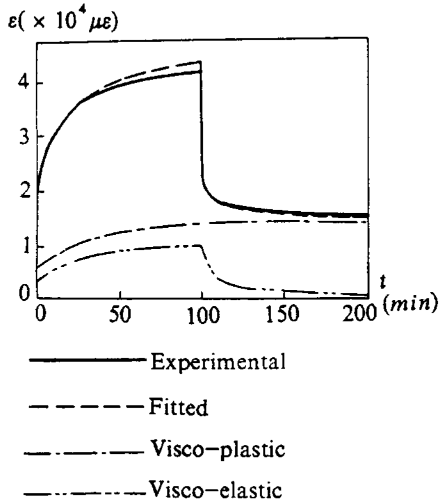

Figure 3. *The fitted creep-recovery strain curves.*

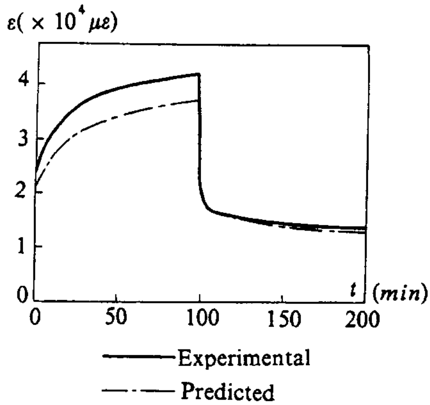

Figure 4. *Comparison between experimental and predicted curves (spec. 7).*

Table 3. Three parametric functions.

g_0	$= 1 + 0.0147\ (\sigma - \sigma^*)$
g_2	$= 1 + 0.0763\ (\sigma - \sigma^*)$
a_σ	$= 1 + 0.0238\ (\sigma - \sigma^*)$
σ^*	$= 36$ MPa

By making use of the values of A_0, C, n, B, s and m in Table 2 and the parametric functions of g_0, g_2 and a_σ in Table 3, a theoretical creep-recovery strain curve was drawn, which is illustrated in Figure 4. The comparison between the experimental curve and the predicted one in Figures 3 and 4 indicates that the modified constitutive Equation (3) and the methodology of determination of material parameters are valid for characterizing the time-dependent behavior of PMC.

4. CONCLUDING REMARKS

From the results presented in this paper, the following conclusions can be drawn:

1. The $[\pm 45°]_s$ glass fiber reinforced epoxy laminate exhibits pronounced creep behavior which consists of instantaneous elastic, delayed viscoelastic and irreversible viscoplastic deformations.
2. The modified Schapery equation proposed in this paper proved to be adequate for describing the creep behavior of PMC.
3. The procedure of determining constitutive parameters of the material described in the paper seems to be effective.

ACKNOWLEDGEMENT

This research work has been supported by the Chinese National Science Foundation and the Nonlinear Continuum Mechanics Laboratory, Institute of Mechanics, Chinese Academy of Sciences. The author also gratefully acknowledges the support of K. C. Wong Education Foundation, Hong Kong.

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