GROWTH OF ZERO-MEAN-SHEAR TURBULENT-MIXED LAYER WITH SUSPENDED PARTICLES IN A TWO-LAYER FLUID SYSTEM*

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ABSTRACT

The growth behaviour of zero-mean-shear turbulent-mixed layer containing suspended solid particles has been studied experimentally and analysed theoretically in a two-layer fluid system. The potential model for estimating the turbulent entrainment rate of the mixed layer has also been suggested, including the results of the turbulent entrainment for pure two-layer fluid. The experimental results show that the entrainment behaviour of a mixed layer with the suspended particles is well described by the model. The relationship between the entrainment distance and the time, and the variation of the dimensionless entrainment rate E with the local Richardson number Ri_l for the suspended particles differ from that for the pure two-layer fluid by the factors $\eta^{-1/3}$ and η^{-1}, respectively, where $\eta = 1 + \sigma_0 \Delta\rho / \Delta\rho_0$.

Keywords: stratified fluid, suspended particles, mixing, turbulent entrainment.

I. INTRODUCTION

Stratification of fluid extensively exists in geophysical and industrial fluids. The turbulent mixing in a stratified fluid is important to controlling the water-quality and tackling pollution in the atmosphere and in water bodies. Thus, it has attracted fluid-dynamists' great attention. If there are suspended solid particles in a turbulent mixed layer (such as dust in the atmosphere near earth and silt in a water), kinetic energy will be consumed to keep the particles in suspension, thus changing the turbulent structure and affecting the behaviour of turbulent entrainment. Therefore, it is necessary to investigate the influence of suspended particle concentration on the turbulent entrainment.

E and Hopfinger[1] investigated turbulent entrainments on settled solid particles on the bottom in a uniform fluid, and obtained a relationship between the distance of turbulent entrainment and the r.m.s. turbulent velocity, and the local buoyancy flux. Barenblatt[2] studied the shear turbulence with the saturated suspended parti-

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cles near the wall by means of dimensional analysis. The influence of particle layer over the density interface on the turbulent entrainment was studied experimentally by E Xue-quan[3] in a two-fluid system. In order to understand the entrainment of a turbulent-mixed layer containing suspended particles, one has first to make clear the characteristics of turbulence. The features of the zero-mean-shear turbulence generated from the planar oscillating grid have been widely studied[4,5]. The grid stirring can simulate the natural processes in which the turbulent energy is put in on a scale much smaller than the layer depth, such as the breaking of waves at the sea surface. Therefore, the oscillating grid was selected as the turbulent energy source. Investigations show that the horizontal (u,v) and vertical (w) r. m.s. turbulent velocity components $u \simeq v \sim w$ and the integral lengthscale of turbulence $l \sim z$ at a distance of z away from the grid midplane, and that there are empirical relations

$$u = C M^{1/2} S^{3/2} f z^{-1}, \tag{1.1}$$

$$l = \beta z, \tag{1.2}$$

where C is the coefficient of proportionality, approximately 0.3, S (cm) is the stroke of grid oscillation, M(cm) is the mesh length, f(Hz) is the frequency of grid oscillation, and β is the constant related to stroke S, $\beta = 0.1$[4]. Relation (1.1) is a good approximation to the r. m. s. turbulent velocity generated by the grid made from square bars with the mesh size $M/d = 5$ oscillated at $f \leqslant 6$Hz (d is the bar size).

The aim of the present work is to study the growth law of a zero-mean-shear turbulent-mixed layer with solid particle concentration (in terms of volume or mass) and external parameters of turbulent source. The experiments have been conducted in a wide range: $S = 2$—5 cm, $f = 2-5$ Hz, initial particle volume concentration $\sigma_0 = 2.98\%-7.24\%$, and the results were compared with those obtained by the potential energy model.

II. Experimental Procedure

The experiments were performed in an oscillating grid turbulence tank 52 cm × 52 cm × 70 cm in volume as shown in Fig.1, (cf. [3] for detail). The oscillating grid supported by a rod 10 cm in diameter was placed horizontally near the top of the tank. The rod was connected to an oscillating mechanism with a maximum stroke $S = 9$ cm. The mechanism was powered by a frequency-modulated motor, whose rotative velocity was controlled by a frequency converter. The frequency f of the grid oscillations could be varied over a range of 0—6 Hz via a reduction gear box.

First, the tank was filled with clean water 9 cm in depth, and then saline water with density of 1.062 was poured in from a capped hole in the bottom face of the tank by a specific fluid stratification set-up. The lower layer of heavy salt water supported the upper layer of pure water and rose gradually till the interface was at the midplane. Thus we obtained an intense stratification. The sampling measurement and shadowgraph showed that the interface layer had characteristics of intensely linear stratification and a thickness of 1.5 cm. Finally, as described

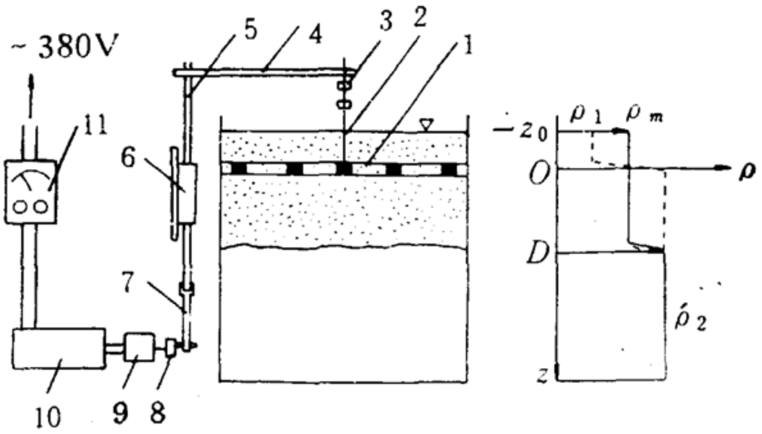


Fig. 1. Schematic diagram of the experimental apparatus and arrangement.
1, Grid; 2, connective rod; 3, localization sleeve; 4, cantilever beam; 5, sliding rod; 6, sliding sleeve; 7, crank; 8, cam; 9, reducing gear; 10, frequency-modulated electric machine; 11. frequency-modulated device,

in Ref [3], the diameter of plastic particles was 100 μm and the density was 1.04 g/cm^3,and the increment of the water level after putting the particles was recorded so as to measure exactly the initial volume concentration of the particles.

Using the method of shadowgraph, we visualized the interface and the front of the turbulent entrainment. On both sides of the front face of the tank, a transparent graduation label was affixed. A digital-clock as accurate as to 0.1 s was installed at the front face. The position of turbulent front and corresponding time were recorded by the photographs. At the beginning of the experiments, we took several photos so as to record the initial position of the interface between the two layers of fluids. The frequency converter and digital clock were simultaneously started, and photos were taken. After the grid oscillation was started, the particles deposited on the interface were mixed at once and suspended homogeneously in the upper turbulent layer due to the stirring of oscillating grid. On the shadowgraph (or film), the mean horizontal position of the turbulent front of the mixed layer (or called the depth of the mixed layer) is D. The reading error was not greater than 2 mm. The time digit on the film was the time at which the front arrived at the corresponding position, and from this, the entrainment rate of the mixed layer could be determined.

III. Experimental Observation

1. *Visualization of the Flow*

By means of the shadowgraph we could clearly observe that in the pure two-layer fluid, the front of the turbulent layer (or called the mixed layer) moved intensely. As soon as grid oscillation started, rapid mixing occurred and the depth of the mixed layer increased quickly. However, when suspended particles appeared in the mixed layer, the motion of the turbulent front was relatively weakened. In order to observe the internal turbulent motion in the mixed layer with suspended particles and the characteristics of the motion inside the interfacial layer, the flow field was visualized by means of tracer particles made of wooden sawdust 200 μm—500μm in diameter, in place of the plastic particles used in the experiment of

Ref. [7]. Fig.2 illustrates the flow pattern visualized by the wood particles. The upper turbulent region and lower undisturbed region were clearly displayed. It could be seen that there was an interfacial layer (or called the translation region) like that of the pure two-layer fluid, above which the zone was a uniformly mixed region composed of a lot of eddies, but the fluid below was still. The internal waves were formed due to the excitation of turbulence above the interfacial layer, and the internal waves were unceasingly broken into active turbulent eddies. The combination of the lower layer fluid entrained by the turbulent eddies with the fluid in the upper turbulent region made the turbulent region expand gradually.

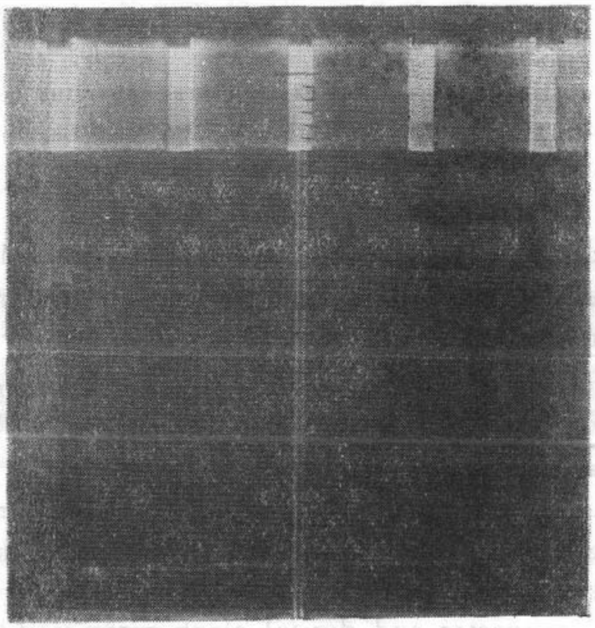
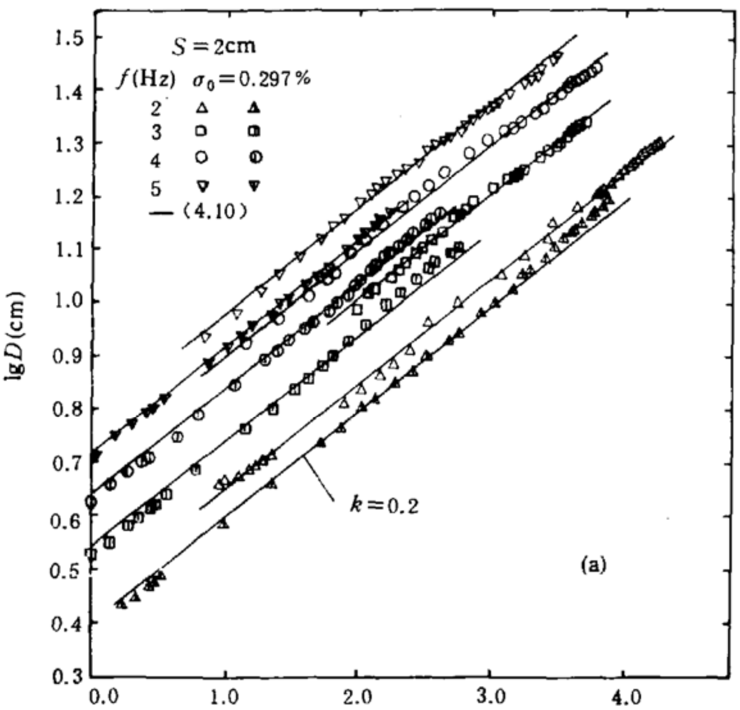


Fig. 2. Flow visualization.



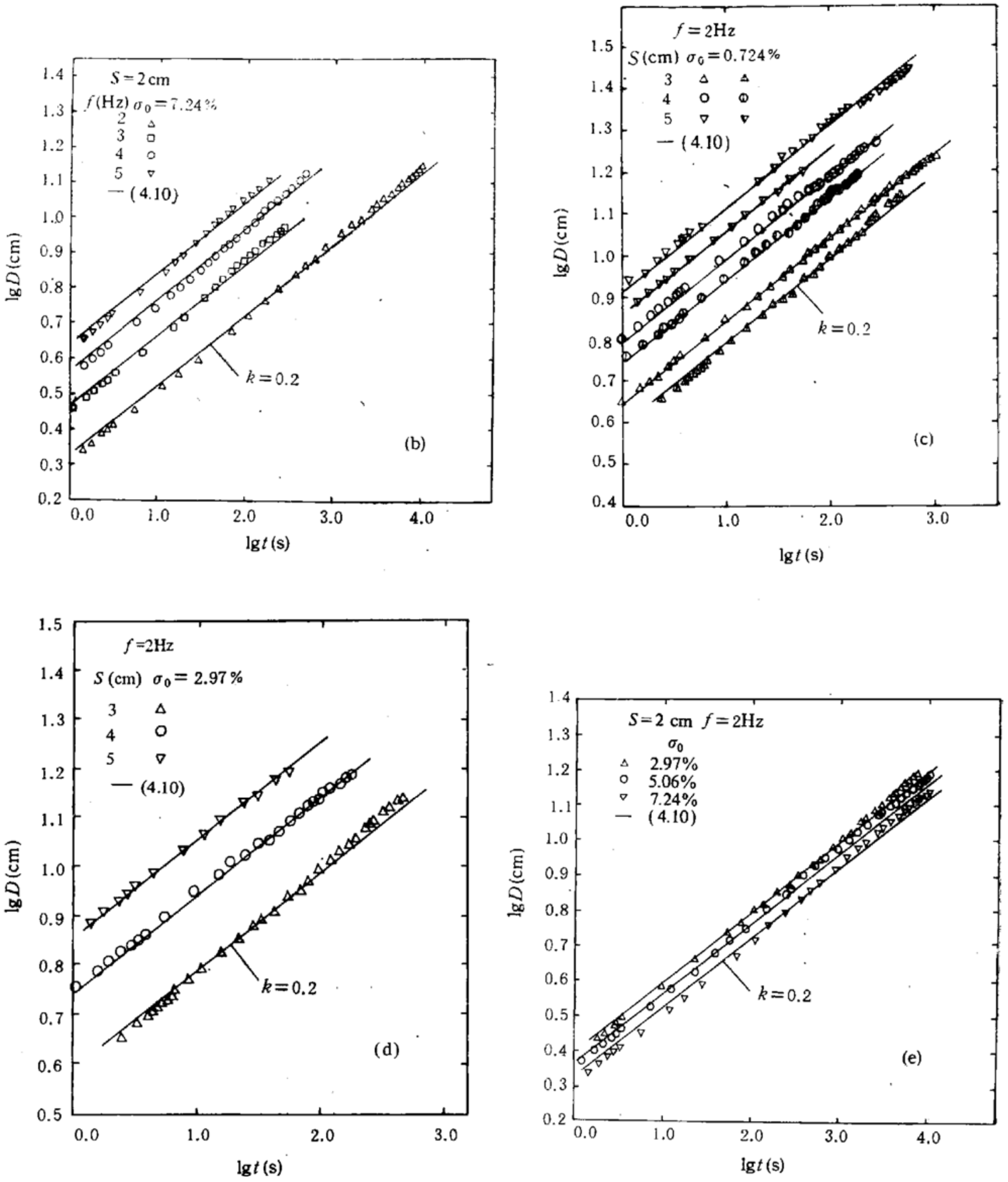


Fig. 3. Variation of the turbulent entrainment D with time t.

2. *Relations Between Entrainment Distance D and Time t*

Under the given conditions (z_0 and $\Delta\rho_0/\rho_2$ were constant, $\Delta\rho_0 = \rho_2 - \rho_1$), for different frequencies f, strokes S and initial volume concentrations of particles σ_0 ($\sigma_0 = 0$, for the pure two-layer fluid), the entrainment distance and the corresponding time were measured as shown in Fig. 3, from which we know that D can be expressed by

$$D = at^k,\ k = 0.20^{+0.034}_{-0.009}, \tag{3.1}$$

where a is a constant related with the frequency f, the stroke S and the initial volume concentration σ_0 determined by experiment, (see Table 1). Fig. 3 shows that the time variation law of the entrainment of the suspended particles in the mixed layer is the same as that of the pure two-layer fluid: $D \propto t^{1/5}$, but the D—t line for the suspended particles is below the line for the pure two-layer fluid under the same initial conditions (Fig. 3(a), (b)), showing that the growth of the mixed layer with suspended particles is slower than that without suspended particles. When $\sigma_0 = 0$, $k = 0.20^{+0.014}_{-0.009}$, which is in good agreement with the results in [8].

Table 1 gives some of the values of the experimental results. They were obtained by using the least-square fitting to the measured datum points (D, t), with the linear correlativity above 0.99.

These results are well consistent with the $D \propto t^{1/5}$ in (4.10) obtained from a potential model which will be described below.

It is noted that in the presence of suspended particles, the values of k obtained at different frequencies were greater than 0.2; the maximum deviation was about 0.034 without any negative deviation. Possibly this is because when the mixed layer arrived at a certain distance D, a part of the particles with smaller density began to go up due to the inhomogeneity of the particle density. We took account of this phenomenon and selected the particles with a uniform density in the exchanging stroke experiments. The results are satisfactory (see Table 1).

3. *Relation Between the Entrainment Distance D and the Oscillation Frequency f*

It is known from Fig. 3(a), (b) that when S and σ_0 are given, the interception $\log a$ has different values for different frequencies f, and a depends only

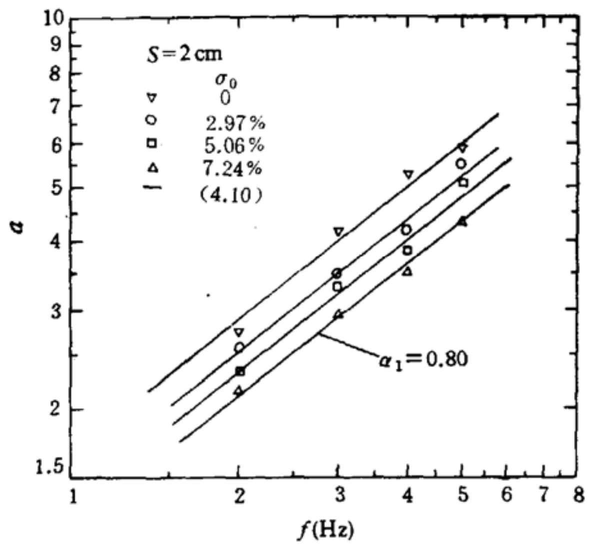


Fig. 4. Relation of the proportional coefficient a with the oscillation frequency f.

on f

$$a = a_1 f^{\alpha_1}, \quad \alpha_1 = 0.8 \pm 0.05, \tag{3.2}$$

where a_1 is the proportional constant related to S and σ_0. The variation of a with f is given in Fig. 4. From (3.1) and (3.2) we obtain

$$D \propto f^{0.8}. \tag{3.3}$$

That is to say, the entrainment distance D changes with 0.8 power of f, which also shows that the fluids with and without suspended particles have the same frequency relation. The corresponding values are given in Table 1.

Table 1

Numerical Values for Relations (3.1), (3.2) and (3.6)

($z = 9$ cm, $\Delta\rho_0/\rho_2 = 0.0584$; $S = 2$ cm)

σ_0	f (Hz)	a ($cm \cdot s^{-k}$)	a_1 ($cm \cdot s^{\alpha_1-k}$)	a_s ($cm \cdot s^{\alpha_1-k}$)	α_1 (0.80)	α_3 (−0.20)	k (0.20)
0.0000	2	2.73	1.57		0.846		0.203
	3	4.14					0.214
	4	5.25					0.195
	5	5.86					0.196
0.0297	2	2.51	1.40		0.824		0.213
	3	3.46					0.234
	4	4.13				−0.240	0.219
	5	5.51		1.56			0.209
0.0506	2	2.34	1.35		0.798		0.219
	3.	3.31					0.230
	4	3.81					0.229
	5	5.06					0.218
0.0724	2	2.14	1.29		0.750		0.206
	3	2.93					0.240
	4	3.49					0.234
	5	4.31					0.229

Note: The values in parentheses are the theoretical values for Relation (4.10). $a_s = a_1/\eta^{\alpha_3}$.

4. *Stroke Dependence*

In order to determine the relation between D and S in the measurements of the turbulent entrainment distance at a fixed frequency ($f = 2$Hz) and the same values of σ_0 as that in the various frequency experiments, the measuring results are given in Fig. 3(c), (d). Evidently, when f is fixed, a in (3.1) is related to S and σ_0. Fig. 5 gives the logarithmic relation between a and S in the case of given σ_0. Consequently we obtain

$$a = a_2 S^{\alpha_2}, \quad \alpha_2 = 1.20 {}^{+0.03}_{-0.04}. \tag{3.4}$$

Here a_2 is a constant related to f and σ_0, but it is only the function of σ_0 if f is

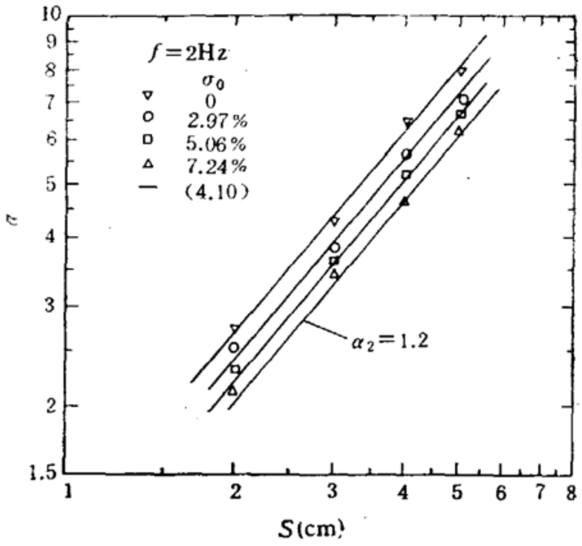


Fig. 5. Relation of the proportional coefficient a with the stroke S.

Table 2

Numerical Values for Relations (3.1), (3.2) and (3.6) ($z = 9\,\text{cm}$, $\Delta\rho_0/\rho_2 = 0.0584$, $f = 2\,\text{Hz}$)

σ_0	S (cm)	a ($\text{cm}\cdot S^{-k}$)	a_2 ($\text{cm}^{1-\alpha}S^{-k}$)	a_f ($\text{cm}^{1-\alpha}S^{-k}$)	α_2 (1.20)	α_3 (−0.20)	k (0.20)
	3	4.27					0.200
0.0000	4	6.46	1.13		1.23		0.191
	5	7.94					0.196
	3	3.85					0.215
0.0297	4	5.62	1.04		1.20		0.199
	5	7.08		1.13		−0.214	0.193
	3	3.60					0.219
0.0506	4	5.15	1.00		1.17		0.201
	5	6.53					0.193
	3	3.41					0.227
0.0724	4	4.61	0.95		1.16		0.209
	5	6.18					0.205

Note: The values in parentheses are the theoretical values for Relation (4.10). $a_f = a_2/\eta^{\alpha_3}$

fixed. The corresponding values are given in Table 2. The mixed layer entrainment of both the pure two-layer fluid and the fluid with suspended particles varies with S of 1.20 power

$$D \propto S^{1.20}. \tag{3.5}$$

This is in excellent agreement with that predicted by our theory.

5. *Variation of Entrainment Distance With Initial Particle Concentration*

After the relationship of D with f and S is obtained, it is not difficult to find the relation of the entrainment distance with the initial particle volume concentration. Fig. 3(e) shows that in the case where f and S are fixed, the interception $\log a$ has different values for different values of σ_0. This means that a is related to σ_0. a_1 and a_2 in Tables 1 and 2 also give some indications of such dependence. But, considering that the D—t relationship must degenerate into that for the pure two-layer fluid, it is reasonable to take $\eta = 1 + \sigma_0\rho_p/\Delta\rho_0$ (here ρ_p is the density of plastic material of the particle) as a parameter to replace σ_0. The variations of a_1 and a_2 with η are given in Fig. 6. Dividing a_1 and a_2 by $S^{1.2}$ and $f^{0.8}$ respectively and making statistic average, we obtain the proportional constant a_3 related only to σ_0. Generally, a_3 is also related to $z_0, \Delta\rho_0/\rho_2$ and M. But these parameters were fixed in our experiments. Fig. 7 shows the variation of a_3 with η, from which we get

$$a_3 = a_4\eta^{\alpha_3}, \quad \alpha_3 = -0.20 \pm 0.027, \tag{3.6}$$

where a_4 is the proportional coefficient related to M, z_0 and $\Delta\rho_0/\rho_2$. From (3.1), (3.2), (3.4) and (3.6) it is known that D is proportional to η^{α_3}:

$$D \propto \eta^{-0.2}. \tag{3.7}$$

From $\sigma_{0\max} = 1$, $\rho_p = 1.04\text{g/cm}^3$ and $\rho_{2\max} \approx 1.18\text{g/cm}^3$, we have $\eta_{\max} = 5.8$ which is impossible. Consequently, $1 \leqslant \eta < 5.8$. $\eta = 1$ for the pure two-layer fluid.

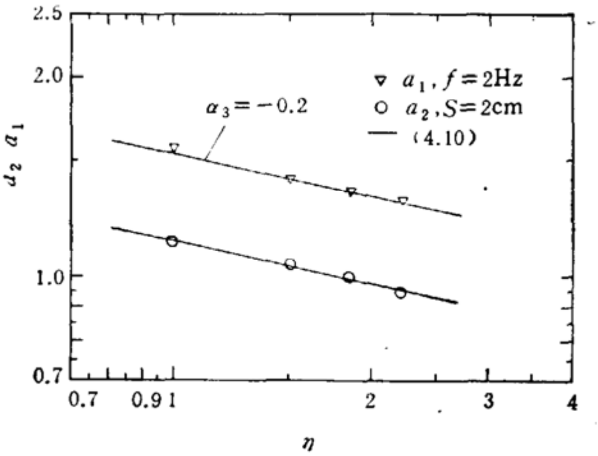


Fig. 6. Dependence of a_1 and a_2 on η.

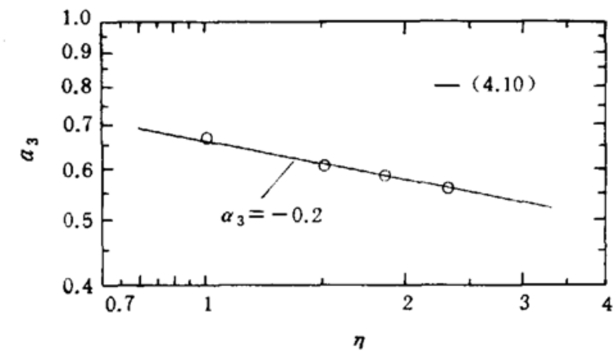


Fig. 7. Dependence of a_3 on η.

6. *Growth Behaviour of the Turbulent Mixed Layer*

The growth of the mixed layer is expressed by the entrainment distance D. As mentioned above, D is directly proportional to $t^{0.20}$, $f^{0.80}$, $S^{1.20}$ and $\eta^{-0.20}$. So D can be written as

$$D = a_5\eta^{-0.20}S^{1.20}f^{0.80}t^{0.20}, \tag{3.8}$$

where a_5 is the proportional constant related to M, z_0 and $\Delta\rho_0/\rho_2$. (3.8) can be obtained from (3.1) by resolving a gradually. (3.8) gives the entrainment law of the mixed layer containing suspended particles. The experimental results agree well with (4.10). Actually, the proportional coefficient a_5 is equal to a_4, so $a_5 =$ 0.664.

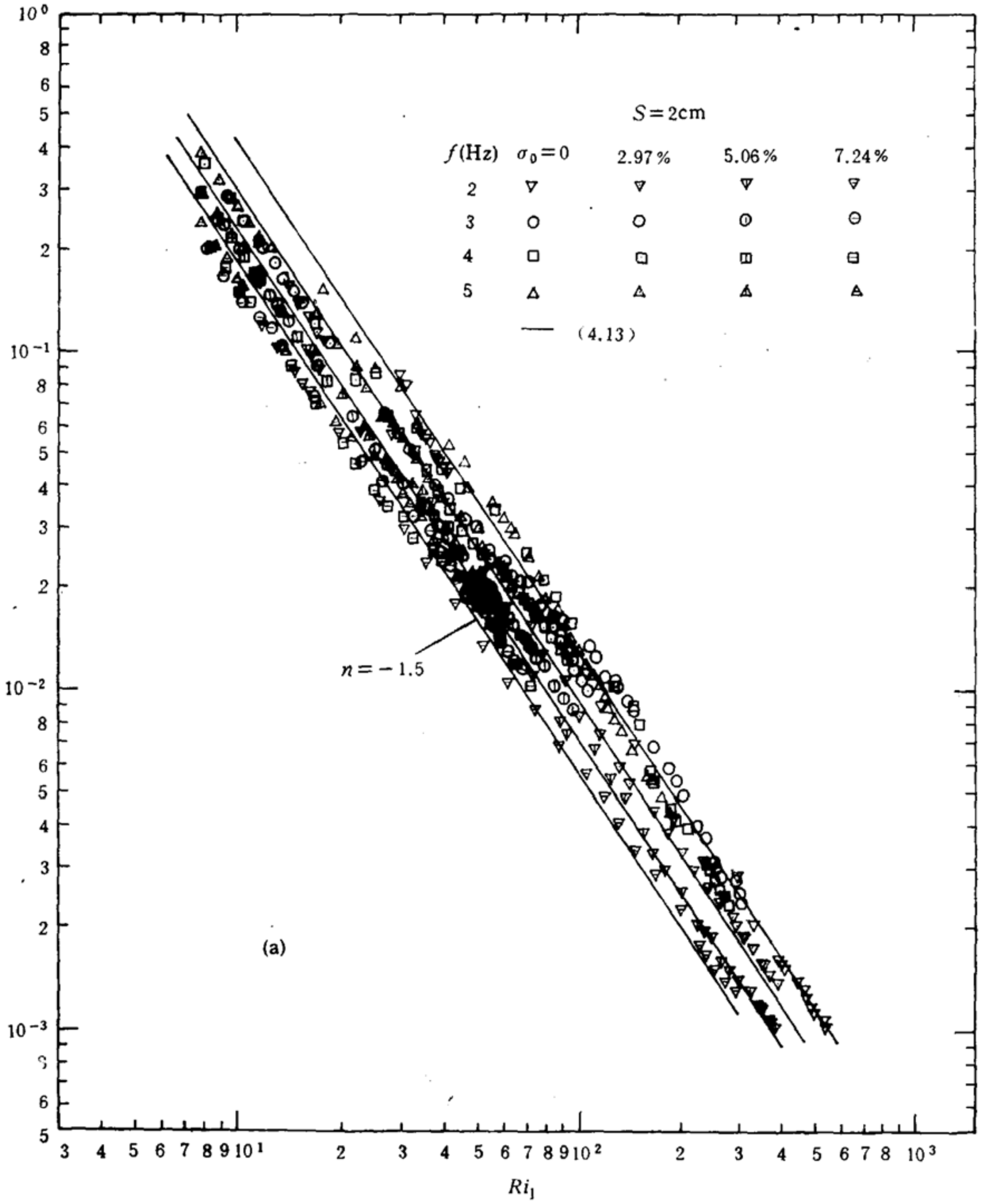
When $\sigma_0 = 0$ and $\eta = 1$, (3.8) becomes

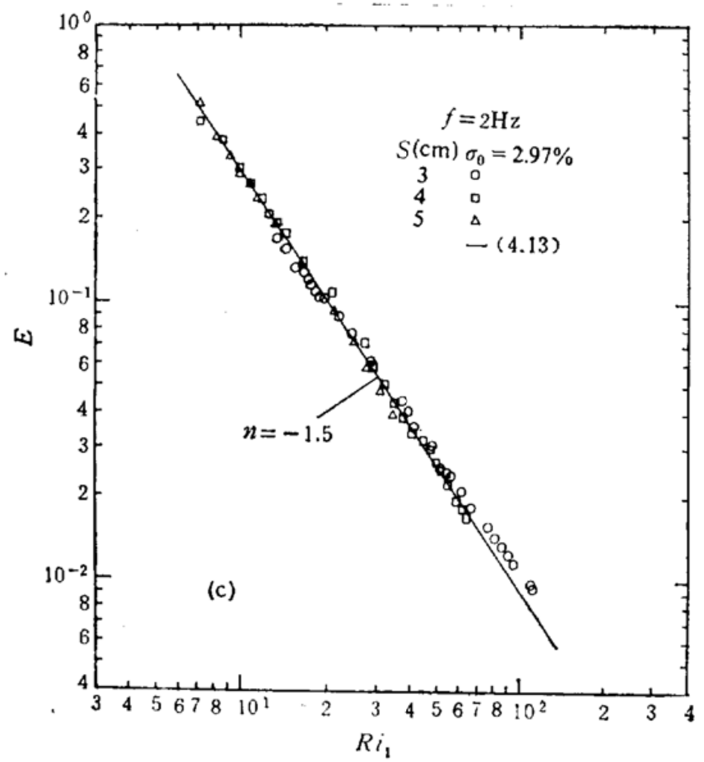
$$D = a_5 S^{1.20} f^{0.80} t^{0.20}. \tag{3.9}$$

(3.9) shows the entrainment behaviour of the pure two-layer, which is consistent with the results of Ref. [7].

7. *Relation Between Entrainment Rate E and Richardson Number Ri_l*

With the D—t relation determined, it is not difficult to find the growth velocity of the mixed layer $u_e = dD/dt$. The dimensionless entrainment rate E and the local Richardson number Ri_l based on the horizontal r. m. s. turbulent velocity at $z = D$





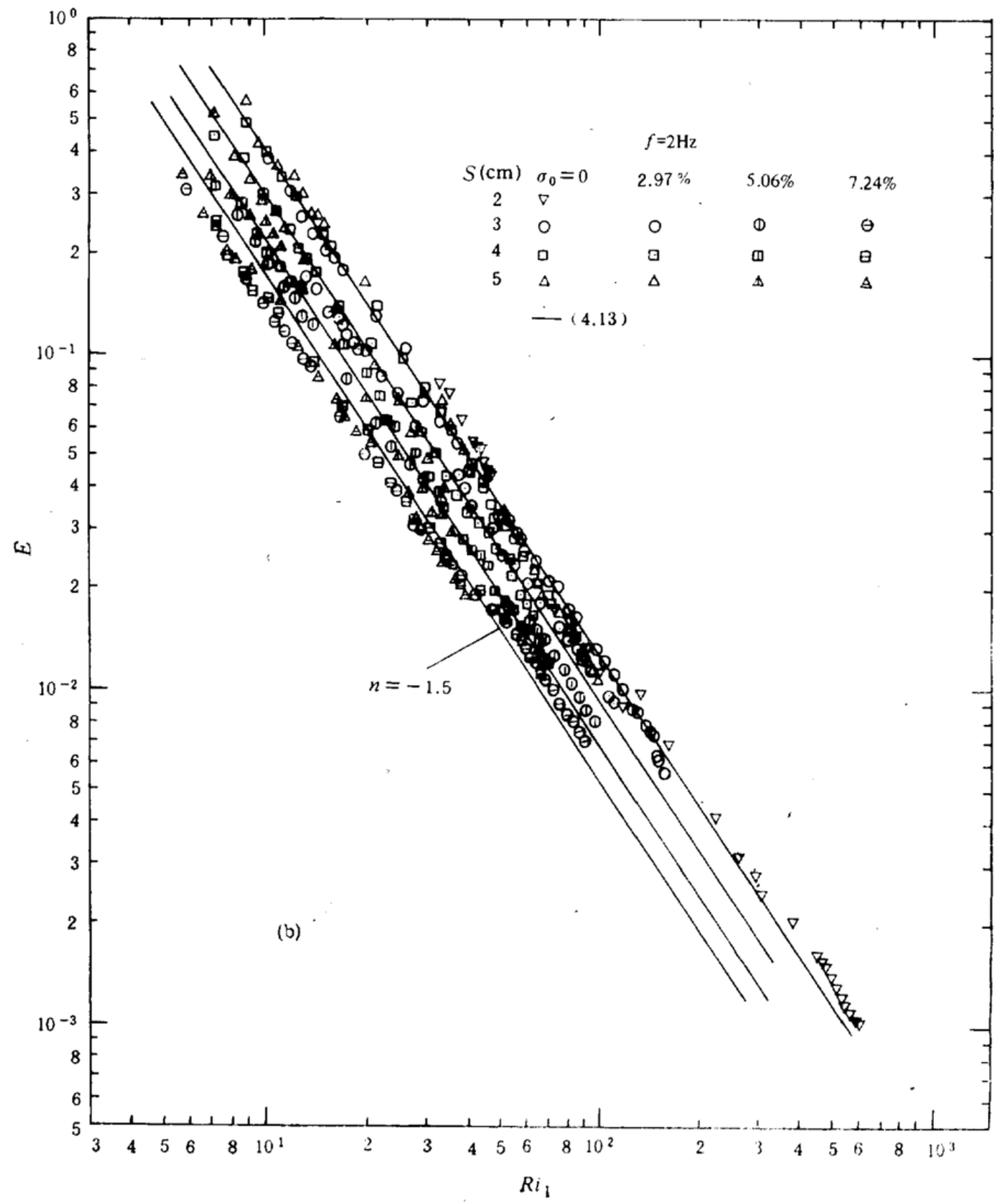


Fig. 8. Entrainment rate E plotted logarithmically as function of local Richardson number Ri_1.

(a) $S = 2$cm, at different grid oscillation frequencies;

(b) $f = 2$Hz, with different strokes;

(c) $f = 2$Hz, $\sigma_0 = 2.97\%$, $S = 3$—5cm.

Table 3

Experimental Values for Relation $K = K_0 \eta^{\alpha_4}$

z (cm)	S (cm)	f (Hz)	σ_0	η	K	K_0	α_4
9	2	2−5	0.0000	1.000	13.213	13.583	−0.984 (−1.000)
			0.0297	1.498	9.638		
			0.0506	1.849	7.447		
			0.0724	2.214	6.026		
9	3—5	2	0.0000	1.000	12.972	13.4[illegible]0	−1.052 (−1.000)
			0.0297	1.498	9.550		
			0.0506	1.849	7.015		
			0.0724	2.214	5.636		

Note: The values in parentheses are the theoretical values for Relation (4.13).

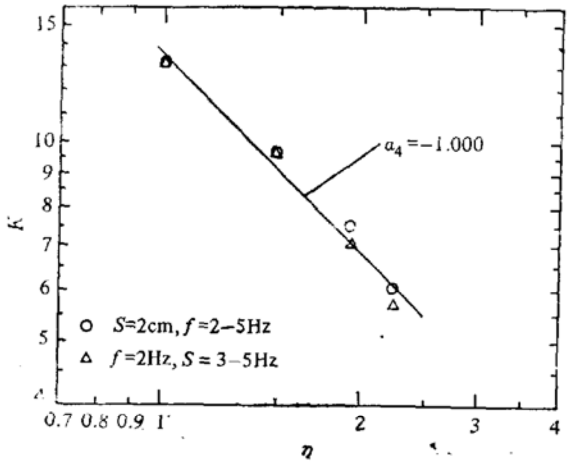


Fig. 9. Relation between K and η.

are defined as[8]

$$E = u_e/u, \tag{3.10}$$

$$Ri_1 = \left(g\,\frac{\Delta\rho}{\rho}\,l\right)\Big/u^2, \tag{3.11}$$

where

$$\Delta\rho = (\rho_2 - \rho_1)\frac{z_0}{z_0 + D}, \tag{3.12}$$

$$\rho = \rho_1\frac{z_0}{z_0 + D} + \rho_2\frac{D}{z_0 + D}. \tag{3.13}$$

In Fig. 8 the relation of entrainment rate E to Richardson number Ri_1 is plotted on a logarithmic scale. The fitting to the experimental data by the equation

$$E = KRi_1^{-1.5} \tag{3.14}$$

is satisfactory. From Fig. 8 we see the dependence of K on σ_0 or η, which can be expressed as

$$K = K_0 \eta^{-1}, \tag{3.15}$$

where K_0 is the constant at $\sigma_0 = 0$, $K_0 = 13.421$ (see Fig. 9). The data are given in Table 3.

Eq. (3.14) shows that the dimensionless entrainment rate E of the mixed layer with suspended particles does not change the power law of Ri_1, differing with the pure two-layer by only a factor η^{-1}. This means that when η (i.e. σ_0) increases, E decreases by the first power ($E \propto 1/\eta$). This result well agrees with Eq. (4.10) obtained in this paper. When $\sigma_0 = 0$, Eq. (3.13) becomes $E = K_0 Ri_1^{-3/2}$, which is excellently consistent with the experimental results in [8].

IV. Theoretical Analysis

It is very difficult to find from the turbulent energy equation the entrainment rate for the zero-mean-shear turbulent-mixed layer with suspended particles(see Fig. 1). So we have to consider some other means.

The grid oscillation unceasingly inputs kinetic energy, changing the potential energy of the water column in the tank. Assuming that the rate of the change in the potential energy (per unit horizontal area) of the water column is proportional to the vertical energy flux available at the interface, we obtain

$$\frac{dV}{dt} = B \frac{1}{2} \rho_2 \omega_1^3, \tag{4.1}$$

where V is the total potential energy of the water column, B is the proportional constant, ω_1 is the vertical velocity at the interface.

In the experiments as illustrated in Fig. 1, the potential energy of the fluid and the suspended particles per unit horizontal area are respectively given by

$$V_f = \frac{1}{2} g \Delta\rho_0 z_0 D + \text{const.} \tag{4.2}$$

and

$$V_p = \frac{1}{2} g \rho_p \sigma_0 z_0 (z_0 + D). \tag{4.3}$$

The total potential energy is the sum of both the energies ($V = V_f + V_p$):

$$V = \frac{1}{2} g z_0 (\Delta\rho_0 + \rho_p \sigma_0) D + \text{const.} \tag{4.4}$$

Differentiating (4.4) with respect to time, we obtain the rate of change in the potential energy:

$$\frac{dV}{dt} = \frac{1}{2} g z_0 (\Delta\rho_0 + \rho_p \sigma_0) \frac{dD}{dt}. \tag{4.5}$$

At the interface separating the turbulent mixed region from the stable linearly stratified fluid, the relation between ω_1 and u_1 is given by [9]

$$\omega_1^2 \propto u_1^2 \left(\frac{u_1}{ND}\right)^{2/3}, \tag{4.6}$$

where N is the buoyancy frequency. In fact, between two-layer fluids there is a linear density interfacial layer with a certain thickness[7]. So (4.6) can be applied to the two-layer fluid system, i. e. N is expressed in terms of the local buoyancy frequency by $N^2=g(\Delta\rho/\rho l)$. Using (1.2), (3.12), and (3.13) and considering $\rho_1 z_0 \ll \rho_2 D$, we have $N^2=\left(g\,\frac{z_0}{\beta}\frac{\Delta\rho_0}{\rho_2}\right)D^{-2}$. Then (4.6) can be written as

$$w_1^2 \propto u_1^2\left[u_1\Big/\left(\frac{g z_0}{\beta}\frac{\Delta\rho_0}{\rho_2}\right)^{1/2}\right]^{2/3}. \tag{4.7}$$

Substituting (4.7) into (4.1), we obtain the entrainment velocity equation for the mixed layer with suspended particles

$$u_e\equiv\frac{dD}{dt}=C_1\beta^{1/2}(gz_0)^{-3/2}\left(\frac{\Delta\rho_0}{\rho_2}\right)^{-3/2}\eta^{-1}u_1^4, \tag{4.8}$$

where $\eta=1+\rho_p\sigma_0/\Delta\rho_0$. Substituting (1.1) into (4.8), we have

$$\frac{dD}{dt}=C_2\left(g\,\frac{\Delta\rho_0}{\rho}\,z_0\right)^{-3/2}\eta^{-1}M^2S^6f^4D^{-4}, \tag{4.9}$$

where C_2 is a proportional coefficient. Considering $D=0$ at $t=0$ and integrating (4.9), we get

$$D=A\left(g\,\frac{\Delta\rho}{\rho}\,z_0\right)^{-3/10}\eta^{-1/5}M^{2/5}S^{6/5}f^{4/5}t^{1/5}, \tag{4.10}$$

where A is a constant determined by experiments. In fact, comparing (3.8) with (4.10), we have $A=1.721$, which indicates that the entrainment behaviour of the mixed layer with suspended particles agrees well with the experimental results. The distance D is inversely proportional to the 1/5 power of η, and the entrainment velocity u_e is inversely proportional to first power of η, showing that the greater the initial concentration the smaller the entrainment distance D and the velocity u_e. If $\eta=1$, i.e. $\sigma_0=0$, then (4.10) can well describe the entrainment law of the grid-turbulence and is in agreement with the results of Ref. [6].

It is easy to find the relation between the dimensionless entrainment E and the local Richardson number Ri_1. It is known from (2.1) that $u=u_1\propto D^{-1}$ when $z=D$. Dividing (4.8) by u, we obtain

$$E\equiv\frac{u_e}{u}=C_1\beta^{1/2}\left(g\,\frac{\Delta\rho_0}{\rho_2}\,z_0\right)^{-3/2}\left(\frac{1}{u_1^2}\right)^{-3/2}\eta^{-1}. \tag{4.11}$$

According to the law of conservation of mass, we have $z_0\Delta\rho_0=D\Delta\rho$ for the two-layer fluid. When $D\gg z_0, \rho_2\sim\rho$ (or defining it by ρ_2). So we obtain

$$E=K_1\left[\left(g\,\frac{\Delta\rho}{\rho}\,l\right)\Big/u_1^2{}^{-3/2}\right]\eta^{-1} \tag{4.12}$$

or

$$E=K_1\eta^{-1}Ri_1^{-3/2}, \tag{4.13}$$

where K_1 is the proportional coefficient determined by the experiments. Making a comparison between (3.14) and (4.13), we find $K_1=K_0$. At $\eta=1$ (i. e. $\sigma_0=0$), (4.13) is the $E - Ri$ relation for the pure two-layer fluid, which is in agreement with the results in Ref. [7]. Eq. (4.13) is in good agreement with the experimental results.

V. Conclusions

In this paper, the entrainment of the turbulent-mixed layer with the suspended particles have been studied theoretically and experimentally in the two-layer fluid system. The potential model and the experimental results are in excellent agreement. The principal conclusions are drawn as follows.

1. The potential model suggested here can well describe the grid-turbulent entrainment of a mixed layer with and without suspended particles in a two-layer fluid system.

2. The time dependence of turbulent entrainment for suspended particles is the same as that for a pure two-layer fluid: the entrainment distance (or depth) D varies with 1/5 power of time, such as $D = at^k$, $k = 1/5$.

3. The entrainment distance D depends on the exterior parameters of the oscillating grid generating a turbulent source, i.e. D is directly proportional to $f^{4/5}$ and $S^{6/5}$.

4. When $D \gg z_0$ (a long time procedure), the entrainment distance D can be estimated by

$$D = A\left(g\,\frac{\Delta\rho_0}{\rho_2}\,z_0\right)^{-3/10}\eta^{-1/5}M^{2/5}S^{6/5}f^{4/5}t^{1/5},$$

which differs from that for the pure two-layer fluid by a factor of $\eta^{-1/5}$.

5. The entrainment velocity u_e of a mixed layer decreases with an increase in η i. e.

$$u_e \propto \eta^{-1}.$$

6. The dimensionless entrainment rate E for a fluid with suspended particles observes the same power law of Ri_1 as that without suspended particles, $E = KRi_1^{-3/2}$, where $K = K_0\eta^{-1}$, only differing by a factor η^{-1}. This shows that E for the former decreases as η increases. When $\eta = 1$, $E = K_0Ri_1^{-3/2}$, which describes the turbulent entrainment of a pure two-layer fluid. Therefore, the potential model is also suitable for the turbulent entrainment of a pure two-layer fluid.

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