

VERTICAL SEDIMENT DISTRIBUTION

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ABSTRACT: So far, various calculation models for the vertical distribution of suspended sediment concentration have been produced by several investigators from different theories. The limitations of all these models imply that it is possible to find a more reasonable model, for which each previous model can be included as special case. The formulation of a reasonable general model is the purpose of this paper.

INTRODUCTION

Many theories have been used to approach the problem of the vertical distribution of suspended-sediment concentration. These include diffusion theory, energy theory, mixture theory, similarity theory, and stochastic theory. A systematic analysis and comparison of these theories shows that although the starting point of each is different, the results are nearly or exactly the same in form as the diffusion equation (Ni and Hui 1988a). The only distinction, if any, is the appearance of slightly different expressions for the sediment exchange coefficient ϵ_s . For each of the aforementioned theoretical formulas, one can find some measured data that show a better fit than for any other formula; and a poorer fit or no fit at all often results when other data are chosen. This raises the following problems for us: Why can each formula be fitted with some data but not with other data? What is the range of applicability for each formula? How can one choose ϵ_s so that a general formula for suspended-sediment concentration distribution can be developed? So far, many kinds of formulas for the turbid-flow-velocity distribution have been introduced into the diffusion equation, and various concentration-distribution formulas have been developed correspondingly. Nonetheless, it is still not clear what the real turbid-flow-velocity distribution law is, and how the mixing length L_o and the velocity distribution change with the concentration distribution (Ni and Hui 1988b). Then, can a reasonable way to determine the concentration distribution without introducing any velocity distribution be found? These problems are discussed here.

CALCULATION MODEL FOR VERTICAL CONCENTRATION DISTRIBUTION

Almost all of the previous theories used to study the law of the vertical-concentration distribution are related to a velocity distribution based on the Prandtl mixing-length hypothesis, but this hypothesis itself is not reasonable in some respects (von Kármán 1937; Chien and Wan 1983) and the precise theoretical expression for the so-called mixing length is still a mystery. On the other hand, the production of turbulent diffusion or the formation of a stable suspended-sediment vertical distribution is mainly caused by the ver-

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tical fluctuating velocity V' and the influence of the concentration variation on the root mean square $\sqrt{V'^2}$ is so small that it can be neglected with little error, especially for the low-concentration condition (Yalin 1972; Hino 1963). Therefore, it is reasonable to approach the concentration-distribution law directly from the probability distribution law of V' (Ni 1989).

For any given point on a vertical line in two-dimensional flow, the diffusion equation can be easily developed if the up-and-down sediment mass exchange is balanced

$$\left(C - \frac{dC}{dy} \frac{L_1}{2}\right) \left(1 - \frac{\omega}{|V'|}\right) = \left(C + \frac{dC}{dy} \frac{L_1}{2}\right) \left(1 + \frac{\omega}{|V'|}\right) \dots\dots\dots (1)$$

or

$$\frac{dC}{dy} = -\frac{\omega}{\epsilon_s} C \dots\dots\dots (2a)$$

where

$$\epsilon_s = \frac{1}{2} L_1 |V'| \dots\dots\dots (2b)$$

where V' = vertical fluctuating velocity; and C = point volumetric concentration. The characteristic length for the vertical sediment movement is L_1 , or the vertical component of the Lagrangian trajectory of the particle, which is different from the so-called mixing length L_0 . For low concentration, the settling velocity ω in (2) is usually replaced by ω_0 , the terminal settling velocity in still clear water. Eq. (3) should be adopted in order to consider the effect of sediment concentration in the case of hyperconcentrated flow

$$\omega = \omega_0(1 - C)^\alpha \dots\dots\dots (3)$$

This was first suggested by Richardson and Zaki (1954) and has further proved to be a very useful relation for the hyperconcentrated flow studies developed in recent years (Chien and Wan 1983). In the present paper, α is an exponent related to such factors as sediment diameter.

If V' obeys the normal distribution, then

$$f(V') = \frac{1}{\sqrt{2\pi}\sigma_{V'}} \exp \left[\frac{-V'^2}{(2\sigma_{V'}^2)} \right] \dots\dots\dots (4a)$$

$$\sigma_{V'} = \sqrt{V'^2} \dots\dots\dots (4b)$$

thus

$$f(|V'|) = \frac{2}{\sqrt{2\pi}\sigma_{V'}} \exp \left[\frac{-V'^2}{(2\sigma_{V'}^2)} \right], \quad V' \geq 0 \dots\dots\dots (5)$$

and

$$|V'| = \int_0^{+\infty} f(|V'|) V' dV' = \sqrt{\frac{2}{\pi}} \sqrt{V'^2} \dots\dots\dots (6)$$

By combining (2), (3), and (6), a diffusion equation in another form is obtained

$$\frac{dC}{dy} = - \frac{\frac{\sqrt{2\pi} \omega_0}{U_*} C(1 - C)^\alpha}{\left(\frac{L_1 \sqrt{V'^2}}{U_*} \right)} \dots \dots \dots (7)$$

Since $\sqrt{V'^2}/U_* = F(y/H) = 1$ is a good approximation in most of the flow region according to several measured results (Chien and Wan 1983; Grass 1971; Nikitin 1963), we can easily see that L is only determined by L_1 if the following relation is defined:

$$L = L_1 \frac{\sqrt{V'^2}}{U_*} \dots \dots \dots (8)$$

Then, the determination of L becomes the key to the problem of vertical concentration distribution. Investigations of the expression of the so-called mean free path of a sediment particle were made by Li and Shen (1975), Bayazit (1973) and others, but none of the ordinary expressions can be used as a general one for different sediment properties, as indicated by Bayazit (1973). In fact, the Reynolds analogy in which the sediment-exchange coefficient ϵ_s is assumed to be proportional to the momentum-transfer coefficient is often used (Vanoni 1946; Karim 1983), although the basis of such an analogy is not sound. So far, many different empirical or semiempirical expressions for L_o have been developed (von Kármán 1937; Chien and Wan 1983; Coleman 1969; Zagustin 1968), and this makes it possible to give a general description for L by an analogy between L_o and L . For including the effect of sediment properties, a typical comprehensive expression for L is

$$\frac{L}{H} = \frac{y}{H} \left(1 - \frac{y}{H} \right)^n \dots \dots \dots (9)$$

where the parameter n = an index reflecting the effect of water and sediment properties on the characteristic length for sediment vertical motion. Its significance to the investigation of sediment-concentration distribution is further explained in the following.

The structure of (9) is very similar to those eddy-viscosity relations used by other authors. However, we cannot expect the relations for sediment-laden flow to be exactly the same as those in clear-water flow, although the similarities between them are apparent. For turbid flows, it is impossible for us to describe the variable L in a single relation without counting the influences of the sediment properties. Thus, (9) can be taken as a first approximation for the improvement of the previous relations by introducing an index n , which makes the concentration equations obtained by other investigators be members of a family of relation given by (9).

Substituting relations (8) and (9) into relation (7), we have the following solution by integration:

$$\int_{C_a}^C = \frac{dC}{C(1-C)^\alpha} = - \int_a^y \frac{\sqrt{2\pi} \frac{\omega_0}{U_*}}{y \left(1 - \frac{y}{H}\right)^n} dy \dots \dots \dots (10)$$

where C_a = sediment concentration by volume at a distance $y = a$ from the bed.

Various kinds of formulas, including most of the well-known concentration-distribution formulas can be deduced directly from (10) as the parameter n changes, so (10) is a generalized model.

COMMENTS ON CALCULATION MODELS OF CONCENTRATION DISTRIBUTION (LOW CONCENTRATION)

Most of the commonly used formulas are derived for low-concentration conditions, that is, where $\alpha = 0$ in (10). With the substitution $\eta = y/H$ and $\eta_a = a/H$, (10) becomes

$$\frac{C}{C_a} = \exp \left[-\sqrt{2\pi} \frac{\omega_0}{U_*} \int_{\eta_a}^{\eta} \frac{d\eta}{\eta(1-\eta)^n} \right] \dots \dots \dots (11)$$

When $n = 0$, the Laursen (1980) formula is obtained from (11)

$$\frac{C}{C_a} = \left(\frac{\eta_a}{\eta} \right)^{\sqrt{2\pi}(\omega_0/U_*)} \dots \dots \dots (12)$$

When $n = 1$, the Rouse (1937) formula is derived

$$\frac{C}{C_a} = \left(\frac{\eta_a}{\eta} \frac{1-\eta}{1-\eta_a} \right)^{\sqrt{2\pi}(\omega_0/U_*)} \dots \dots \dots (13)$$

If $\alpha = 0$, $n = 3$ is inserted into (11) and the vertical concentration distribution will be more nonuniform, which is similar to Velikanov's (1958) result from gravitation theory (Ni 1989).

For the case of n taking positive integer values, a progressional solution is developed

$$\frac{C}{C_a} = \left(\frac{\eta_a}{\eta} \frac{1-\eta}{1-\eta_a} \right)^{\sqrt{2\pi}(\omega_0/U_*)} \cdot \exp \left\{ \sqrt{2\pi} \frac{\omega_0}{U_*} \left[\sum_{m=1}^{n-1} \frac{1}{m(1-\eta_a)^m} - \sum_1^{n-1} \frac{1}{m(1-\eta)^m} \right] \right\} \dots \dots \dots (14)$$

For the case of n taking integer values ≤ 1 , another progressional expression can be obtained

$$\frac{C}{C_a} = \left(\frac{\eta_a}{\eta} \right)^{\sqrt{2\pi}(\omega_0/U_*)} \cdot \exp \left\{ \sqrt{2\pi} \frac{\omega_0}{U_*} \sum_{i=1}^n \left[\frac{(-1)^i}{i} \cdot \frac{n(n-1) \dots (n-i+1)}{i!} (\eta_a^i - \eta^i) \right] \right\} \dots \dots \dots (15)$$

where n takes absolute values.

A progressional method is also used because of the difficulty in integrating (11) for any given value of n , i.e.

$$F(1 - \eta) = \int \frac{d\eta}{\eta(1 - \eta)^n} = \sum_0^m \frac{\eta^{1+m-n}}{1+m-n} \dots \dots \dots (16)$$

$$F(\eta) = - \int \frac{\eta^{-n} d\eta}{1 - \eta} = - \sum_0^m \frac{(1 - \eta)^{1+m-n}}{1+m+n} \dots \dots \dots (17)$$

So, the concentration distribution can be described by the following general equation:

$$\frac{C}{C_a} = \exp \left\{ \sqrt{2\pi} \frac{\omega_0}{U_*} [F(\eta) - F(\eta_a)] \right\} \dots \dots \dots (18)$$

The integration in (11) is very easy for a few n values; for example, when $n = 0.5$, the Tanaka-Sugimoto formula (1958) is derived

$$\frac{C}{C_a} = \left[\left(\frac{1 + \sqrt{1 - \eta}}{1 + \sqrt{1 - \eta_a}} \right) \left(\frac{1 - \sqrt{1 - \eta}}{1 - \sqrt{1 - \eta_a}} \right) \right]^{\sqrt{2\pi}(\omega_0/U_*)} \dots \dots \dots (19)$$

Noting the equivalent substitution of $2(1 - \eta)(1 - \sqrt{1 - \eta})$ and $\eta(1 - \eta)^{0.8}$, the Barenblatt (1956) formula from energy theory is also obtained, with $n = 0.8$ in (11)

$$\frac{C}{C_a} = \left[\frac{\sqrt{1 - \eta}}{\sqrt{1 - \eta_a}} \left(\frac{1 - \sqrt{1 - \eta}}{1 - \sqrt{1 - \eta_a}} \right) \right]^{\sqrt{2\pi}(\omega_0/U_*)} \dots \dots \dots (20)$$

There is a little difference in structure between the Hunt (1954) formula

$$\frac{C}{C_a} \cdot \frac{1 - C_a}{1 - C} = \left[\frac{\sqrt{1 - \frac{y}{H}}}{\sqrt{1 - \frac{a}{H}}} \left(\frac{B - \sqrt{1 - \frac{a}{H}}}{B - \sqrt{1 - \frac{y}{H}}} \right) \right]^{\sqrt{2\pi}(\omega_0/U_*)} \dots \dots \dots (21)$$

and other formulas. The parameter B is defined as $0.995 < B < 1$ by Hunt. It is easily proved (Ni 1989) that the functions $2(1 - y/H) \cdot (B - \sqrt{1 - y/H})$ and $(y/H) \cdot (1 - y/H)^{0.8}$ are approximately identical over the whole flow depth under this condition. Therefore, the Hunt formula is also a special case of (10), with $\alpha = 1$ (Hunt properly considered the concentration effect on ω) and $n = 0.8$, in which a judiciously introduced equivalent substitution makes it possible to obtain a dominant form after integration, i.e. (21).

From the preceding discussion, one can see that (10), as a generalized formula of the vertical concentration distribution, can at least supply us the following interesting or new understandings.

Using the stable characteristic of $\sqrt{V^{1/2}}$, the most difficult work of determining the so-called universal von Kármán coefficient k for turbid flow and the amended coefficient β in the suspension index $\omega_0/(\beta k U_*)$ is improved. A very interesting result is that the suspension index $Z = \sqrt{2\pi} (\omega_0/$

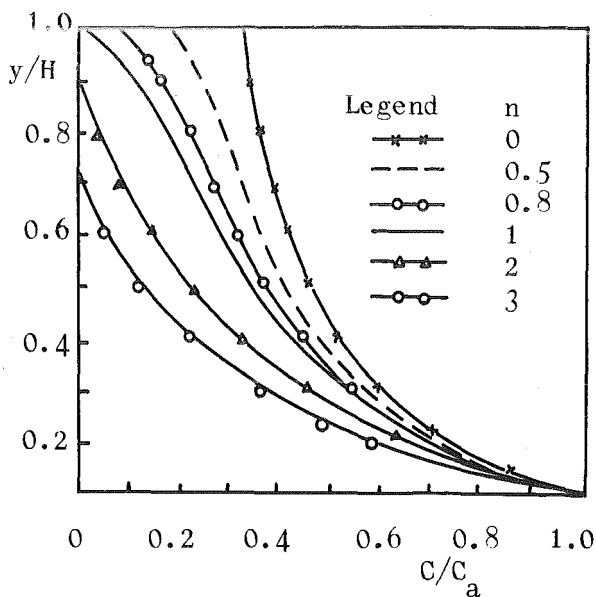


FIG. 1. Comparison of Different Models with Same $Z (= 0.5)$

U_*) in (10) is exactly the same as the form appearing in earlier equations, i.e., ω_0/kU_* , because the equality $\sqrt{2\pi} = 1/k \doteq 2.5$ is always true for the commonly used $k = 0.4$.

Almost all of the formulas from various theories are merely special cases of (10), in which the parameter n takes certain specific values (Ni 1989). The variation law of the parameter n is the key to the problem of concentration distribution (Ni 1989). Various theoretical models can be obtained

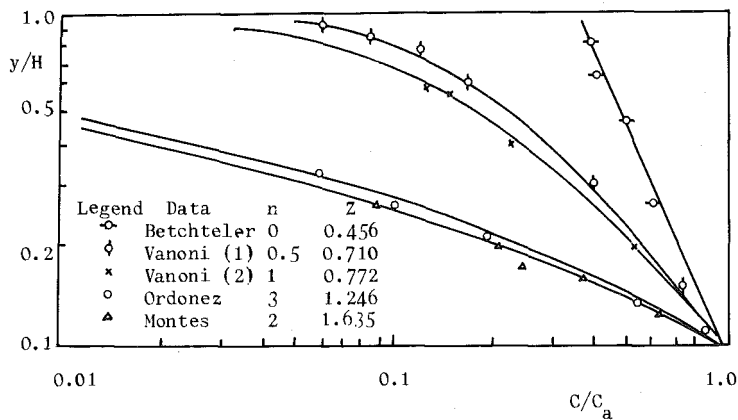


FIG. 2. Variation of n Values with Different Water and Sediment Conditions

from (10) with different n , even for the same suspension index, e.g., $Z = \sqrt{2\pi} (\omega_0/U_*) = 0.5$, and these formulas are quite different from each other, as shown in Fig. 1.

The mathematical structure of the concentration distribution derived from (10) always changes with n , so it is meaningless to revise the suspension index or the von Kármán universal constant k for different formulas. In addition, k itself has been argued to be the same (about 0.4) for both clear and sediment-laden flow with an added wake function (Coleman 1981). It is because each formula has been compared only with the data corresponding to a given n for a given water and sediment condition that none of the previous formulas are universal. Some n values corresponding to different water-sediment conditions are given in Fig. 2. [Data are from Bechteler and Vetter (1983), Vanoni (1946), and Montes and Ippen (1973).] More systematic measured data are still needed for the precise determination of the parameter n .

It is indicated by (10) that when n is not small (usually for coarser sand), the surface concentration is zero, just as the measured data show (Chien 1983), and nonzero for small n values (usually for finer sand).

CALCULATION MODEL FOR HYPERCONCENTRATION DISTRIBUTION

More factors should be considered for the calculation of concentration distribution in turbulent two-phase flow with hyperconcentration. Among these factors the effect of concentration on the settling velocity is a very important one. For the circumstances of hyperconcentration, the terminal velocity ω in (1) is replaced by $(1 - C)^\alpha \cdot \omega_0$ according to the (3), so the hyperconcentration distribution is still described by (10) with nonzero α .

Noting that both integrations in (10) are very similar, the left-hand term can be treated the same way as the right-hand term, which was discussed in the previous section. Strictly speaking, the parameter α should be related to the grain Reynolds number $R = \omega_0 D/\nu$; here ν = kinematic viscosity coefficient; and D = mean diameter of the particles. Two examples from both the earlier results of Richardson and Zaki (1954) and the data given by Chien and Wan (1983) are shown in Fig. 3. However, just as the later study (Chien and Wan 1983) shows, the parameter α can be precisely determined by its relation to the particle diameter owing to the fact that when the common temperature is considered, the parameter R is actually only the function of the particle diameter for a given fluid. Thus, the α - R relation can be reduced to a relatively simple relation between α and D . Some of the measured data (Chien and Wan 1983; Minze and Schubert 1957; Hsia and Wang 1982) with different particle properties are summarized in Fig. 4, from which we can see that the experimental data are well approximated by the dotted line.

If $\alpha = 0$, (10) is exactly the same as (11), i.e. a generalized model for low concentration.

When $\alpha = 1$, (10) becomes

$$\frac{C}{C_a} \cdot \frac{1 - C_a}{1 - C} = \exp \left[- \int_{\eta_a}^{\eta} \frac{\sqrt{2\pi} \frac{\omega_0}{U_*}}{\eta(1 - \eta)^n} d\eta \right] \dots \dots \dots (22)$$

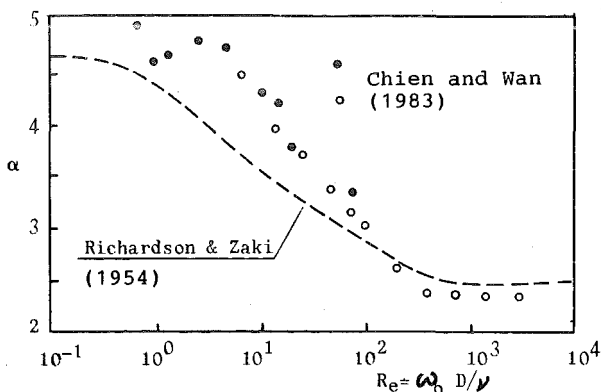


FIG. 3. α -R Relation

If α takes integer values greater than 1, the calculation model for concentration distribution is described by

$$\left(\frac{C}{C_a} \cdot \frac{1 - C_a}{1 - C} \right) \cdot \exp \left\{ \sum_{i=1}^{\alpha-1} \frac{1}{i} \left[\frac{1}{(1 - C)^i} - \frac{1}{(1 - C_a)^i} \right] \right\}$$

$$= \exp \left[- \int_{\eta_a}^{\eta} \frac{\sqrt{2\pi} \frac{\omega_0}{U_*}}{\eta(1 - \eta)^\alpha} d\eta \right] \dots \dots \dots (23)$$

and if α takes any given positive value, an expression similar to (10) can be used

$$F(C) = \int \frac{dC}{C(1 - C)^\alpha} = - \sum_0^i \frac{(1 - C)^{1+i-\alpha}}{1 + i - \alpha} \dots \dots \dots (24)$$

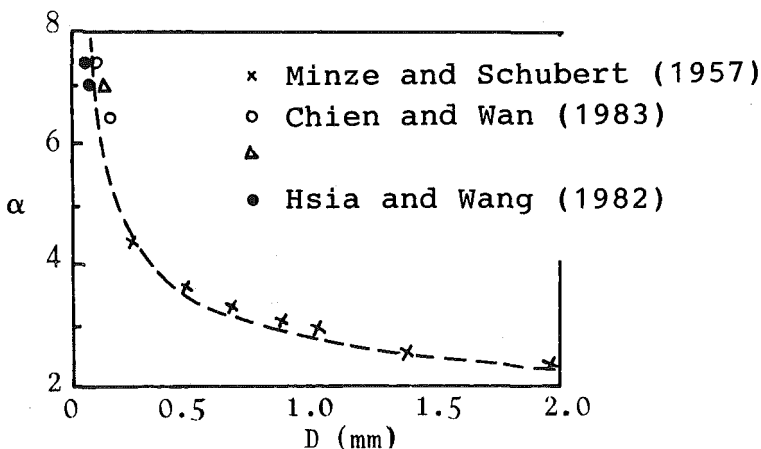


FIG. 4. α -D Relation

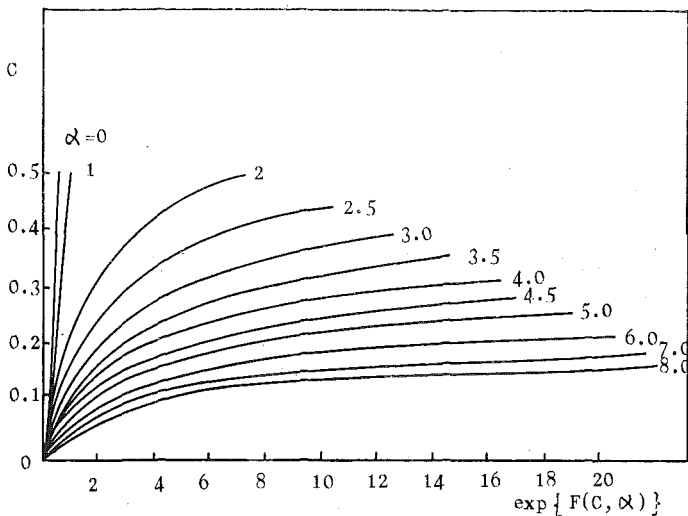


FIG. 5. Relations between C and $\exp [F(C, \alpha)]$

as a result, a simplified form of (10) is obtained

$$F(C) - F(C_a) = \sqrt{2\pi} \frac{\omega_0}{U_*} [F(\eta) - F(\eta_a)] \dots \dots \dots (25)$$

It is laborious to calculate the concentration distribution directly from (25), so the writers have given calculation results in terms of a “ $C \sim \exp [F(C, \alpha)]$ ” relation (as shown in Fig. 5) with different α values. Once α and C_a are given, $\exp [F(C, \alpha)]$ can subsequently be determined from

$$\frac{\exp [F(C, \alpha)]}{\exp [F(C_a, \alpha)]} = \exp \left[- \int_{\eta_a}^{\eta} \frac{\sqrt{2\pi} \frac{\omega_0}{U_*}}{\eta(1-\eta)^n} d\eta \right] \dots \dots \dots (26)$$

and so the sediment concentration is thus obtained from Fig. 5.

CONCLUSION

The calculation model for the vertical distribution of suspended sediment concentration presented in this paper is a succinct summary and an advance beyond the previous models because of the following.

For flow with low sediment concentration, almost all the previous formulas for the vertical concentration distribution can be derived from (11), and the limitations of the previous formulas can also be explained to some extent.

The calculation model for the concentration distribution can be obtained as indicated in the paper, without introducing any velocity-distribution model. In such a circumstance, the commonly used suspension index $\omega_0/(kU_*)$ is replaced by $\sqrt{2\pi} \omega_0/U_*$. This clearly implies a theoretical definition of k as 0.39, which corresponds to the usual experimental value of 0.4.

For flows with hyperconcentration, the foregoing model is also usable if the influence of the concentration on the settling velocity is included.

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APPENDIX II. NOTATION

The following symbols are used in this paper:

- a = vertical distance at $y = a$ from bottom;
- B = parameter defined in Hunt's (1954) formula;
- C = sediment concentration by volume;
- C_a = reference concentration at $y/H = a$;
- D = grain diameter;
- H = depth of flow;
- k = von Kármán universal constant;
- L = characteristic Eulerian length related to sediment vertical motion, defined in (8);
- L_0 = mixing length for clear water;
- L_1 = vertical mean free path of a particle in turbulent flow;
- n = exponent defined in (9);
- R = $\omega_0 D / \nu$, grain Reynolds number;
- U_* = shear velocity;
- V' = vertical fluctuating velocity;
- V'^2 = mean square value of V' ;
- y = vertical coordinate from the bottom;
- Z = $\sqrt{2\pi} \omega_0 / U_*$, suspension index;
- α = exponent in (3);
- β = modification factor of suspension index;
- ϵ_s = sediment exchange coefficient;
- η = y/H , relative flow depth;
- η_a = a/H ;
- ν = kinematic viscosity coefficient;
- ω = settling velocity of sediment particles; and
- ω_0 = terminal velocity of a particle in still, clear water.