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Analysis of K_{ic} and Its Temperature Dependence of Metals by a Simplified Dislocation Model[2])

By

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The relative K_{ic} values of metals are calculated with a simplified dislocation model. It is found that the ratio of K_{IIc} to K_{Ic} and the temperature dependence of fracture toughness of some metals estimated with this model are consistent with the experimental results.

Die Bezugswerte K_{ic} für Materialien werden mit einem vereinfachten Versetzungsmodell berechnet. Es wird gefunden, daß der Quotient K_{IIc}/K_{Ic} und die Temperaturabhängigkeit der Bruchhärte für einige Metalle, abgeschätzt mit diesem Modell, im Einklang mit Versuchsergebnissen ist.

1. Introduction

It is well-known that the contribution of the energy consumed by the plastic process in the plastic zone at a crack tip is much larger than the true surface energy of materials. In the small scale yielding case, the stress field near the crack tip can still be approximately described by the linear elastic fracture mechanics, but the fracture toughness of materials cannot be simply estimated by their true surface energy of them. (i) Up till now, the relationship between K_{IIc} and K_{Ic} calculated by linear elastic fracture mechanics are: $K_{IIc}/K_{Ic} = 0.87$, 0.96, and 0.724 with respect to various theoretical calculations [1]. These relations can only be true for the ideal brittle case. This is not consistent with the experimental data of K_{IIc}/K_{Ic} under small scale yielding. Experimental results of ultra high strength steels and other materials showed that K_{IIc} is larger than K_{Ic} even under common brittle fracture [2]. (ii) Linear fracture mechanics does not explain the temperature dependence of fracture toughness of materials which is of importance to practical engineering design and new material development.

This paper will suggest a simplified dislocation model to calculate the energy consumed by dislocation motion during the formation of the plastic zone. Dividing by the surface area created in the first step of crack propagation, we get the relative values of K_{ic}.

2. A Simplified Dislocation Model

Under the action of stress near the crack, edge dislocations will be created at the crack tip and move into the material reaching a certain location of stress balancing (Fig. 1). The dislocations will have a distribution in the plastic zone and this distribution will reach a critical value as the applied stress reaches the critical stress of fracture.

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[2]) The notation is generally the same as in the book by J. F. Knott, Fundamentals of Fracture Mechanics Butterworths, London 1973.

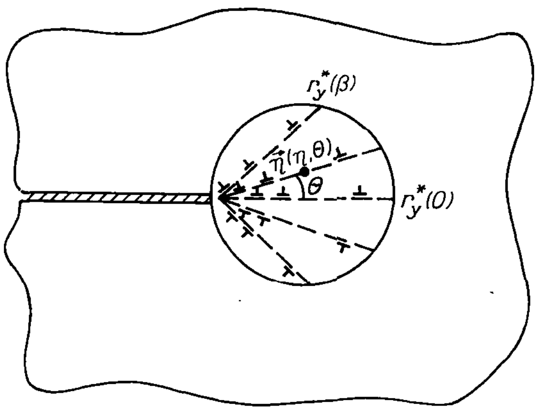
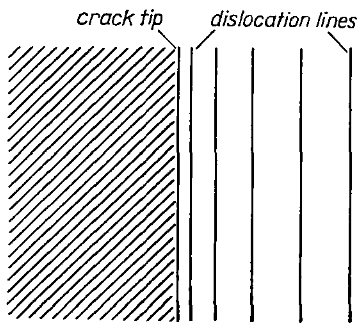


Fig. 1. Dislocations in the plastic zone of a crack tip. (The plastic zone is simplified to be a circle.)



Suppose a dislocation line of length L moves from the crack tip to a position $\boldsymbol{\eta}$. The energy consumed is

$$L \int_0^{\eta} \tau(\boldsymbol{r})\, \boldsymbol{b}\, \mathrm{d}\boldsymbol{r}\,,$$

where τ is the shear stress acting on the dislocation line and $\boldsymbol{b}$ the Burgers vector of the dislocation. Let the dislocation density be $D^{\mathrm{p}}(\eta)$ which is the equilibrium distribution function of stress balance, so the energy consumed by $D^{\mathrm{p}}(\eta)$ dislocations from $r = 0$ to $r = \eta$ is

$$LD^{\mathrm{p}}(\boldsymbol{\eta}) \int_0^{\eta} \tau(\boldsymbol{r})\, \boldsymbol{b}\, \mathrm{d}\boldsymbol{r}\,.$$

If the applied stress reaches the value of fracture σ_{F}, then the farthest dislocation will reach a distance r^*, and the total energy consumed by all the dislocations moving to their equilibrium positions will be

$$W_i = 2L \int_0^{\pi/2} \mathrm{d}\beta \int_0^{r_y^*(\beta)} D_c^{\mathrm{p}}(\boldsymbol{\eta})\, \mathrm{d}\boldsymbol{\eta} \int_0^{\eta} \tau(\boldsymbol{r})\, \boldsymbol{b}\, \mathrm{d}\boldsymbol{r}\,, \tag{1}$$

where we have assumed that the dislocations are continuously distributed in the whole plastic zone. The critical crack extension force becomes

$$G_{\mathrm{ic}}^{\mathrm{p}} \approx 2\gamma_{\mathrm{p}} = \frac{W_i}{L\Delta}\,; \qquad i = \mathrm{I}, \mathrm{II}, 1, 2\,, \tag{2}$$

where L is the length of the dislocation line, Δ the first step of crack propagation which is determined by the microstructure of the material. In particular, $\Delta = w - a$ for homogeneous continuous materials, where w is the width of the specimen and a the crack length. In order to calculate W_i in (1), $D^{\mathrm{p}}(\boldsymbol{\eta})$ and $\tau(\boldsymbol{r})$ should be analyzed further.

3. The Dislocation Distribution Function in the Plastic Zone at a Crack Tip

Up till now, what we can take as a basis of $D^{\mathrm{p}}(\boldsymbol{\eta})$ is only the BCS crack dislocation model. According to this model [3],

$$g(x_1) = \frac{\pi^2 A}{\sigma_1}\, D^{\mathrm{p}}(x_1) = \cosh^{-1}\left(\left|\frac{m}{a - x_1} + n\right|\right) - \cosh^{-1}\left(\left|\frac{m}{a + x_1} + n\right|\right). \tag{3}$$

where

$$m = r_y^* \frac{(2a + r_y^*)}{(r_y^* + a)}, \qquad n = \frac{a}{(r_y^* + a)}.$$

As to our small scale yielding process, we can make some approximations to get an analytical form of the solution. Let $x_1 = a + s$, $s \approx 0$ ($s < r_y^* \ll a$),

$$g(s) \approx \cosh^{-1} \left|\frac{m}{s}\right| \qquad (m \approx 2ar_y^*/(r_y^* + a) \approx 2r_y^* .$$

We notice that $(s/m)^{1/2} \cosh^{-1} (m/s) \approx 1$, as s/m is between 0.1 and 0.5. As $s/m < 0.1$, $D\sigma s \to 0$, due to $s \to 0$; on the other side, as $s/m > 0.5$, $D\sigma s \to 0$, due to $D \to 0$. We can approximate $D^p(x)$ to (4) without large error in calculating W_i according to (1),

$$D^p(\eta) \approx \text{const}\, s^{-1/2} . \tag{4}$$

4. The Stress Field Near the Crack Tip

There will actually be a dislocation shielding effect on the stress field near the crack tip but in our small scale yielding case, we may neglect this effect. We use the linear elastic stress field approximately. Then,

for mode I,

$$\tau_{r,\theta} = \frac{K_I^0}{\sqrt{2\pi r}} F_1(\theta) ,$$

for mode II,

$$\tau_{r,\theta} = \frac{K_{II}^0}{\sqrt{2\pi r}} F_2(\theta) , \tag{5}$$

where

$$F_1(\theta) = F_I(\theta) = \frac{1}{4} \sin \frac{\theta}{2} + \frac{1}{4} \sin \frac{3\theta}{2},$$

$$F_2(\theta) = F_{II}(\theta) = \frac{1}{4} \cos \frac{\theta}{2} + \frac{3}{4} \cos \frac{3\theta}{2}.$$

The r_y^*'s have been chosen such that the area of the circle is approximately proportional to the area of the original critical plastic zone of the material under various modes of loading and temperatures. The boundary of the critical plastic zone was determined according to Von Mises criterion, that is

$$\bar{\sigma} = \sqrt{\tfrac{1}{2} [(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6\tau_{xy}^2]} = \sigma_y , \tag{6}$$

where σ_{ys} is the unaxial tension yield stress; σ_x, σ_y, σ_z, and τ_{xy} are the components of stress at a point on the boundary of the plastic zone.

5. Results and Discussions

From (1), (4), and (5) we get

$$W_i = E_0(K_{ic}^0)^2 F_i(\theta_0) r_i^*(\theta_0)^{1/2} , \tag{7}$$

where

$$E_0 = \frac{3}{4}\frac{bLa^{1/2}}{A\pi}, \tag{8}$$

$$K_{ic} = (E'G^{p}_{ic})^{1/2},$$

$$\frac{K_{ic}}{K_{jc}} \approx \left(\frac{K^0_{ic}}{K^0_{jc}}\right)\left(\frac{F_i(\theta_0)}{F_j(\theta_0)}\right)^{1/2}\left(\frac{r^*_i(\theta_0)}{r^*_j(\theta_0)}\right)^{1/4}. \tag{9}$$

5.1 The ratio of K_{2c} to K_{1c}

In order to compare our calculations with experimental results, we took a GC-4 steel specimen as an example. The respective data of θ_0, $F(\theta_0)$, and $r^*_y(\theta_0)$ are shown in Table 1. θ_0 is the direction of r^*_{max} of the plastic zone. Choosing the values of K^0_{IIc}/K^0_{Ic} and K^0_{2c}/K^0_{1c} according to the result calculated by energy release theory, 0.724, we get

$$\frac{K_{2c}}{K_{1c}} \approx 2.08 \quad \text{for plane stress,}$$

$$\frac{K_{IIc}}{K_{Ic}} \approx 2.4 \quad \text{for plane strain.}$$

The latter is roughly consistent with the experimental plane strain results of complex mode fracture tests [2].

Table 1
Calculated results for GC-4 steel [2]

mode of loading	θ_0	$F(\theta_0)$	$r^*_y(\theta_0)$ (mm)	A^* (mm²)
I plane stress	70.5°	0.385	0.52	0.113
plane strain	90°	0.354	0.332	0.033
II plane stress	0°	1	6.824	11.869
plane strain	0°	1	5.19	8.173

5.2 Temperature dependence of K_{ic}

From (9) we see that the main strongly temperature-dependent factor is r^*_y; then

$$K_{ic}(T) \sim r^{*\,1/4}_y \sim \left(\frac{K_{ic}}{\sigma_y}\right)^{1/2}, \qquad K_{ic}(T)\,\sigma_y(T) = \text{const}\,, \tag{10}$$

where σ_y is the yield stress of the material. This equation is true only under the condition that the microstructure of the material is simple and unchanged.

Fig. 2 shows the linear relationship between G_{1c} and σ_y^{-2}. The data were taken from [4] and [5]. We assume them to be of similar microstructures due to their similar heat treatments.

It is well known that [6 to 8]

$$\sigma_y(T) = \sigma_0 \exp\,(-BT)\,, \tag{11}$$

where B is a parameter which can be determined by yield stress measurement and also can be analyzed by interatomic potential function [7]. From (10) and (11) we get

$$K_{1c} = \text{const}\exp\,(BT)\,. \tag{12}$$

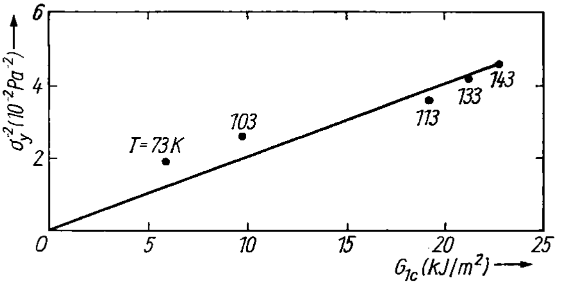


Fig. 2. The G_{1c} values (quenched and tempered at 650 °C for 2 h) [4] and σ_y^{-2} values (800 °C annealed in vacuum) [5] of Fe–C alloys (0.03 wt% C)

We may now compare relative values of K_{ic} under different temperatures

$$\frac{K_{ic}(T_1)}{K_{ic}(T_2)} = \exp\,[B(T_1 - T_2)]\,.$$

The most important parameter is B which controls the temperature dependence of the material. For example, for low carbon steels $B \approx 5 \times 10^{-3}\,K^{-1}$ [7]; therefore, $K_{Ic}(273\,K)/K_{Ic}(77\,K) \approx e$, $a_c(273\,K)/a_c(77\,K) \approx e^2 \approx 10$!! This means that if the critical crack length of a material is 10 mm at room temperature, then, at 77 K, the critical crack length decreases to 1 mm. Low carbon steels at 77 K are much brittler than at room temperature.

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