第25卷 第2期
2008年04月

工 程 数 学 学 报
CHINESE JOURNAL OF ENGINEERING MATHEMATICS

Vol. 25 No. 2
Apr. 2008

文章编号:1005-3085(2008)02-0267-06

求解载流薄壳二维磁弹性的差分法*

边宇虹[1,2], 田振国[1,2], 白象忠[1,2]

(1- 燕山大学建筑工程与力学学院，秦皇岛 066004; 2- 中国科学院力学研究所，
国家非线性连续介质力学重点实验室(LNM)，北京 100080)

摘 要: 本文针对在电磁场和机械场耦合作用下的载流薄壳的非线性变形问题进行了研究。给出了载流薄壳在耦合场作用下的二维电动力学方程、磁弹性非线性运动方程和 Lorentz 力表达式，通过变量代换将描述载流薄壳的磁弹性状态方程整理成含有10个基本未知函数的标准 Cauchy 型。并通过差分法及准线性化方法，将标准 Cauchy 型非线性偏微分方程组，变换成为能够用正交离散法编程求解的准线性微分方程组，实现了载流薄壳的磁弹性应力与变形的数值解。

关键词: 载流；薄壳；磁弹性；非线性；差分法

分类号: AMS(2000) 34K28 **中图分类号**: O441; O482.6 **文献标识码**: A

1 引言

磁弹性理论的研究在最近三十年中得到了快速发展[1]。近些年来，许多学者在这方面进行了积极的探索，Hasanyan[2,3]给出了理想导体板在倾斜磁场中非线性振动的数学模型，并研究了有限导电性板条在磁场中的非线性振动问题。Hany 和 Kamal[4]借助于 Laplace 及指数 Fourie 变换方法解决了二维热磁弹性问题，给出了半空间导体中的温度、应力以及感应磁场的分布。Ezzat 和 El-Karamany[5]借助于状态空间和 Laplace 变换解决导电介质在两次热松弛情况下的二维热磁弹性问题。这些成果为磁弹性理论及其应用的研究打下了良好基础。

但是，目前非线性理论研究还不够完备，除梁、板、壳在电磁场中的振动和稳定性问题外，其应力应变状态的分析非常少见，而且仅仅停留在解一维[6-8](或轴对称)问题的数值解上。对于二维或空间磁弹、塑性问题，只处于建立方程式的理论研究阶段，尚未见到载流板壳在电磁场中的应力应变二维及三维问题解的研究，也很少见到实际应用方面的研究成果。国内外一些学者尽管在理论上部分解决了对可变形物体的电磁-力耦合问题的描述，然而由于其数学模型具有高度的非线性性质，即便有简单力学解的结构，其对于电磁场的确定也是相当复杂的。因而，严格应用连续介质力学的理论求得二维乃至三维问题的解析解，仍具有难以克服的困难。因为工程与生产实际要求给出各个参变量的具体数值，例如，改进 Tokamak 装置中 TF 构件和核反应堆防护壳的设计方案时，就需要计算变化的电磁场引起上述构件的应力应变状态变化的具体数值，这就使此类问题数值解的研究显得非常必要。

本文针对在电磁场和机械场耦合作用下的载流薄壳的磁弹性应力与变形问题进行了研究，得到了载流薄壳的磁弹性应力与变形的数值解。为改变在电磁场环境下的工程结构中板壳的工作状态提供了理论分析和数值计算方法。

收稿日期: 2006-03-14. **作者简介**: 边宇虹(1962年5月生)，女，博士生，教授. 研究方向：板壳磁弹性理论及应用.
***基金项目**: 国家自然科学基金(50275128)；河北省自然科学基金(A2006000190).

2 基本方程

如图1所示，在时变磁场中，载流柔性体满足直法线假设及纵向纤维无挤压的 Kirchhoff-Love 假设。在满足磁弹性假说[9]的基础上，根据弹性力学理论、电磁场理论[10]中的欧姆定律和 Maxwell 方程，导出了载流薄壳的二维电动力学方程、磁弹性运动方程和 Lorentz 力表达式。

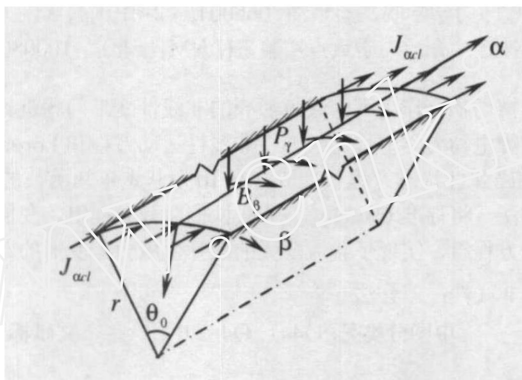

图 1: 条形薄壳上电流磁场分布图

2.1 二维电动力学方程

$$
\begin{aligned}
&-\frac{\partial B_\gamma}{\partial t}=\frac{\partial E_\beta}{\partial \alpha}-\frac{1}{r}\frac{\partial E_\alpha}{\partial \beta},\\
&\sigma\Big[E_\alpha+\frac{\partial v}{\partial t}B_\gamma-\frac{1}{2}\frac{\partial w}{\partial t}(B_\beta^{+}+B_\beta^{-})\Big]=\frac{1}{r}\frac{\partial H_\gamma}{\partial \beta}-\frac{H_\beta^{+}-H_\beta^{-}}{h},\\
&\sigma\Big[E_\beta-\frac{\partial u}{\partial t}B_\gamma+\frac{1}{2}\frac{\partial w}{\partial t}(B_\alpha^{+}+B_\alpha^{-})\Big]=-\frac{\partial H_\gamma}{\partial \alpha}+\frac{H_\alpha^{+}-H_\alpha^{-}}{rh},
\end{aligned}
\tag{1}
$$

式中 u, v, w 分别为相应方向的位移；E_α，E_β，E_γ 分别为相应方向的电场强度；H_α，H_β，H_γ 分别为相应方向的磁场强度；B_α，B_β，B_γ 分别为相应方向的磁感应强度；h 为壳的厚度；r 为壳的半径；t 为时间变量；σ 为壳的电导率；$B_i^{\pm}$，$H_i^{\pm}$ $(i=\alpha,\beta)$ 分别为相应量在壳上、下表面的量值。

2.2 运动方程

$$
\begin{aligned}
&\frac{\partial N_\alpha}{\partial \alpha}+\frac{1}{r}\frac{\partial S}{\partial \beta}+P_\alpha+n_\alpha+\rho f_\alpha=\rho h\frac{\partial^2 u}{\partial t^2},\\
&\frac{1}{r}\frac{\partial N_\beta}{\partial \beta}+\frac{\partial S}{\partial \alpha}+\frac{1}{r}\frac{\partial M_{\alpha\beta}}{\partial \alpha}+\frac{Q_\beta}{r}+P_\beta+n_\beta+\rho f_\beta=\rho h\frac{\partial^2 v}{\partial t^2},\\
&\frac{\partial(Q_\alpha-N_\alpha\theta_\alpha-S\theta_\beta)}{\partial \alpha}+\frac{\partial(Q_\beta-N_\beta\theta_\beta-S\theta_\alpha)}{r\partial \beta}-\frac{1}{r}N_\beta+P_\gamma+n_\gamma+\rho f_\gamma=\rho h\frac{\partial^2 w}{\partial t^2},\\
&\frac{\partial M_\alpha}{\partial \alpha}+\frac{1}{r}\frac{\partial M_{\alpha\beta}}{\partial \beta}-Q_\alpha=\frac{\rho h^3}{12}\frac{\partial^2 \theta_\alpha}{\partial t^2},\quad \frac{1}{r}\frac{\partial M_\beta}{\partial \beta}+\frac{\partial M_{\alpha\beta}}{\partial \alpha}-Q_\beta=\frac{\rho h^3}{12}\frac{\partial^2 \theta_\beta}{\partial t^2},
\end{aligned}
\tag{2}
$$

式中

$$\rho f_\alpha = J_{\beta cl}hB_\gamma + \sigma hE_\beta B_\gamma - \sigma h\frac{\partial u}{\partial t}B_\gamma^2 + \frac{\sigma h}{2}\frac{\partial w}{\partial t}(B_\alpha^+ + B_\alpha^-)B_\gamma,$$

$$\rho f_\beta = -J_{\alpha cl}hB_\gamma - \sigma hE_\alpha B_\gamma - \sigma h\frac{\partial v}{\partial t}B_\gamma^2 + \frac{\sigma h}{2}\frac{\partial w}{\partial t}(B_\beta^+ + B_\beta^-)B_\gamma,$$

$$\begin{aligned}\rho f_\gamma = {} & \frac{h}{2}\left[J_{\alpha cl}(B_\beta^+ + B_\beta^-) - J_{\beta cl}(B_\alpha^+ + B_\alpha^-)\right] + \frac{\sigma h}{2}E_\alpha(B_\beta^+ + B_\beta^-) \\ & -\frac{\sigma h}{2}E_\beta(B_\alpha^+ + B_\alpha^-) + \sigma h\left[\frac{1}{2}\frac{\partial v}{\partial t}(B_\beta^+ + B_\beta^-) + \frac{h}{12}\frac{\partial \theta_\beta}{\partial t}(B_\beta^+ - B_\beta^-)\right]B_\gamma \\ & -\sigma h\frac{\partial w}{\partial t}\left[\frac{1}{4}(B_\beta^+ + B_\beta^-)^2 + \frac{1}{12}(B_\beta^+ - B_\beta^-)^2 + \frac{1}{4}(B_\alpha^+ + B_\alpha^-)^2 + \frac{1}{12}(B_\alpha^+ - B_\alpha^-)^2\right] \\ & +\frac{\sigma h}{2}\frac{\partial u}{\partial t}B_\gamma(B_\alpha^+ + B_\alpha^-) + \frac{\sigma h^2}{12}\frac{\partial \theta_\alpha}{\partial t}B_\gamma(B_\alpha^+ - B_\alpha^-),\end{aligned} \tag{3}$$

式中 N_α，N_β，Q_α，Q_β，S，M_α，M_β 和 $M_{\alpha\beta}$ 分别为壳内相应方向上的内力和力矩；P_α，P_β，P_γ 分别为机械载荷；n_α，n_β，n_γ 分别为体积力；ρf_α，ρf_β，ρf_γ 分别为相应方向上的 Lorentz 力；θ_α，θ_β 分别为相应方向的转角；$J_{\alpha cl}$，$J_{\beta cl}$ 分别为相应方向侧向电流密度；ρ 为壳的质量密度。

3　载流薄壳的磁弹性非线性方程

选择 u，v，w，θ_β，N_β，$\hat{Q}_\beta$，S，M_β，E_α 和 B_γ 作为基本未知函数，同时考虑薄壳的几何方程和物理方程[11]，可得如下偏微分方程组

$$\frac{\partial u}{\partial \beta} = \frac{2rS}{D_N(1-\nu)} - r\frac{\partial v}{\partial \alpha} + r\frac{\partial w}{\partial \alpha}\theta_\beta, \qquad \frac{\partial v}{\partial \beta} = \frac{r}{D_N}N_\beta - w - \frac{r}{2}\theta_\beta^2 - \frac{r\nu}{2}\left(\frac{\partial w}{\partial \alpha}\right)^2 - r\nu\frac{\partial u}{\partial \alpha},$$

$$\frac{\partial w}{\partial \beta} = -r\theta_\beta, \qquad \frac{\partial \theta_\beta}{\partial \beta} = \frac{r}{D_M}M_\beta + r\nu\frac{\partial^2 w}{\partial \alpha^2},$$

$$\frac{\partial N_\beta}{\partial \beta} = r\rho h\frac{\partial^2 v}{\partial t^2} - r\frac{\partial S}{\partial \alpha} - (\hat{Q}_\beta + N_\beta\theta_\beta + S\theta_\alpha) - r(P_\beta + n_\beta + \rho f_\beta),$$

$$\begin{aligned}\frac{\partial \hat{Q}_\beta}{\partial \beta} = {} & \frac{r\rho h^3}{12}\cdot\frac{\partial^2}{\partial t^2}\left(\frac{12}{h^2}w - \frac{\partial^2 w}{\partial \alpha^2}\right) - r(P_\gamma + n_\gamma + \rho f_\gamma) - r\nu\frac{\partial N_\beta}{\partial \alpha}\cdot\frac{\partial w}{\partial \alpha} - r\nu N_\beta\frac{\partial^2 w}{\partial \alpha^2} \\ & -\frac{\partial w}{\partial \alpha}rEh\left(\frac{\partial^2 u}{\partial \alpha^2} + \frac{\partial w}{\partial \alpha}\cdot\frac{\partial^2 w}{\partial \alpha^2}\right) - \frac{\partial^2 w}{\partial \alpha^2}rEh\left[\frac{\partial u}{\partial \alpha} + \frac{1}{2}\left(\frac{\partial w}{\partial \alpha}\right)^2\right] + rS\frac{\partial \theta_\beta}{\partial \alpha} \\ & +r\theta_\beta\frac{\partial S}{\partial \alpha} - r\nu\frac{\partial^2 M_\beta}{\partial \alpha^2} + \frac{rEh^3}{12}\cdot\frac{\partial^4 w}{\partial \alpha^4} + N_\beta,\end{aligned} \tag{4}$$

$$\frac{\partial S}{\partial \beta} = r\rho h\frac{\partial^2 u}{\partial t^2} - rEh\left(\frac{\partial^2 u}{\partial \alpha^2} + \frac{\partial w}{\partial \alpha}\cdot\frac{\partial^2 w}{\partial \alpha^2}\right) - r\nu\frac{\partial N_\beta}{\partial \alpha} - r(P_\alpha + n_\alpha + \rho f_\alpha),$$

$$\frac{\partial M_\beta}{\partial \beta} = r\hat{Q}_\beta + rN_\beta\theta_\beta + rS\theta_\alpha + \frac{r\rho h^3}{12}\cdot\frac{\partial^2 \theta_\beta}{\partial t^2} - 2r\frac{\partial}{\partial \alpha}\left(D_M(1-\nu)\frac{\partial \theta_\beta}{\partial \alpha}\right),$$

$$\frac{\partial E_\alpha}{\partial \beta} = -\frac{r}{\sigma\mu}\cdot\frac{\partial^2 B_\gamma}{\partial \alpha^2} + rB_\gamma\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial \alpha}\right) + r\frac{\partial u}{\partial t}\cdot\frac{\partial B_\gamma}{\partial \alpha} - \frac{(B_\alpha^+ + B_\alpha^-)}{2}r\cdot\frac{\partial}{\partial t}\left(\frac{\partial w}{\partial \alpha}\right) + r\frac{\partial B_\gamma}{\partial t},$$

$$\frac{\partial B_\gamma}{\partial \beta}=r\sigma\mu\Big[E_\alpha+\frac{\partial v}{\partial t}B_\gamma-\frac{\partial w}{\partial t}\cdot\frac{(B_\beta^+ + B_\beta^-)}{2}\Big]+\frac{B_\beta^+ - B_\beta^-}{h}r,$$

式中 $D_N(=Eh/(1-\nu^2))$ 和 $D_M(=Eh^3/[12(1-\nu^2)])$ 分别为壳的抗拉刚度和抗弯刚度；E 为弹性模量；ν 为泊松比；μ 为板的绝对磁导率；且

$$\hat{Q}_\beta=Q_\beta-N_\beta\theta_\beta-S\theta_\alpha+\frac{\partial M_{\alpha\beta}}{\partial\alpha}. \tag{5}$$

4 计算方法

为求解式(4)的非线性问题，通常采用迭代法。在采用迭代法解方程组(4)之前，先应用差分通项[11]对方程组(4)的右侧部分进行差分，得到如下差分方程

$$\frac{\partial u^i}{\partial\beta}=\frac{2rS^i}{D_N(1-\nu)}-r\frac{v^{i+1}-v^{i-1}}{2\lambda}+r\frac{w^{i+1}-w^{i-1}}{2\lambda}\theta_\beta^i,$$

$$\frac{\partial v^i}{\partial\beta}=\frac{r}{D_N}N_\beta^i-w^i-\frac{r}{2}(\theta_\beta^i)^2-\frac{r\nu}{2}\Big(\frac{w^{i+1}-w^{i-1}}{2\lambda}\Big)^2-r\nu\Big(\frac{u^{i+1}-u^{i-1}}{2\lambda}\Big),$$

$$\frac{\partial w^i}{\partial\beta}=-r\theta_\beta^i,\qquad \frac{\partial\theta_\beta^i}{\partial\beta}=\frac{r}{D_M}M_\beta^i+r\nu\frac{w^{i+1}-2w^i+w^{i-1}}{\lambda^2},$$

$$\frac{\partial N_\beta^i}{\partial\beta}=r\rho h\frac{\partial^2 v^i}{\partial t^2}-r\frac{S^{i+1}-S^{i-1}}{2\lambda}-\Big(\hat{Q}_\beta^i+N_\beta^i\theta_\beta^i-S^i\frac{w^{i+1}-w^{i-1}}{2\lambda}\Big)-r(P_\beta^i+n_\beta^i+\rho f_\beta^i),$$

$$\begin{aligned}\frac{\partial\hat{Q}_\beta^i}{\partial\beta}=&\frac{r\rho h^3}{12}\cdot\frac{\partial^2}{\partial t^2}\Big(\frac{12}{h^2}w^i-\frac{w^{i+1}-2w^i+w^{i-1}}{\lambda^2}\Big)-r\nu\frac{N_\beta^{i+1}-N_\beta^{i-1}}{2\lambda}\cdot\frac{w^{i+1}-w^{i-1}}{2\lambda}-rP_\gamma^i\\&-r\rho f_\gamma^i-\frac{w^{i+1}-w^{i-1}}{2\lambda}rEh\Big(\frac{u^{i+1}-2u^i+u^{i-1}}{\lambda^2}+\frac{w^{i+1}-w^{i-1}}{2\lambda}\cdot\frac{w^{i+1}-2w^i+w^{i-1}}{\lambda^2}\Big)\\&-r\nu N_\beta^i\frac{w^{i+1}-2w^i+w^{i-1}}{\lambda^2}-\frac{w^{i+1}-2w^i+w^{i-1}}{\lambda^2}rEh\Big[\frac{u^{i+1}-u^{i-1}}{2\lambda}+\frac{1}{2}\Big(\frac{w^{i+1}-w^{i-1}}{2\lambda}\Big)^2\Big]\\&+\frac{rEh^3}{12}\cdot\frac{w^{i+2}-4w^{i+1}+6w^i-4w^{i-1}+w^{i-2}}{\lambda^4}-r\nu\frac{M_\beta^{i+1}-2M_\beta^i+M_\beta^{i-1}}{\lambda^2}\\&+rS^i\frac{\theta_\beta^{i+1}-\theta_\beta^{i-1}}{2\lambda}+r\theta_\beta^i\frac{S^{i+1}-S^{i-1}}{2\lambda}-rn_\gamma^i+N_\beta^i,\end{aligned} \tag{6}$$

$$\begin{aligned}\frac{\partial S^i}{\partial\beta}=&r\rho h\frac{\partial^2 u^i}{\partial t^2}-r(P_\alpha^i+n_\alpha^i+\rho f_\alpha^i)-rEh\frac{u^{i+1}-2u^i+u^{i-1}}{\lambda^2}-r\nu\frac{N_\beta^{i+1}-N_\beta^{i-1}}{2\lambda}\\&-rEh\frac{w^{i+1}-w^{i-1}}{2\lambda}\cdot\frac{w^{i+1}-2w^i+w^{i-1}}{\lambda^2},\end{aligned}$$

$$\frac{\partial M_\beta^i}{\partial\beta}=r\hat{Q}_\beta^i+rN_\beta^i\theta_\beta^i-rS^i\frac{w^{i+1}-w^{i-1}}{2\lambda}-2D_M(1-\nu)r\frac{\theta_\beta^{i+1}-2\theta_\beta^i+\theta_\beta^{i-1}}{\lambda^2}+\frac{r\rho h^3}{12}\cdot\frac{\partial^2\theta_\beta^i}{\partial t^2},$$

$$\begin{aligned}\frac{\partial E_\alpha^i}{\partial\beta}=&r\frac{\partial B_\gamma^i}{\partial t}-\frac{r}{\sigma\mu}\cdot\frac{B_\gamma^{i+1}-2B_\gamma^i+B_\gamma^{i-1}}{\lambda^2}-\frac{B_\alpha^+ + B_\alpha^-}{2}r\cdot\frac{\partial}{\partial t}\Big(\frac{w^{i+1}-w^{i-1}}{2\lambda}\Big)\\&+\frac{\partial}{\partial t}\Big(\frac{u^{i+1}-u^{i-1}}{2\lambda}\Big)rB_\gamma^i+r\frac{\partial u^i}{\partial t}\cdot\frac{B_\gamma^{i+1}-B_\gamma^{i-1}}{2\lambda},\end{aligned}$$

$$\frac{\partial B_\gamma^i}{\partial \beta} = r\sigma\mu\Big[E_\alpha^i + \frac{\partial v^i}{\partial t}B_\gamma^i - \frac{1}{2}\frac{\partial w^i}{\partial t}(B_\beta^+ + B_\beta^-)\Big] + \frac{B_\beta^+ - B_\beta^-}{h}r,$$

式中 Lorentz 力分量的差分表达式为

$$\rho f_\alpha^i = J_{\beta cl}hB_\gamma^i - \frac{h}{\mu}B_\gamma^i\Big(\frac{B_\gamma^{i+1} - B_\gamma^{i-1}}{2\lambda} - \frac{B_\alpha^+ - B_\alpha^-}{h}\Big),$$

$$\rho f_\beta^i = -J_{\alpha cl}hB_\gamma^i - \sigma hE_\alpha^iB_\gamma^i - \sigma h\frac{\partial v^i}{\partial t}(B_\gamma^i)^2 + \frac{\sigma h}{2}\frac{\partial w^i}{\partial t}(B_\beta^+ + B_\beta^-)B_\gamma^i,$$

$$\begin{aligned}\rho f_\gamma^i = {} & \frac{h}{2}\Big[J_{\alpha cl}(B_\beta^+ + B_\beta^-) - J_{\beta cl}(B_\alpha^+ + B_\alpha^-)\Big] + \frac{\sigma h}{2}E_\alpha^i(B_\beta^+ + B_\beta^-) \\ & + \frac{\sigma h}{2}(B_\alpha^+ + B_\alpha^-)\Big[\frac{1}{\sigma\mu}\Big(\frac{B_\gamma^{i+1} - B_\gamma^{i-1}}{2\lambda} - \frac{B_\alpha^+ - B_\alpha^-}{h}\Big) + \frac{1}{2}\frac{\partial w^i}{\partial t}(B_\alpha^+ + B_\alpha^-)\Big] \\ & - \sigma h\frac{\partial w^i}{\partial t}\Big[\frac{1}{4}(B_\beta^+ + B_\beta^-)^2 + \frac{1}{12}(B_\beta^+ - B_\beta^-)^2 + \frac{1}{4}(B_\alpha^+ + B_\alpha^-)^2 + \frac{1}{12}(B_\alpha^+ - B_\alpha^-)^2\Big] \\ & + \sigma h\Big[\frac{1}{2}\frac{\partial v^i}{\partial t}(B_\beta^+ + B_\beta^-) + \frac{h}{12}\frac{\partial \theta_\beta^i}{\partial t}(B_\beta^+ - B_\beta^-)\Big]B_\gamma^i \\ & - \frac{\sigma h^2}{12}\frac{\partial}{\partial t}\Big(\frac{w^{i+1} - w^{i-1}}{2\lambda}\Big)B_\gamma^i(B_\alpha^+ - B_\alpha^-),\end{aligned} \tag{7}$$

式中 λ 为沿 α 方向的差分步长。将式(7)代入式(6)，在相应的初始条件及边界条件下，采用数值计算中的正交离散法即可求出基本未知函数的值。对于时变电磁场和机械载荷，式(4)需按瞬态求解。采用 Newmark N M 稳定有限等差式[12]进行计算。

5 结论

本文针对在电磁场和机械场耦合作用下的载流薄壳的磁弹性应力与变形问题进行了研究。通过变量代换将描述载流薄壳的磁弹性状态方程整理成含有10个基本未知函数的标准 Cauchy 型，并通过差分法及准线性化方法，将标准 Cauchy 型非线性偏微分方程组，变换成为能够用正交离散法求解的准线性微分方程组。可以求解载流薄壳的非线性磁弹性问题。

参考文献：

[1] 周又和，郑晓静. 电磁固体结构力学[M]. 北京：科学出版社，1999, 14, 15

[2] Hasanyan D J, Khachaturyan G M, Piliposyan G T. Mathematical modeling and investigation of nonlinear vibration of perfectly conductive plates in an inclined magnetic field[J]. Thin-walled Structures, 2001, 39(1): 111-123

[3] Hasanyan, Davresh, Librescu, Liviu. Nonlinear vibration of finitely electro-conductive plate-strips in a magnetic field[C]// Collection of Technical Papers-AIAA/ASME/ ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference, 2003, 4: 2827-2837

[4] Hany H Sherie, Kamal A Hekmy. A two-dimensional problem for a half-space in magneto thermoelasticity with thermal relaxation[J]. Int J Eng Sci, 2002, (40): 587-604

[5] Magdy A Ezzat, Ahmed S El-Karamany. Magnetothermoelasticity with two relaxation times in conducting medium with variable electrical and thermal conductivity[J]. Applied Mathematics and Computation, 2003, (142): 449-467

[6] Molchenko L V. Nonlinear deformation of shells of rotation of an arbitrary meridian in a nonstationary magnetic field[J]. International Applied Mechanics, 1996, 32(3): 173-179

[7] Molchenko L V, Loos I I. Nonlinear deformation of conical shells in magnetic fields[J]. International Applied Mechanics, 1997, 33(3): 221-226

[8] Molchenko L V. Influence of an external electric current on the stress state of an annular plate of variable stiffness[J]. Prikladnaya Mekhanika, 2001, 37(12): 108-112

[9] Мольченко Л В. Магнитоупругость нелинейных токонесущих облочек[M]. Киев: Вишашкола, 1989

[10] 虞福春，郑春开. 电动力学[M]. 北京：北京大学出版社，2004, 8: 54-55

[11] 徐芝纶. 弹性力学[M]. 北京：高等教育出版社，1990

[12] Newmark N M. A method of computation for structural dynamics[J]. J End Mech Div Proc ASCE, 1959, 85(7): 67-97

Difference Method for Solving Two-dimensional Magneto-elasticity of Thin Current-carrying Shells

BIAN Yu-hong[1,2], TIAN Zhen-guo[1,2], BAI Xiang-zhong[1,2]

(1- College of Civil Engineering and Mechanics, Yanshan University, Qinhuangdao 066004;
2- The State Key Laboratory of Nonlinear Continuum Mechanical (LNM), Institute of Mechanics, Chinese Academy of Science, Beijing 100080)

Abstract: In this paper, the problems of nonlinear deformation of thin current-carrying shell under the coupled action of the electromagnetic field and mechanical field are studied. Derived are the two-dimensional electrodynamics equations, the nonlinear magneto-elastic kinetic equations and the expressions of Lorentz force of thin current-carrying shell under the action of the coupled field, the normal Cauchy form nonlinear differential equations, which includes ten basic unknown functions in all, are obtained by means of the variable replacement method. Using the difference method and quasi-linearization method, the nonlinear magneto-elastic equations are reduced to a sequence of quasi-linear differential equations, which can be solved by the orthogonal discrete method. The numerical solution of the stresses and deformations in thin current-carrying shell is realized.

Keywords: current-carrying; thin shell; magneto-elastic; nonlinear; difference method