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Finite length microchannel flow in transitional regime
(—Analytical prediction compared with experiment)

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Abstract: In a steady finite length channel of gas flow, analytical expressions of the pressure distribution along the channel axis and of mass flow rate are obtained by the global mass conservation law for continuum and slip flow regimes. In the transitional flow regime a strict kinetic theoretical solution of the channel flow pressure distribution is also provided, when the kinetic theoretical solution of the mass flux for the Poiseuille flow is known, the equation governing the pressure distribution is shown to be the degenerated Reynolds equation transplanted by the senior author from the bearing lubrication problem to the microchannel problem. Analytical expression of the flow mass rate is also obtained and compared with the available experimental data and excellent agreement is obtained.

Key words: microchannel; global mass conservation law; kinetic theoretical solution

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0 Introduction

Microchannel is the basic constituent of the MEMS devices, being regular and simple in form, it can reveal the specific features of the low speed micro internal flows. The flow in the microchannel has been studied intensively both experimentally and theoretically.

In the experimental study the technique of combined surface-bulk silicon micro-machining was used to fabricate integrated microchannel/pressure sensor systems, the pressure distribution along the channel axis, the flow rate and the friction factor were obtained[1-8]. The theoretical analyses accompanying these experimental investigations have been based without exception on continuum models, sometimes with slip velocity boundary conditions involved. But the gas flow in the experimental systems is not restricted in slip flow regime, when helium is used as the media in the channel the Knudsen number can reach 0.4[6,7], 0.8[8] and 2.5[4,5], and under such *Kn* numbers the flow is certainly in the transitional regime. Theoretical investigation of channel flow in transitional regime is highly desirable.

In the classical fluid mechanics the steady Poiseuille flow is considered as a flow in long tube or channel having constant pressure gradient $G = -\partial p/\partial x$ (see, e.g., Batchelor[9]). In the kinetic theoretical consideration a constant pressure gradient is also proposed for the linearized Boltzmann solution of the Poiseuille flow of gas (see e.g. Ohwada et al.[10]). But the constancy of pressure gradient in a tube or channel is true only for the flow of liquid (with constant density) and, as shown in the following, in any realistic tube or channel gas flow the pressure gradient is never a constant but varies from one cross section to another and is determined by the global mass conservation law. The assumption of the constancy of the pressure gradient in the Poiseuille flow is a hypothetical necessity, in fact, the solution of the Poiseuille flow determines the local flow characteristics in dependence of the local pressure gradient.

In a steady finite length channel the global mass conservation law determines the pressure distribution along the

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channel axis and analytical expressions of this pressure distribution are obtained for continuum and slip flow regimes[11]. Of course, as implied in [11] and [12], the analytical expressions of mass flow rate can also be simply obtained and are given here. In the transitional flow regime a strict kinetic theoretical solution of the channel flow pressure distribution is also provided by the global mass conservation across the channel, when the kinetic theoretical solution of the mass flux for the Poiseuille flow is known, the equation governing the pressure distribution is shown to be the degenerated Reynolds equation transplanted by the senior author from the bearing lubrication problem to the microchannel problem[11,12]. Analytical expression of the flow rate in the transitional flow regime is also obtained for the diffuse reflection case. The analytical predictions are compared with the available experimental data and excellent agreement is obtained.

1 The pressure distribution determined by the global mass conservation

Consider the gas flow in a channel between two plane plates at a cross section x with the local pressure gradient $\mathrm{d}p/\mathrm{d}x$, which can vary from one cross section to another (see Fig. 1). First we consider the continuum flow case. In the case of low speed the temperature variation is neglected, μ is a constant, the Navier-Stokes equation attains the form

$$\frac{\mathrm{d}^2 u}{\mathrm{d}y^2} = \frac{\mathrm{d}p/\mathrm{d}x}{\mu} \tag{1}$$

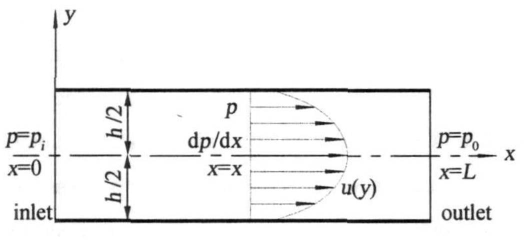

Fig. 1 The schematic of the Poiseuille flow

图 1 平面 Poiseuille 流动示意图

Equation (1) with the slipless boundary condition $u = 0$ at $y = h/2$ yields

$$u = -\frac{1}{8\mu}(h^2 - 4y^2)\frac{\mathrm{d}p}{\mathrm{d}x} \tag{2}$$

The mass flux $Q_{m,C}$ in unit time through the gap between the two plates (with unit length in z) is obviously

$$Q_{m,C} = 2\rho\int_0^{\frac{h}{2}} u\,\mathrm{d}y = \frac{-p}{RT}\frac{\mathrm{d}p}{\mathrm{d}x}\frac{h^3}{12\mu} \tag{3}$$

The global mass conservation across any cross section requires that

$$p(\mathrm{d}p/\mathrm{d}x) = \text{constant} \tag{4}$$

or in the dimensionless variables $P = p/p_0$ and $X = x/L$ (p_0: the pressure at the outlet, L: the length of the channel)

$$P(\mathrm{d}P/\mathrm{d}X) = C_{m,C} \tag{5}$$

From the above equation with the inlet and outlet boundary conditions

$$P = P_i = p_i/p_0 \text{ at } X = 0, \text{ and } P = 1 \text{ at } X = 1 \tag{6}$$

the pressure distribution is easily determined

$$P = P_i\left[1 - \left(1 - \frac{1}{P_i^2}\right)X\right]^{1/2} \tag{7}$$

with the pressure gradient

$$\frac{\mathrm{d}P}{\mathrm{d}X} = -\frac{1}{2}P_i\left(1 - \frac{1}{P_i^2}\right)\left[1 - \left(1 - \frac{1}{P_i^2}\right)X\right]^{-1/2} \tag{8}$$

Obviously, the pressure distribution is approximately linear if the value $(1 - 1/P_i^2)$ is small, and its slope is negative with $P_i^2 > 1$, and the absolute value of the pressure slope monotonously increases with increasing X.

So, the constant

$C_{m,C} = -P_i^2(1 - 1/P_i^2)/2 = P\mathrm{d}P/\mathrm{d}X$. And substitute it into equation (3), then get the mass flux

$$Q_{m,C} = \frac{-p}{RT}\frac{\mathrm{d}p}{\mathrm{d}x}\frac{h^3}{12\mu} = -\frac{h^3}{12\mu RT}C_{m,C}\frac{p_0^2}{L}$$

$$= \frac{h^3}{24\mu RTL}(p_i^2 - p_0^2) \tag{9}$$

In the case of slip flow the basic equation (1) remains the same, the velocity profile in the case of slip boundary condition $u|_{y=h/2} = -\zeta\frac{\mathrm{d}u}{\mathrm{d}y}$, with $\zeta = \frac{2-\sigma}{\sigma}\lambda$, is

$$u = -\frac{1}{8\mu}(h^2 - 4y^2 + 4\zeta h)\frac{\mathrm{d}p}{\mathrm{d}x} \tag{10}$$

And the mass flux instead of Eq. (3) is obtained as

$$Q_{m,SLIP} = 2\rho\int_0^{\frac{h}{2}} u\,\mathrm{d}y = \frac{-p}{RT}\frac{\mathrm{d}p}{\mathrm{d}x}\frac{1}{4\mu}\left(\frac{1}{3}h^3 + 2\zeta h^2\right) \tag{11}$$

Note that ζ is dependent on p: $\zeta = (2-\sigma)\lambda/\sigma$ and as $\rho\lambda$ = constant, one can write $\zeta = \zeta_0/p$, where

$$\zeta_0 = \frac{2-\sigma}{\sigma}\lambda p = \frac{2-\sigma}{\sigma}\lambda_0 p_0 \qquad (12)$$

So from Eq. (11) the mass conversation can be written as

$$p\frac{\mathrm{d}p}{\mathrm{d}x} + 6\frac{\zeta_0}{h}\frac{\mathrm{d}p}{\mathrm{d}x} = \text{constant}$$

or in dimensionless variables

$$P\frac{\mathrm{d}P}{\mathrm{d}X} + 6\frac{2-\sigma}{\sigma}Kn_0\frac{\mathrm{d}P}{\mathrm{d}X} = C_{m,SLIP} \qquad (13)$$

where $Kn_0 = \lambda_0 / h$ is Knudsen number at the outlet of the channel. The solution of the above equation with the integration constants determined from the inlet and outlet boundary conditions, Eq. (6), is

$$P = -6\frac{2-\sigma}{\sigma}Kn_0 + \sqrt{\left(6\frac{2-\sigma}{\sigma}Kn_0\right)^2 + P_i^2 + 12\frac{2-\sigma}{\sigma}Kn_0 P_i + \left[1 - P_i^2 + 12\frac{2-\sigma}{\sigma}Kn_0(1-P_i)\right]X} \qquad (14)$$

and $P\frac{\mathrm{d}P}{\mathrm{d}X} + 6\frac{2-\sigma}{\sigma}Kn_0\frac{\mathrm{d}P}{\mathrm{d}X} = C_{m,SLIP} = (1-P_i^2)/2 + 6\frac{2-\sigma}{\sigma}Kn_0(1-P_i)$. Note that the mass flux Eq. (11) in slip flow can also be written as

$$Q_{m,SLIP} = \frac{-h^3}{12\mu RT}\left(1 + 6\frac{2-\sigma}{\sigma}Kn\right)\frac{p\mathrm{d}p}{\mathrm{d}x} = \frac{-h^3}{12\mu RT}\widetilde{Q}_{m,SLIP}\frac{p\mathrm{d}p}{\mathrm{d}x},$$

$$\widetilde{Q}_{m,SLIP} = 1 + 6\frac{2-\sigma}{\sigma}Kn \qquad (15)$$

or

$$Q_{m,SLIP} = \frac{-h^3}{12\mu RT}C_{m,SLIP}\frac{p_0^2}{L},$$

$$C_{m,SLIP} = (1-P_i^2)/2 + 6\frac{2-\sigma}{\sigma}Kn_0(1-P_i) \qquad (16)$$

if the first half of Eq. (15) is used for ζ, and here $Kn = \lambda / h$ is the local Knudsen number. This is the well known solution of the mass flow rate of the Poiseuille flow in the slip flow regime, where p, $\mathrm{d}p/\mathrm{d}x$ and Kn are local values of pressure, pressure gradient and Knudsen number.

The above expression of pressure distribution in the slip flow regime seems to be obtained the earliest by Arkilic through a different derivation as a first approximation with accuracy of order $\varepsilon = H/L$ (see Ph. D. Thesis of E. B. Arkilic [5] and its publication version [13]), and the similar results have been also published and recorded in [14-16].

In the transitional flow regime the flow rate of the plane Poisuille flow has been solved by the linearized Boltzmann equation or its BGK[17] model version by many authors (see e. g. by Cercignani and Daneri [18] and Ohwada et al. [10]). Fukui and Kaneko[19] in derivation of the generalized Reynolds lubrication equation in the lubrication problem also calculated the flow rate of Poiseuille flow numerically, and later[20] they had used this flow rate calculated rigorously to generate a database for rapid calculation of the generalized Reynolds equation for high Knudsen numbers. The flow rate of the Poiseuille flow can be expressed in the form

$$Q_{m,TR} = \frac{-h^3}{12\mu RT}\widetilde{Q}_{m,TR}(Kn)\frac{p\mathrm{d}p}{\mathrm{d}x} \qquad (17)$$

where Kn is the local Knudsen number and $\widetilde{Q}_{m,TR}$ is the flow rate in transitional regime (normalized by the slip-less value $Q_{m,C}$) calculated from the linearized Boltzmann equation for Poiseuille flow. A tabled database of the calculated values of $\widetilde{Q}_{m,TR}$ for $\sigma = 1$, $\sigma = 0.9$, $\sigma = 0.8$ and $\sigma = 0.7$ is provided in [20], and a fitted formula approximation for diffuse reflection ($\sigma = 1$) by Robert is recorded in [21] (there the second term on the right hand side is misprinted as $6A\sqrt{\pi}Kn$)

$$\widetilde{Q}_{m,TR}(Kn) = 1 + 6AKn + \frac{12}{\pi}Kn\log(1 + BKn) \qquad (18)$$

where $A = 1.318889$ and $B = 0.387361$ (Yang [22] suggests that

$$\widetilde{Q}_{m,TR}(Kn) = 1 + 6aKn + \frac{12}{\pi}bKn\log(1 + Kn)$$

$a = 1.223401$ and $b = 0.624958$. For $Kn < 5$, the difference of above two fitting formula is small). The global mass conservation, i. e., the constancy of the mass flux being equal at any cross section, requires

$$\widetilde{Q}_{m,TR}(Kn)\frac{p\mathrm{d}p}{\mathrm{d}x} = C_{m,TR} \qquad (19)$$

or in the dimensionless form

$$\frac{\mathrm{d}}{\mathrm{d}X}\left[\widetilde{Q}_{m,TR}(Kn)P\frac{\mathrm{d}P}{\mathrm{d}X}\right] = 0 \qquad (20)$$

With the database incorporated this equation is valid for any surface conditions of the plates and can be integrated numerically. But for the illustrative purpose only the case of complete diffuse reflection, $\sigma = 1$, is expounded here, then the mass conservation is (from Eqs. (18) and (20))

$$\frac{d}{dX}\{[1 + 6AKn + \frac{12}{\pi}Kn\log(1 + BKn)]P\frac{dP}{dX}\} = 0 \tag{21}$$

For the ease of integration the local Knudsen number Kn is most conveniently expressed through P

$$Kn = \frac{\lambda}{h} = \frac{\lambda_0}{h}\frac{p_0}{p} = Kn_0/P, \tag{22}$$

and Eq. (21) becomes

$$[(P + 6AKn_0 + \frac{12}{\pi}Kn_0\log(1 + \frac{BKn_0}{P})]\frac{dP}{dX} = C_{m,TR,CDR} \tag{23}$$

where $C_{m,TR,CDR}$ is an unspecified constant to be determined from the integration. The equations determining the pressure distribution (20) and (21) are in fact the generalized Reynolds equation of the gas lubrication theory degenerated for the microchannel flows (see [20,21]). Equation (23) is integrated and it becomes

$$\frac{1}{2}(P^2 - P_i^2) + 6AKn_0(P - P_i) + \frac{12Kn_0}{\pi}\times [P\log(1 + \frac{BKn_0}{P}) - P_i\log(1 + \frac{BKn_0}{P_i}) + BKn_0\log(\frac{P + BKn_0}{P_i + BKn_0})] = C_{m,TR,CDR}X \tag{24}$$

here

$$C_{m,TR,CDR} = \frac{1}{2}(1 - P_i^2) + 6AKn_0(1 - P_i) + \frac{12Kn_0}{\pi}\times [\log(1 + BKn_0) - P_i\log(1 + \frac{BKn_0}{P_i}) + BKn_0\log(\frac{1 + BKn_0}{P_i + BKn_0})] \tag{25}$$

And the corresponding mass flow rate is

$$Q_{m,TR,CDR} = \frac{-h^3}{12\mu RT}C_{m,TR,CDR}\frac{p_0^2}{L} \tag{26}$$

2 Comparison with the experimental data and the simulation results

To compare the above results with the experimental data of flow in the microchannel with pressure sensors imbedded in the channel accomplished by Zohar et al.[7], we calculate the pressure distribution for argon in the $0.53\times 40\times 4000\mu m^3$ channel[7]. Under the experimental condition $T_0 = 293K$ the value of Kn_0 for argon is 0.1333. The pressure drops $\Delta p = p_i - p_0$ of the channel in kPa are given in [7] (measured from Fig. 13 in [7]), and the corresponding values of $P_i = P|_{X=0}$ are listed in Table 1. In Eq. (14) and (24) Kn_0 is taken equal to 0.1333, and $\sigma = 1$. Both are under the following boundary condition $P_i = P|_{X=0} = p_i/p_0$ as given in Table 1, and $P|_{X=1} = p_0/p_0 = 1$. The comparison shows that the global mass conservation predicts the pressure distribution quite well and the results obtained by Eq. (14) and (24) improve the agreement of the continuum prediction with the experimental data (see Fig. 2).

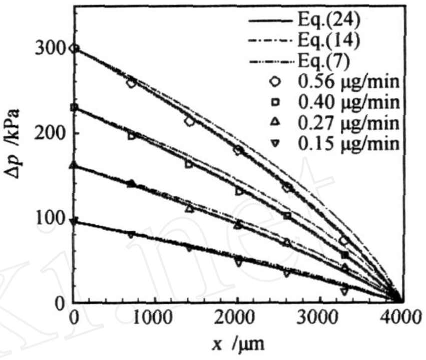

Fig. 2 Comparison of the pressure distribution obtained from Eq. (24) (solid line), Eq. (14) (dash-dot line), and Eq. (7) (dash-dot-dot line) with erimental data in a $0.53\times 40\times 4000\mu m^3$ microchannel for argon [7] (symbols) at $Kn_0 = 0.1333$

图2 在 $Kn_0 = 0.1333$ 时,由式(24)(实线),式(14)(点划线)和式(7)(双点划线)和在 $0.53\times 40\times 4000\mu m^3$ 槽道内氩气试验[7](符号)得到的压力分布的比较

We calculate the mass flow rate for helium in the $0.53\times 40\times 4000\mu m^3$ channel[7]. Under the experimental condition $T_0 = 293K$ the value of Kn_0 for helium is 0.3684. The comparison shows that the global mass conservation predicts the mass flow rate also quite well and the results obtained by Eq. (26) improve the agreement of the continuum Eq. (9) and slip flow (16) prediction with the experimental data (see Fig. 3).

About the comparison with the simulation results obtained by IP method [23-27] can be seen in [11,12].

Table 1 The experimental pressure drop data in kPa and corresponding values of P_i for Argon

表1 氩气的实验压力数据(单位 kPa)和相应的 P_i 值

for argon[7]				
p_i (in kPa)	300	230	161	95.3
P_i	3.96	3.27	2.59	1.94

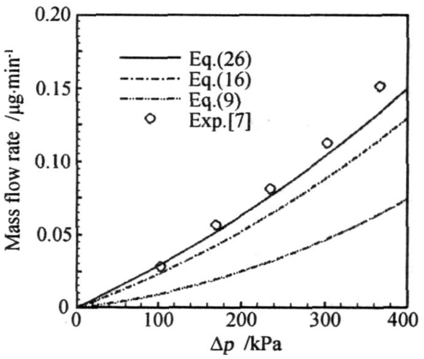

Fig. 3　Comparison of the mass flow rate obtained rom Eq. (26) (solid line) , Eq. (16) (dash-dot line) , and Eq. (9) (dash-dot-dot line) with experimental data in a $0.53 \times 40 \times 4000 \mu m^3$ microchannel for helium [7] (symbols) at $Kn_o = 0.3684$

图 3　在 $Kn_o = 0.3684$ 时 ,由式(26) (实线) ,式(16) (点划线) 和式(9) (双点划线) 和在 $0.53 \times 40 \times 4000 \mu m^3$ 槽道内氦气试验[7] (符号) 得到的质量流率的比较

3　Concluding remarks

In the present paper the finite length microchannel pressure distribution for continuum flow , slip flow and transitional flow regimes is determined by the global mass conservation equation. In the transitional flow regime this equation is the Reynolds equation for the gas film lubrication problem degenerated by the suggestion of the senior author[12] for solving the microchannel flow. The comparison shows excellent agreement with the experimental data. The global mass conservation equation (the degenerated Reynolds equation) provides a means with the merit of strict kinetic theory to test various methods in solving the micro scale rarefied gas dynamics flows in transitional regime. From the practical application point of view database for the flow rates of the Poiseuille flow with various combinations of possible surface properties obtained from strict kinetic theory is an actual task for the solution of the microchannel flow , and also for the thin film air bearing problem and the gas damping problem[28] in micromechanical accelerometers. The database in the form of fitting formulas is especially desirable.

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有限长度的过渡领域微槽道流动 (—分析预测和试验的比较)

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摘 要: 在连续介质和滑流区有限长度槽道内的稳定气流 ,通过整体质量守恒规律得到了沿槽道轴向的压力分布和质量流率的分析表达式。在过渡领域区 ,也提供了槽道流动压力分布的严格的动理学理论解。在 Poiseuille 流动的质量流率动理学理论解给定的情况下 ,压力分布的控制方程被证实是被前一作者从轴承润滑问题移植到解微槽道问题的退化 Reynolds 方程。同时得到了质量流率分析表达式 ,它们和现有试验数据的比较相符很好。

关键词: 微槽道 ;整体质量守恒规律 ;动力学理论解