第 15 卷 第 12 期 强 激 光 与 粒 子 束 Vol. 15, No. 12
2003 年 12 月 HIGH POWER LASER AND PARTICLE BEAMS Dec., 2003

Article ID: 1001-4322(2003)12-1195-05

Numerical study on the Rayleigh-Taylor instability with various initial length scale*

WANG Li-li[1], LI Jia-chun[2]

(1. *Institute of Applied Physics and Computational Mathematics, P. O. Box* 8009-12, *Beijing* 100088, *China*)

(2. *Institute of Mechanics, the Chinese Academy of Sciences, Beijing* 100080, *China*)

Abstract: The Rayleigh-Taylor instability caused by a density gradient is very important in the inertial confinement fusion. With the passive scalar transport model, behaviors of the interface with various initial length scales are simulated. It appears that the initial form has a significant influence upon the interfacial motion. Long grooves grow more slowly than square shaped perturbations. And generally the mixing process in the vertical direction may be decreased by the interaction between bubbles with different scales.

Key words: Rayleigh-Taylor instability; Passive scalar transport modal; Inertial confinement fusion (ICF); Multi-scale; Interface

CLC number: TN241; O354 **Document code:** A

In the inertial confinement fusion (ICF), when the deuterium-tritium capsule is irradiated with an intense laser, the shell of the capsule is spherically accelerated inwards to compress the DT fuel. The light fluid formed by ablation of the shell pushes the heavy fluid (the rest of the shell) across a surface, which will cause the Rayleigh-Taylor instability. Evolution of the Rayleigh-Taylor instability disrupts the spherical symmetry and may cause the failure of the ignition of DT. Therefore, it is very important to understand this instability for improving the design of the shells. As a key problem in both fundamental research and technological applications, the Rayleigh-Taylor instability has attracted great attention in the past two decades. Several numerical methods have been performed, such as the level set method[1,2], the front tracking scheme[3], the lattice Boltzmann scheme[4] and so on.

The physical interest of this paper is focused on the effect of the length scale in the initial perturbation on the evolution of the instability. Evolution from two kinds of initial perturbation forms is achieved. One is a single sinusoidal bubble with different length scales in x and y directions. The other is with several adjacent bubbles in different length scale, with consideration of both the wavelength and the amplitude.

1 The numerical method

For the sake of simplicity, the passive scalar modal is adopted in which a dissolving diluted passive scalar is applied to simulate two fluids with different density. With consideration of the Boussinesq flow, the fluid is assumed to be incompressible and only the variation of density in the buoyancy term is taken into account. The governing equations and the passive scalar transportation equation are as follows

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}=v\zeta_z-w\zeta_y-\frac{\partial P}{\partial x}+\frac{\partial \tau_{xx}}{\partial x}+\frac{\partial \tau_{xy}}{\partial y}+\frac{\partial \tau_{xz}}{\partial z} \tag{2}$$

$$\frac{\partial v}{\partial t}=w\zeta_x-u\zeta_z-\frac{\partial P}{\partial y}+\frac{\partial \tau_{xy}}{\partial y}+\frac{\partial \tau_{yy}}{\partial x}+\frac{\partial \tau_{yz}}{\partial z} \tag{3}$$

$$\frac{\partial w}{\partial t}=u\zeta_y-v\zeta_x-\frac{g\rho}{\rho_0}-\frac{\partial P}{\partial z}+\frac{\partial \tau_{xz}}{\partial x}+\frac{\partial \tau_{yz}}{\partial y}+\frac{\partial \tau_{zz}}{\partial z} \tag{4}$$

* **Received date:** 2003-09-24; **Revised date:** 2003-10-14
Foundation item: Supported by National Natural Science Project (10176032)
Biography: WANG Li-li (1968-), female, postdoctor, vice-research fellow; mainly studies on fluid mechanics.

$$\frac{\partial \rho}{\partial t} = - u\frac{\partial \rho}{\partial x} - v\frac{\partial \rho}{\partial y} - w\frac{\partial \rho}{\partial z} + \frac{\partial \tau_{\rho x}}{\partial x} + \frac{\partial \tau_{\rho y}}{\partial y} + \frac{\partial \tau_{\rho z}}{\partial z} \tag{5}$$

where a is the acceleration, ρ is the density of the computational cell, ρ_0 is the mean density of the cells around it, ζ_i is the vorticity component. And

$$\tau_{ij} = \nu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), \quad \tau_{\rho i} = k\frac{\partial \rho}{\Delta x_i}$$

where ν and k are the molecular diffusivities of momentum and mass diffusion, correspondingly.

Periodic boundary conditions are applied on the four sides, while the non-slip boundary conditions are applied at top and bottom walls. Each of the two fluids occupies half of the computational domain. A mixed scheme of Fourier expansion in the horizontal direction and finite differencing in the vertical direction is therefore appropriate for the simulation. The time advancement is solved using the Adams-Bathoforth scheme. The results are presented in terms of dimensionless variables, with the length of the shortest side of the computational domain as the length scale, L, and $\sqrt{L/(Aa)}$ as the time scale. A is the Atwood number.

2 Numerical results

2.1 Initial perturbation with different length scale in x and y directions

The interest of this section is to compare the evolutions of bubbles with different shapes. The perturbation we discussed here is of a 3-dimensional sinusoidal form as follows:

$$\zeta = A\left(\cos\frac{2\pi x}{\lambda_x} + \cos\frac{2\pi y}{\lambda_y}\right) \tag{6}$$

where λ_i $(i = x, y)$ denotes the wavelength in the i direction. To study the effect of the different length scale in x and y directions, we define the perturbation's aspect ratio as $\alpha = \lambda_y / \lambda_x$.

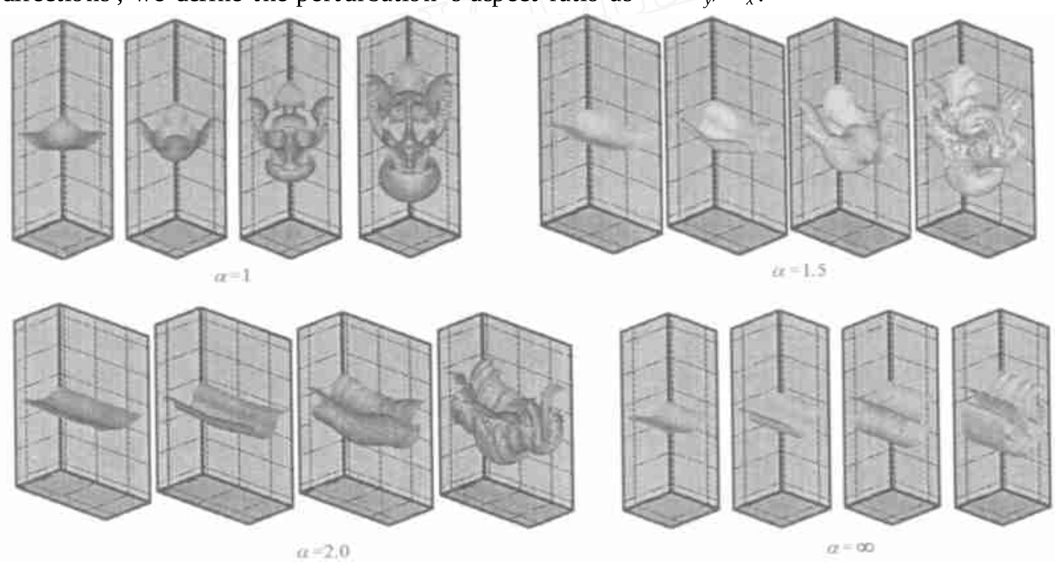

Fig. 1 Evolutions of the interface with different perturbation aspect ratios

Four cases with different value of α are simulated here. Evolutions of the interface are shown as Fig. 1. We can see that the profile of the interface grows more quickly with smaller aspect ratio. From previous study we know that the growth of the bubble and spike is consistent with the theoretically predicted exponential growth in the early stage. At the nonlinear stage the bubble tends to grow at a constant speed, in which the terminal velocity of the bubble can be expressed with the following form

$$v_{\mathrm{b}} = C\sqrt{g\lambda/2} \tag{7}$$

where C is a constant. Table 1 shows the variation of C with perturbation aspect ratio α. As expected, C decreases when α increases. In other words, the long troughs grow more slowly than the square shaped bubbles. The spike front and bubble front versus time is plotted in Fig. 2.

Table 1 Variation of C with perturbation aspect ratio α

α	1	1.5	2	∞
C	0.39	0.33	0.31	0.23
$C/\sqrt{A}$	0.67	0.57	0.54	0.40

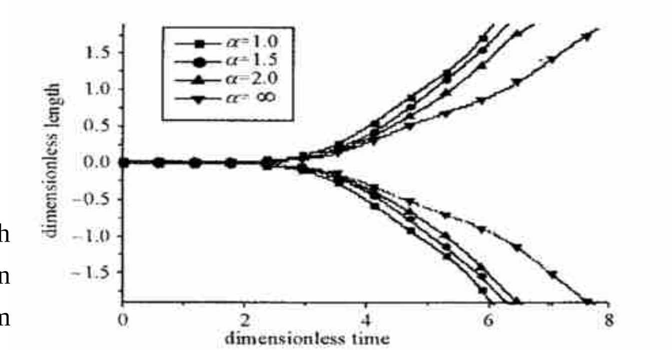

Fig. 2 Bubble and spike fronts versus time

2.2 Initial perturbation with multi-scales

In this section we deal with initial perturbationwith multi-scales. The initial perturbation contains bubbles in two length scales. The first bubble is of sinusoidal form with wavelength λ_1 and amplitude A_1, while the bubbles around it with λ_2 and A_2. Evolution of the instability is plotted in Fig. 3.

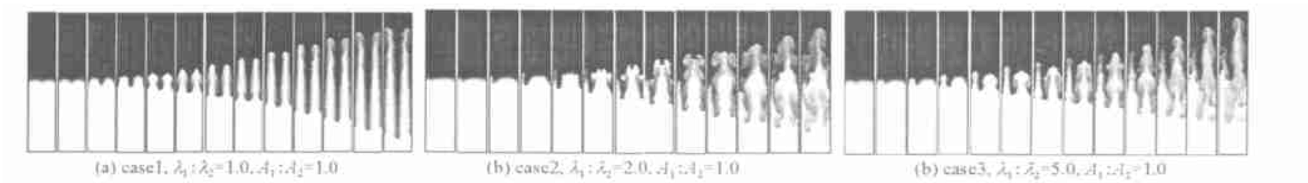

Fig. 3 Evolution of the instability with different length scales

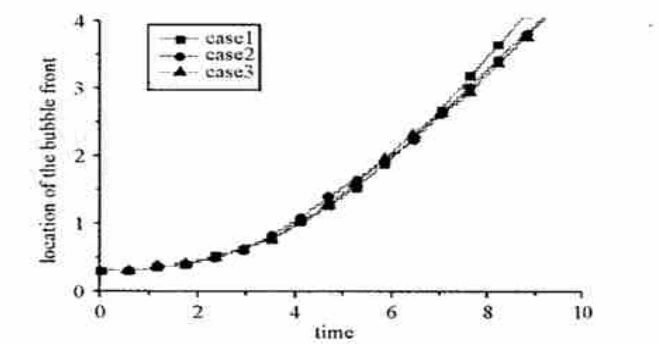

Fig. 4 Location of the bubble front versus time

The result shows that bubbles with the same length scale evolve almost independently with little influence to each other before they contact each other. This is understandable for the two bubbles have the same velocity. For bubbles with different length scales bubble merger can be observed. With the elapse of time, the small length scale bubbles will inosculate with the adjacent large one. At the later stage the characteristic length of the interfacial front is dominated by the large bubble. The competition among bubbles is caused by the variety of velocity between different bubbles. At the linear stage, the smaller the bubble is, the more quickly the perturbation evolves. But during the nonlinear stages, velocity of the bubble is proportional to the square root of the wavelength to Layzer's theory. Thus the larger bubble grows more and more quickly until it moves far ahead of the smaller ones. Location of the interface front is an important characteristic to describe the instability. Fig. 4 shows the location of interfacial front versus time. At the final stage, the interfacial front moves more slowly in cases contain different scales than that in the case with only one scale.

With regard to the Rayleigh-Taylor instability in ICF, sometimes evolution of the mixing in the vertical direction is much more critical than that in the horizontal direction. To learn the influence of the bubble competition, we studied the mean kinetic energy, in both the horizontal and vertical directions. Fig. 5 shows that generally the kinetic energy in the horizontal direction significantly increases when the initial perturbation contains bubbles with different scales. Meanwhile the kinetic energy in the vertical direction decreases. In other words, due to the interaction between bubbles with different scales, the mixing is enhanced in the horizontal direction.

3 Conclusions

The behavior of Rayleigh-Taylor instability with multi-scale pertubation is simulated in this paper. From the simulation we get the following conclusions:

The shape of the initial perturbation affects the behavior of Rayleigh-Taylor instability in a marked way. Long grooves grow more slowly than square shaped perturbations. Both the velocity of bubble and the mixing rate de-

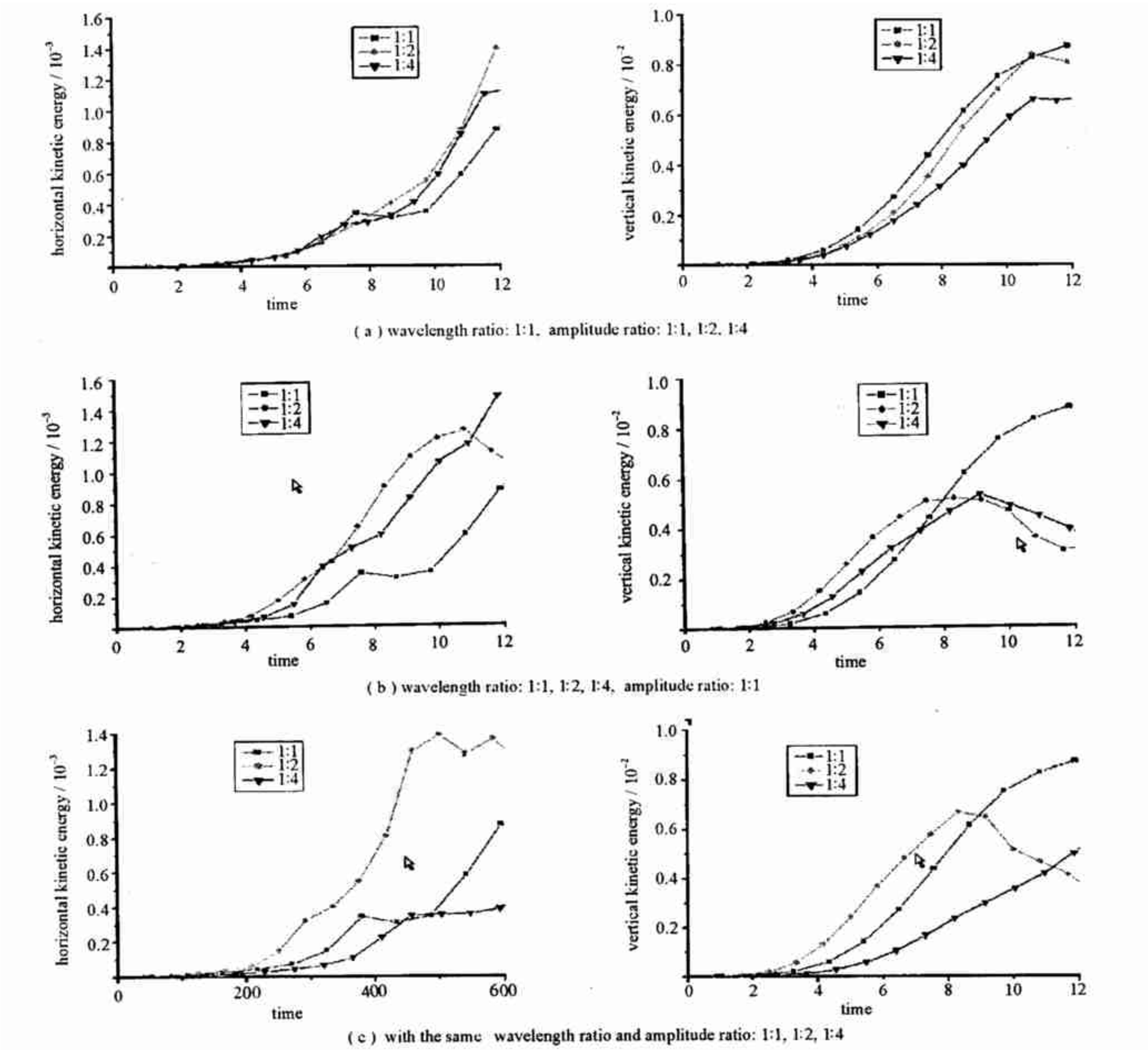

Fig. 5 Mean kinetic energy of per unit mass in the horizontal and vertical motions

crease when the bubble aspect ratio increases.

Simulation of Rayleigh-Taylor instability developed from several adjacent bubbles is performed. Bubble merger is observed when the two bubbles are in different length scale. The results show that the mixing process in the vertical direction is decreased by the bubble competition. This conclusion may be useful in the design of the capsule in ICF.

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瑞利-泰勒不稳定性中尺度效应的数值模拟研究*

王丽丽[1]， 李家春[2]

(1. 北京应用物理与计算数学研究所, 北京 100088; 2. 中国科学院 力学研究所, 北京 100080)

摘要： 瑞利-泰勒不稳定性是一种由于密度梯度引起的界面不稳定性,在惯性约束聚变中具有重要的意义。利用被动标量输运模型对包含不同尺度初始扰动的界面演化过程进行数值模拟。计算结果表明界面的初始形状对不稳定性的发展具有很大的影响,狭长型扰动比正方型扰动发展慢。另外,不同尺度扰动的相互作用一般会减小沿界面发展方向运动的动能,使能量更多地用于平行于界面方向的运动。

关键词： 瑞利-泰勒不稳定性； 被动标量输运模型； 惯性约束聚变； 多尺度； 界面

* 第七届全国激光科学技术青年学术交流会优秀论文。