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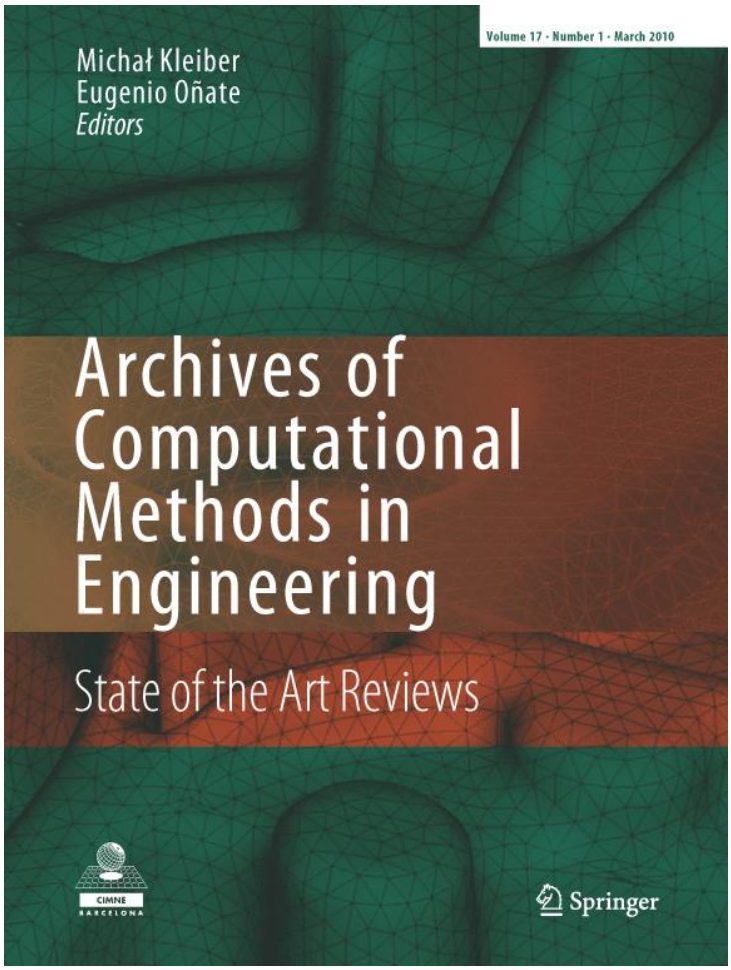

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Smoothed Particle Hydrodynamics (SPH): an Overview and Recent Developments

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Abstract Smoothed particle hydrodynamics (SPH) is a meshfree particle method based on Lagrangian formulation, and has been widely applied to different areas in engineering and science. This paper presents an overview on the SPH method and its recent developments, including (1) the need for meshfree particle methods, and advantages of SPH, (2) approximation schemes of the conventional SPH method and numerical techniques for deriving SPH formulations for partial differential equations such as the Navier-Stokes (N-S) equations, (3) the role of the smoothing kernel functions and a general approach to construct smoothing kernel functions, (4) kernel and particle consistency for the SPH method, and approaches for restoring particle consistency, (5) several important numerical aspects, and (6) some recent applications of SPH. The paper ends with some concluding remarks.

1 Introduction

1.1 Traditional Grid Based Numerical Methods

Computer simulation has increasingly become a more and more important tool for solving practical and complicated problems in engineering and science. It plays a valuable role in providing tests and examinations for theories, offering insights to complex physics, and assisting in the interpretation and even the discovery of new phenomena. Grid or mesh based numerical methods such as the finite difference methods (FDM), finite volume methods (FVM) and the finite element methods (FEM) have been widely applied to various areas of computational fluid dynamics (CFD) and computational solid mechanics (CSM). These methods are very useful to solve differential or partial differential equations (PDEs) that govern the concerned physical phenomena. For centuries, the FDM has been used as a major tool for solving partial differential equations defined in problem domains with simple geometries. For decades, the FVM dominates in solving fluid flow problems and FEM plays an essential role for solid mechanics problems with complex geometry [1–3]. One notable feature of the grid based numerical models is to divide a continuum domain into discrete small subdomains, via a process termed as discretization or meshing. The individual grid points (or nodes) are connected together in a pre-defined manner by a topological map, which is termed as a mesh (or grid). The meshing results in elements in FEM, cells in FVM, and grids in FDM. A mesh or grid system consisting of nodes, and cells or elements must be defined to provide the relationship between the nodes before the approximation process for the differential or partial differential equations. Based on a properly pre-defined mesh, the governing equations can be converted to a set of algebraic equations with nodal unknowns for the field variables. So far the grid based numerical models have achieved remarkably, and they are currently the dominant methods in numerical simulations for solving practical problems in engineering and science [1–5].

Despite the great success, grid based numerical methods suffer from difficulties in some aspects, which limit their

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