

Initiation and Development of Water Film by Seepage

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Abstract: When water seeps upwards through a saturated soil layer, the soil layer may become instability and water films occur and develop. Water film serves as a natural sliding surface because of its very small friction. Accordingly, debris flow may happen. To investigate this phenomenon, a pseudo-three-phase media is presented first. Then discontinuity method is used to analyze the expansion velocity of water film. Finally, perturbation method is used to analyze the case that a water flow is forced to seep upwards through the soil layer while the movement of the skeleton may be neglected relative to that of water. The theoretical evolutions of pore pressure gradient, effective stress, water velocity, the porosity and the eroded fine grains are obtained. It can be seen clearly that with the erosion and re-deposited of fine grains, permeability at some positions in the soil layer becomes smaller and smaller and, the pore pressure gradient becomes bigger and bigger, while the effective stress becomes smaller and smaller. When the effective stress equals zero, e.f. liquefaction, the water film occurs. It is shown also that once a water film occurs, it will be expanded in a speed of $U(t)/(1-\varepsilon)$.

Keywords: Slope; Seepage; Debris flow; Water film

Introduction

It is occurred often in the mountain area that soil layer or grain flow translates to debris flow

(CUI 1992, CUI et al. 2009, HU et al. 2009, CHEN et al. 2007). When saturated soil layer stays on a slope, it may become instability and water films occur and develop during water seepage. Accordingly, debris flow happens.

The possible existence of “water film” in soil containing an impermeable layer was first suggested by Seed (1987) in attempting to explain slope failure observed in earthquakes. The “water film” may serve as a sliding surface for post-liquefaction failure. Thereafter, landslide or debris flow may occur on a slope with very gentle slope-angle. The “water film” in saturated soil is a water gap which forms when the permeability is nonuniform and therefore the pore water is trapped by relatively low permeable layers. The soil grains do not support one another and are therefore suspended in the condition of zero effective stresses (Scott 1986). The soil grains in suspension eventually settle because they are heavier than water. The rate of such settlement is restricted by the fact that water must flow upward around the soil grains. If liquefiable soil deposits are overlain by less permeable soils in a stratified or layered deposit, the overlaying deposit can restrict the pore water to pass through. If there is no downward drainage through the deposit, this relative flow at the interface, by continuity, must be equal to the velocity of settlement at the upper liquefied sand surface (Feigel et al. 1994). Thus an accumulation of water in the form of a water gap at the interface forms. Feigel and Kutter (1994)

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performed a centrifuge shake table test to demonstrate the formation of water films in layered sand. More recently, Kokusho (1999) performed shake table tests using sand samples containing a seam of nonplastic silt and showed that water films were formed beneath the silt layer. In this case the column was subjected to horizontal dynamic loadings to simulate earthquakes. The mechanism was explored subsequently (Kokusho, 2000, 2002). Experimental observations on the formation of water films in vertical columns of saturated sand contained in circular cylinders have also been reported by ZHANG et al. (1999) and PENG et al (2000). In both cases care is taken in preparing the sample by feeding wetted uniform sand continuously into a column of water to avoid intentional stratification. However, small inhomogeneity still exists due to uneven settling velocity.

However, the formation mechanism of the “water film” in a soil layer with the porosity distributed continuously is not clear. The formation of “water film” is one reason that soil layer translates to debris flow. As debris flow may flow through a much more long distance than landslide or grain flow because of the lower obstruction, so debris flow may cause heavier damage.

According to the viewpoints above, analysis is presented in this paper to obtain the formation mechanism and conditions of the water film.

1 Formulation of the Problem

It is considered that a saturated soil layer stays on a slope and the water is not over the surface. The fine grains are assumed to be eroded from the skeleton, the eroding relation is assumed to be the following type (CHENG et al. 2000, LU et al. 2006). The x axis is upward and y in horizontal direction.

$$\frac{1}{\rho_s} \left(\frac{\partial Q}{\partial t} + u_s \frac{\partial Q}{\partial x} \right) = \frac{\lambda}{T} \left(\frac{u - u_s}{u^*} - q \right) \quad (1)$$

$$\text{if } -\varepsilon(x,0) \leq \frac{Q}{\rho_s} \leq \frac{Q_c(x)}{\rho_s}$$

$$\frac{1}{\rho_s} \left(\frac{\partial Q}{\partial t} + u_s \frac{\partial Q}{\partial x} \right) \leq 0 \quad \text{in other conditions} \quad (2)$$

in which the first term on the right side of the first equation shows that how the fine grains is being transferred to water, the second term describing deposition places a limit on the amount of soil that can be carried in the field, Q is the mass of fine grains eroded per unit volume of the soil/water mixture, ρ_s is the density of the grains, u and u_s are the velocities of the seepage fluid containing fine soil particles and the skeleton, q is the volume fraction of soil carried in the seepage fluid, T and u^* are physical parameters, λ is a small dimensionless parameter used to arrive at a perturbation solution, $\varepsilon(x,t)$ is the porosity, $Q_c(x)$ is the maximum Q that can be eroded at x .

Considering the erosion of fine grains, a pseudo-three-phase model is presented here. The mass conservation equations are as follows:

$$\frac{\partial(\varepsilon - q)\rho}{\partial t} + \frac{\partial(\varepsilon - q)\rho u}{\partial x} = 0 \quad (3)$$

$$\frac{\partial q \rho_s}{\partial t} + \frac{\partial q \rho_s u}{\partial x} = \frac{\partial Q}{\partial t} + u_s \frac{\partial Q}{\partial x} \quad (4)$$

$$\frac{\partial(1 - \varepsilon)\rho_s}{\partial t} + \frac{\partial(1 - \varepsilon)\rho_s u_s}{\partial x} = -\frac{\partial Q}{\partial t} - u_s \frac{\partial Q}{\partial x} \quad (5)$$

in which ρ is the density of water. A general equation which denotes the volume conservation may be obtained from the above three equations, which is

$$\varepsilon u + (1 - \varepsilon)u_s = U(t) \quad (6)$$

in which $U(t)$ is the flow rate of water and grains per unit cross the sectional area of soil column.

The momentum equations can be written as follows:

$$[(\varepsilon - q)\rho + q\rho_s] \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} \right) = -\varepsilon \frac{\partial p}{\partial x} - \frac{\varepsilon^2(u - u_s)}{k(\varepsilon, q)} - [(\varepsilon - q)\rho + q\rho_s]g \quad (7)$$

$$[(\varepsilon - q)\rho + q\rho_s] \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} \right) + (1 - \varepsilon)\rho_s \left(\frac{\partial u_s}{\partial t} + u_s \frac{\partial u_s}{\partial x} \right) = -\frac{\partial p}{\partial x} - \frac{\partial \sigma_\varepsilon}{\partial x} - [(\varepsilon - q)\rho + q\rho_s]g - (1 - \varepsilon)\rho_s g - \left(\frac{\partial Q}{\partial t} + u_s \frac{\partial Q}{\partial x} \right) (u - u_s) \quad (8)$$

in which eqn.(7) denotes the momentum conservation of water, eqn.(8) denotes the total momentum conservation, the last term on the right hand side of eqn.(8) denotes the momentum caused by the eroded fine grains, p is the pore

pressure, k is the permeability, σ_e is the effective stress. Here k is assumed to be a function of ε and q in the following form

$$k(\varepsilon, q) = k_0 \left(-\alpha \frac{q}{\varepsilon_0} + \beta \frac{\varepsilon}{\varepsilon_0} \right) \quad (9)$$

in which ε_0 is the initial porosity, α, β are parameters and $1 < \beta \ll \alpha$, we choose to let α much greater than β , so that changes in q overweighs that of ε .

2 Expansion Velocity of Water Film

Now, the expansion velocity of water film is analyzed by the discontinuity theory. Assuming the velocity of the discontinuity is w and translating the coordinate with the relation $\xi = x - wt$, which is often adopted in the analysis of discontinuity, e.g. shock wave (Courant 1948), the equations (4)~(8) become

$$\begin{cases} \frac{d}{d\xi}(\varepsilon - q)(u - w) = 0 \\ \frac{d}{d\xi} q(u - w) = G \\ \frac{d}{d\xi} (1 - \varepsilon)(u_s - w) = -G \\ \frac{d}{d\xi} [(\varepsilon - q)\rho + q\rho_s]u(u - w) + \frac{d}{d\xi} (1 - \varepsilon)\rho_s u_s (u_s - w) \\ + \frac{d}{d\xi} (p + \sigma_e) = -[(\varepsilon - q)\rho + (1 - \varepsilon + q)\rho_s] \\ \frac{1}{\varepsilon} \frac{d}{d\xi} [(\varepsilon - q)\rho + q\rho_s]u(u - w) + \frac{d}{d\xi} p \\ = -\frac{H}{\varepsilon} - \left[\rho + \frac{q}{\varepsilon}(\rho_s - \rho) \right] \end{cases} \quad (10)$$

in which $H = \varepsilon^2(u - u_s)/k$, σ_e is the effective stress.

By eq. (10), five discontinuity relations may be obtained by integral at the discontinuity:

$$\begin{cases} \rho_u^+ = \rho^- \\ (\varepsilon^+ - q^+) (u^+ - w^+) = (\varepsilon^- - q^-) (u^- - w^-) \\ (1 - \varepsilon^+) (u_s^+ - w^+) = (1 - \varepsilon^-) (u_s^- - w^-) \\ \varepsilon^+ \rho_u^+ u^+ (u^+ - w^+) + (1 - \varepsilon^+) \rho_s^+ u_s^+ (u_s^+ - w^+) + p^+ + \sigma_e^+ = \\ \varepsilon^- \rho_u^- u^- (u^- - w^-) + (1 - \varepsilon^-) \rho_s^- u_s^- (u_s^- - w^-) + p^- + \sigma_e^- \\ \frac{1}{2} \rho_u^+ (u^+ - w^+)^2 + p^+ = \frac{1}{2} \rho_u^- (u^- - w^-)^2 + p^- \end{cases} \quad (11)$$

in which $\rho_u = \rho + \frac{q}{\varepsilon}(\rho_s - \rho)$, assuming that q/ε and ρ_u are constants at the discontinuity. On the side of the water film ($\varepsilon = 1$), the velocity of water is

u_1 , the pore pressure is p_1 , there are no grains. Outside the water film ($\varepsilon \neq 1$), the parameters are the porosity, the velocity of water, the pore pressure, the velocity of grains and the effective stress $\varepsilon, u, p, u_s, \sigma_e$. Let $u - u_1 = U_1$, $w - u_1 = W$, W is the velocity of discontinuity relative to the water film.

Then we can obtain the following relations according to the discontinuity relations

$$\begin{cases} u_s = w \text{ or } u_s - u_1 = W \\ \varepsilon U_1 + (1 - \varepsilon)W = U(t) \\ \Delta p = p_1 - p = \frac{1}{2} \rho_u [(U_1 - W)^2 - W^2] \\ \Delta p - \sigma_e = -\rho_u W U_1 \\ \sigma_e = \frac{1}{2} \rho_u U_1^2 = \frac{1}{2} \rho_u \left[\frac{U(t) - (1 - \varepsilon)W}{\varepsilon} \right]^2 \end{cases} \quad (12)$$

It is clear that

$$W = \frac{U(t) - \varepsilon \sqrt{2\sigma_e / \rho_u}}{1 - \varepsilon} \quad (13)$$

This equation means that the expansion velocity of water film is proportional to the flow rate of water per unit cross the sectional area deducting the velocity corresponding to the gravity. In case $\sigma_e = 0$

$$W = U(t)/(1 - \varepsilon) \quad (14)$$

3 Initiation of Water Film

By the above analysis, it can be seen that the water film will expand with a velocity W once it initiates in saturated soils. How, where and when water films initiate? As we all know, if there is an undrained over-layer, the water film will initiate just below the over-layer. However, water in a soil column without undrained over-layer are still can form (LU et al. 2006) when a flow of water is forced to percolate through the soils. The mechanism may be that the finer grains are washed away to become part of the percolating fluid and re-deposited later on somewhere down stream, thus causing the decrease of permeability or even blocking of soil column. A perturbation method is used here to obtain an approximate solution to explain the initiation mechanism of water film.

A perturbation solution will be sought using λ as the small parameter. All solutions will be expanded in the following form, e.g.,

$$u(x,t) = \sum_0^{\infty} \lambda^n u_n(x,t) \tag{15}$$

The initial porosity distribution $\varepsilon_0(x)$ is taken as

$$\varepsilon_0(x) = a - b \sin\left(\frac{cx - dL}{L} \pi\right) \tag{16}$$

in which L is the length of the soil column, a,b,c,d are coefficients.

In this case, it is assumed that the skeleton is static relative to seepage fluid so that

$$\varepsilon u = U(t) \tag{17}$$

The initial and boundary conditions are given as follows

$$\varepsilon(0,t)u(0,t) = U(t), \quad q(0,t) = 0 \tag{18}$$

Using perturbation method, eq.(1) ~ eq.(5) yields

$$u = \frac{U(t)}{\varepsilon_0(x)} - \lambda \frac{U(t)}{Tu^* \varepsilon_0^3(x)} \int_0^x U(\tau) d\tau + O(\lambda^2) \tag{19}$$

$$\varepsilon = \varepsilon_0(x) + \frac{\lambda}{Tu^* \varepsilon_0(x)} \int_0^x U(\tau) d\tau + O(\lambda^2) \tag{20}$$

$$Q = \frac{\lambda \rho_s}{Tu^* \varepsilon_0(x)} \int_0^x U(\tau) d\tau + O(\lambda^2) \tag{21}$$

$$q = q_0 + q_1 + O(\lambda^2) \tag{22}$$

in which $q_0(x,t) = 0$ while $q_1(x,t)$ satisfies the following equation

$$\frac{\partial q_1}{\partial t} + \frac{U(t)}{\varepsilon_0(x)} \frac{\partial q_1}{\partial x} - q_1 \frac{U(t)}{\varepsilon_0^2(x)} \frac{d\varepsilon_0(x)}{dx} = \frac{U(t)}{Tu^* \varepsilon_0(x)} \tag{23}$$

This equation describes how $q_1(x,t)$ is gathered as each percolating fluid element moves along its own trajectory, defined at this order of approximation by

$$\frac{dx}{dt} = \frac{U(t)}{\varepsilon_0(x)} \tag{24}$$

Depending on where the trajectory originates, the families of characteristics curves may be determined by direct integration of eqn.(23) using initial and boundary conditions.

Integrating eqn.(23) along the characteristics originating from x axis, $q_1(x,t)$ is obtained as follows

$$q_1 = \frac{1}{Tu^*} \varepsilon_0(x) \int_{x_0}^x \frac{d\xi}{\varepsilon(\xi)} \tag{25}$$

A numerical example about porosity ε is given in Figure 1. It is shown that the porosity will develop in a periodic and increasing way if it is

periodic initially. The reason is that the finer particles are eroded by pore flow. The erosion is heavier at the places with larger porosity because of the larger velocity to make the porosity at these places become larger and larger. However, the formation of water film is not induced by erosion since the erosion in anywhere has a limit and so the increase of porosity is finite. The distribution of permeability, changed by the evolution of porosity and eroded finer particles according to eq. (9), is a control factor of the pore pressure and effective stress which determine the initiation and development of water film.

The momentum equations yield an expression of the effective stress as follows

$$\sigma_e = \int_x^H \frac{g(1-\varepsilon)(\rho_s - \rho)(\varepsilon - q)}{\varepsilon} dx - U(t) \int_x^H \frac{dx}{k} \tag{26}$$

Liquefaction will occur and develop when $\sigma_e = 0$ and $\frac{\partial \sigma_e}{\partial t} < 0$ is satisfied, which leads to the following equations:

$$\sigma_e = \int_x^H \frac{g(1-\varepsilon)(\rho_s - \rho)(\varepsilon - q)}{\varepsilon} dx - U(t) \int_x^H \frac{dx}{k} = 0 \tag{27}$$

$$\left. \frac{\partial \sigma}{\partial t} \right|_{x,t} = g \int_{x,t}^H \frac{\partial}{\partial t} \frac{(1-\varepsilon)(\rho_s - \rho)(\varepsilon - q)}{\varepsilon} dx + U(t) \int_{x,t}^H \frac{dx}{k^2} \left(\frac{\partial k}{\partial \varepsilon} \frac{\partial \varepsilon}{\partial t} + \frac{\partial k}{\partial q} \frac{\partial q}{\partial t} \right) \tag{28}$$

$$\begin{aligned} &\approx -\frac{\alpha \lambda U(t)}{k_0} \int_{x,t}^H (\alpha q_1 - \beta \varepsilon_0(x)) \frac{\partial q_1}{\partial t} dx \\ &= -\frac{\alpha \lambda U^2(t) H}{Tu^*} \int_{\varepsilon(0,t)}^1 \frac{d\xi}{k} \left(1 - \frac{Tu^*}{H} \frac{\partial(q/\varepsilon_0(\xi))}{\partial \xi} \right) \end{aligned}$$

Instituting eq. (20) and eq. (22) into eq. (9), the development of permeability k in soil layer can be obtained.

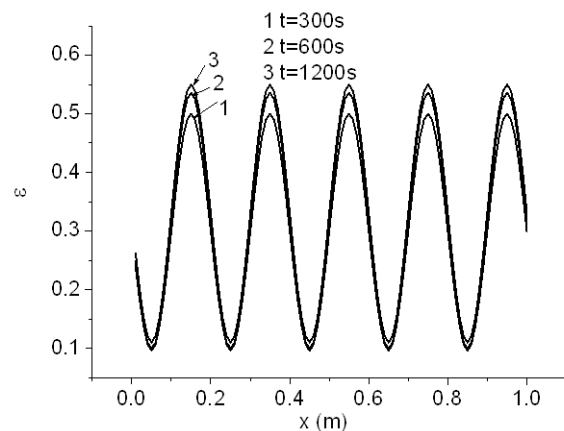


Figure 1 Changes of the porosity with time

Figure 2 shows the development of permeability in the soil layer with time. It can be seen that the permeability decreases gradually to zero at some positions. The zone with zero permeability increases with time. That means, it is blocked at this zone. With the erosion and re-deposit of finer particles, the permeability at some places decrease to zero, e.g., block occurs. Once somewhere is blocked, much more finer particles are re-deposited leads the blocked zone expand because the velocity nearby becomes small.

Figure 3 shows the evolution of k and the pore pressure gradient with time. It is shown that the pore pressure gradients at the interfaces between the blocked places (zone with zero permeability) and the permeable places increase sharply, e.f., discontinuity. Because the flow rate of water and grains per unit cross the sectional area of soil column is unchanged (eq. (6)), the pore pressure gradients at the interfaces with the smallest permeability are the largest. The high pore pressure, which provides the power to push the soil layer above to move upwards, will leads to the initiation of water film.

It is shown in Figure 4 that the effective stress will develop periodic if the initial porosity is periodic. The effective stress at some points in the soil layer will become zero at some time. Thus, the failure of liquefaction will occur at these points. The liquefied soils suspend first and then settle. The above part of the soil column will be pushed upwards under high pore pressure. As a result, water film occurs and expands.

4 Conclusions

It is shown that the occurrence of water films in saturated soils under seepage conditions is due to the jamming of the finer component of the soil. A theory of pseudo-three-phase media is presented to explain the basic formation mechanism of the water films. The primary analysis indicates that the theory does catch the main features. It is shown that if the distribution of ε_0 is periodic, the water film will occur periodic also. The effective stress at some points in the soil layer will become zero (liquefaction) and the gradient of pore pressure becomes sharp with the decrease of permeability. Water films will initiate at these liquefaction zone.

Once a water film initiates, it will expand with a velocity proportional to the flow rate of water per unit cross the sectional area deducting the velocity corresponding to the gravity. After water film occurs, the slop may be unstable and debris flow will form. However, many factors such as the effects of the inertial force are not considered in

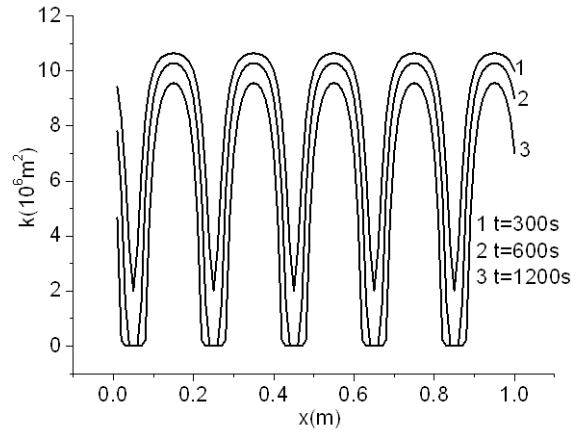


Figure 2 Development of the permeability

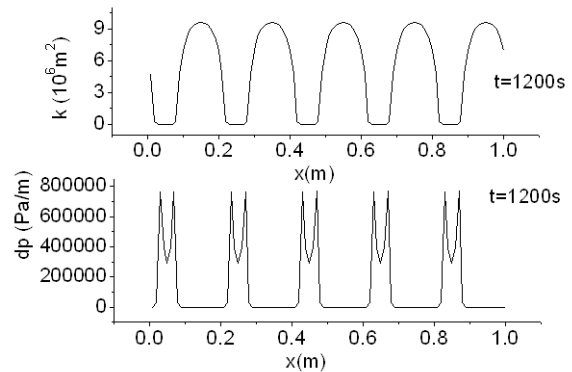


Figure 3 Comparison of the development of permeability and pore pressure gradient ($dp = \partial p / \partial x$)

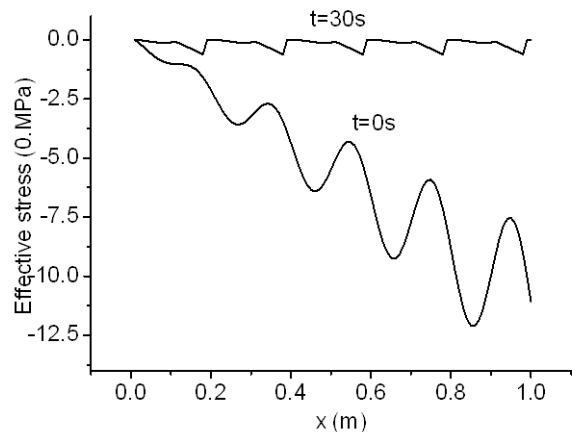


Figure 4 Distribution of effective stress ($t = 30s$)

this paper.

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