International Journal of Heat and Mass Transfer 53 (2010) 2951–2954

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Thermal effects on Rayleigh–Marangoni–Bénard instability in a system of superposed fluid and porous layers

Si Cheng Zhao [a], Qiu Sheng Liu [a,*], Rong Liu [a], Henri Nguyen-Thi [b], Bernard Billia [b]

[a] Key Laboratory of Microgravity (National Microgravity Laboratory), Institute of Mechanics, Chinese Academy of Sciences, Beijing 100190, China
[b] IM2NP, UMR CNRS 6137, Universit d'Aix-Marseille III, 13397 Marseille Cedex 20, France

ARTICLE INFO

Article history:
Received 27 July 2009
Received in revised form 4 March 2010
Accepted 22 March 2010
Available online 21 April 2010

Keywords:
Porous medium
Instability
Rayleigh–Marangoni–Bénard convection
Biot number

ABSTRACT

Thermal effects of the heat transfer at free surface (represented by Biot number) on the Rayleigh–Marangoni–Bénard instability in a system of liquid–porous layers with top free surface are investigated numerically. The results indicate that this thermal effect can evidently lead to the mode transition of convection, which is overlooked in previous works.

© 2010 Elsevier Ltd. All rights reserved.

1. Introduction

The convective motion in a fluid layer [1] can be triggered by buoyancy force [2] (Rayleigh convection), and it has its equivalence for the fluid in the porous media [3,4]. Besides, the convection can also be driven by the surface tension force [5], and the same thing happens to the liquid saturated in the porous layer [6], in which the motion of fluid can be described by using the Brinkman model [7], which can obtain the similar results as those of Darcy's law in the low porosity case.

The onset of pure Rayleigh convection in the superposed liquid–porous layers, sandwiched by two horizontal infinite rigid and thermal conductive wall, heated from below, was first investigated by Chen and Chen [8]. The Darcy's law is applied together with the Beavers and Joseph condition [9] at the liquid–porous interface. They indicate that the neutral instability curves for the onset of instability is bimodal which possess two local minima. The key parameter is the ratio between the depth of liquid layer and that of porous layer. Its critical value $h_c = 0.13$, below which the instability is called the long-wave mode, and above which the instability is called the short-wave mode. After their work, several papers [10–14] studied the coupled gravity and surface tension driven instability problems in a similar system, and all focused on the depth ratio as the crucial parameter which can determine the mode of convection.

The continuity of heat flux across top free surface, written as Newton's cooling law [5], is often employed as a thermal boundary condition. After selecting proper scales (to be mentioned latter), a dimensionless Biot number Bi, which measures the efficiency of heat transfer, is introduced. Desaive and Lebon [15] study the effect of the Biot number on pure buoyancy driven instability. They have a somewhat simple conclusion that the variation of Bi from 0 to 10 significantly affects the magnitude of both critical wavenumber and critical Rayleigh number. The role of Bi is limited when it is higher. Accordingly, we aim to make a thorough investigation and discussion on the onset of convection driven by the gravity and surface tension, under the influence of the Biot number, in a superposed liquid–porous system. The complicated phenomena are overlooked by Desaive and Lebon [15].

2. Mathematical model

We consider a homogeneous porous layer of thickness H_m underlying an incompressible liquid layer of thickness H_l. Cartesian coordinates are used with origin at the liquid–porous interface. The combined system, which is heated from below, is infinite in the horizontal direction x. The z direction is opposite to the gravitational acceleration. The bottom of the porous layer is a rigid, flat and well thermal conductive wall. The upper boundary of liquid layer is a free surface without any deformation. The temperature difference of fluid layer is $\Delta T_l = T_{int} - T_u$, of porous layer is $\Delta T_m = T_b - T_{int}$, and that of the total system is $\Delta T_{tot} = \Delta T_l + \Delta T_m$.

* Corresponding author. Tel.: +86 10 82544095.
E-mail address: liu@imech.ac.cn (Q.S. Liu).

0017-9310/$ - see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijheatmasstransfer.2010.04.003

The continuity, momentum, energy equations for fluid layer are:

$$\nabla \cdot v_l = 0 \tag{1}$$

$$\rho_0\left[\frac{\partial v_l}{\partial t} + (v_l \cdot \nabla) v_l\right] = -\nabla p_l + \mu_l \nabla^2 v_l - \rho_0 g[1 - \alpha(T_l - T_0)]e_z \tag{2}$$

$$\frac{\partial T_l}{\partial t} + (v_l \cdot \nabla) T_l = \kappa_l \nabla^2 T_l \tag{3}$$

The equations for porous layer are:

$$\nabla \cdot v_m = 0 \tag{4}$$

$$\frac{\rho_0}{\phi}\frac{\partial v_m}{\partial t} = -\nabla p_m - \rho_0 g[1 - \alpha(T_m - T_0)]e_z - \frac{\mu_l}{K} v_m \tag{5}$$

$$(\rho c)_m \frac{\partial T_m}{\partial t} + (\rho_0 c_l)(v_m \cdot \nabla) T_m = \kappa_m (\rho_0 c_l) \nabla^2 T_m \tag{6}$$

where $(\rho c)_m = \phi(\rho_0 c_l) + (1 - \phi)(\rho_s c_s)$.

In these equations above, the subscripts l, m and 0 denote quantities of the liquid layer, the porous layer and the ambience, respectively. v denotes the velocity, p the pressure, T the temperature, t the time, ρ the density, $\mu_l = \rho_0 \nu_l$ the dynamic viscosity, κ the heat diffusivity, α the thermal heat expansion of the liquid, ϕ the porosity of the porous layer, c the specific heat capacity, K the permeability, g gravitational acceleration.

We introduce perturbations of velocities, pressure and temperature in order to linearize the equations, and different scales for the non-dimensional form. For fluid layer, the temperature is scaled by $\Delta T_l \nu_l/\kappa_l$, the length by H_l, the time by H_l^2/κ_l and the velocity by ν_l/H_l. For porous layer, they are $\Delta T_m \nu_l/\kappa_m$, H_m, H_m^2/κ_m and ν_l/H_m. According to the normal mode technique, we seek solutions for the vertical velocity component and temperature of the form:

$$(w_k, T_k)^T = [W_k(z_k), \Theta_k(z_k)]^T \exp(\lambda_k t_k + i a_k x_k)$$

The amplitudes W_k and Θ_k (where $k = l, m$) describe the variation of the vertical velocity and the temperature. $a_{l,m}$ are the dimensionless wavenumbers in the x-direction, and $\lambda_{l,m}$ are the complex growth rates of the disturbance. Then the equations (where $D_l = d/dz_l$, $D_m = d/dz_m$) become:

$$\lambda_l Pr_l^{-1}\left(D_l^2 - a_l^2\right)W_l = \left(D_l^2 - a_l^2\right)^2 W_l - a_l^2 Ra_l \Theta_l \tag{7}$$

$$\lambda_l \Theta_l = W_l + \left(D_l^2 - a_l^2\right)\Theta_l \tag{8}$$

$$\lambda_m \frac{\delta^2}{\phi Pr_m}\left(D_m^2 - a_m^2\right)W_m = -\left(D_m^2 - a_m^2\right)W_m - a_m^2 Ra_m \Theta_m \tag{9}$$

$$\lambda_m G_m \Theta_m = W_m + \left(D_m^2 - a_m^2\right)\Theta_m \tag{10}$$

At $z_m = -1$, the boundary is a rigid and perfectly heat conduction wall:

$$W_m = 0, \quad \Theta_m = 0 \tag{11}$$

At $z_l = 1$, the top surface with surface tension is assumed to be non-deformable and heat insulating:

$$W_l = 0, \quad D_l\Theta_l + Bi\Theta_l = 0, \quad D_l^2 W_l + a_l^2 Ma_l \Theta_l = 0 \tag{12}$$

At the liquid–porous interface ($z_m = z_l = 0$), continuity of normal velocity, temperature, heat flux, normal momentum, and Beaver–Joseph condition [9] (with an empirical parameter β) are expressed as following:

$$W_l = hW_m, \quad h\Theta_l = X^2\Theta_m, \quad D_l\Theta_l = XD_m\Theta_m \tag{13}$$

$$D_l^2 W_l - \beta\frac{h}{\delta}D_l W_l + \beta\frac{h^3}{\delta}D_m W_m = 0 \tag{14}$$

$$\frac{\lambda_l}{Pr_l}D_l W_l - \frac{\lambda_m h^4}{\phi Pr_m}D_m W_m = D_l^3 W_l - a_l^2 D_l W_l + \frac{h^4}{\delta^2}D_m W_m \tag{15}$$

The dimensionless parameters in the above equations include: for the fluid layer, the Rayleigh number, Marangoni number and Prandtl number are defined as: $Ra_l = \alpha g \Delta T_l H_l^3/(\nu_l \kappa_l)$, $Ma_l = \gamma \Delta T_l H_l/(\mu_l \kappa_l)$, $Pr_l = \nu_l/\kappa_l$, $Bi_l = \chi H_l/(\kappa_l \rho_0 c_l)$. For the porous layer, the Rayleigh number, the Prandtl number and Darcy number are defined as: $Ra_m = \alpha \rho_0 g \Delta T_m H_m K/(\mu_l \kappa_m)$, $Pr_m = \nu_l/\kappa_m$, $\delta = \sqrt{K}/H_m$. Besides, $h = H_l/H_m$, $G_m = (\rho c)_m/\rho_0 c_l$ and $X = \kappa_l/\kappa_m$. For the whole system, $Ra = Ra_l \cdot (1 + 1/h)^2(1 + X/h)^2$, $Ma = Ma_l \cdot (1 + X/h)^2$, $Bi = Bi_l \cdot (1 + X/h)$, and $a = a_l(1 + 1/h)$. These equations and boundary conditions determine an eigenvalue problem which can be solved by Chebyshev-tau method. In our calculation, we choose: $\delta = 3 \times 10^{-3}$, $\phi = 0.3$, $\beta = 0.1$, and $X = 0.7$.

3. Results and discussion

Fig. 1(a) presents the marginal curves of the pure Marangoni case with different Biot numbers. If Bi is small, the curves are bimodal, and the long-wave instability is dominant, i.e., the convection is initiated in and dominated by the porous layer. With the increase of Bi, the marginal curve becomes single modal, and the short-wave mode convection takes place, i.e., the convection occurs only in the liquid layer. When the upper surface becomes well thermal conductive, i.e., $Bi \to \infty$, the critical Marangoni number is also infinite. This means that the surface tension effect disappears. In the case of the pure Rayleigh convection when $Ma = 0$, the marginal curves shown in Fig. 1(b) are always bimodal evidently. The upper thermal boundary conditions can only determine the values of critical Rayleigh number, while the Rayleigh effect exists constantly.

As pointed out by previous works [8,10–14], the depth ratio h is a major parameter determining the stability modes of the liquid–porous system. Fig. 2(a) represents the variation of critical depth ratio h_R for pure Rayleigh case and h_M for pure Marangoni case with increase of Bi, respectively. With a given Bi, the Rayleigh effect is dominant in the long wave region when $h < h_R$, and in the short wave region when $h > h_R$. For the Marangoni effect, similar situation exists. If the system has a depth ratio $h = 0.1$, which locates in the open interval (h_M, h_R), we can know that the Rayleigh effect operates constantly in the long wave region, while the Marangoni effect mainly operates in the short wave region. Therefore, the coupling instability trends to short-wave mode when reinforcing the surface tension effect. However, the Marangoni effect can be precluded gradually if the top surface becomes more and more thermal conductive. Therefore, the phenomena of coupling instability influenced by the Biot number are complicated and of some abundance.

As the results shown in Fig. 1(a) and (b), the marginal curves of liquid–porous system always have a bimodal characteristic, except in the case of pure Marangoni instability with a too large Biot number. Fig. 2(b)–(d) present the variation of Rayleigh numbers with respect to the increase of Biot number for different Ma. In Fig. 2(b), $Ma = 1000$, the solid curve (long-wave mode) is constantly lower than the dashed one (short-wave mode). This means that the coupling instability is of always long-wave mode and corresponding convection is constantly dominated by the combined layers. In this case, the Marangoni effect is too weak to enhance the convection in liquid layer, and the coupling mode is mainly determined by buoyancy effect.

In Fig. 2(c), $Ma = 1750$, the solid curve and the dashed one have two intersections. This means that the mode transits twice. This phenomenon can be explained from Fig. 2(a). When the Biot number keeps small enough, its increase can make the depth ratio h approach to h_R gradually, caused by the diminution of h_R. Hence, the buoyancy convection in the porous layer is weakened. At the same time, the small Biot number cannot evidently preclude the suffi-

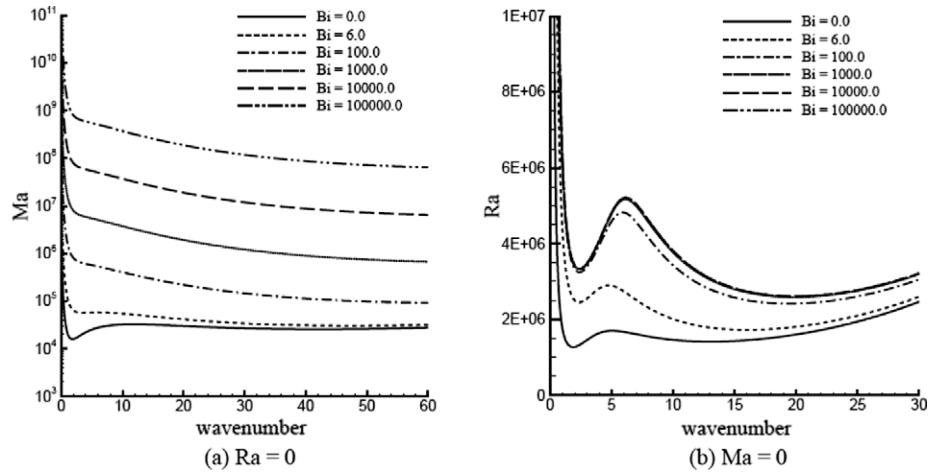

Fig. 1. (a) Marginal curves of pure Marangoni instability for different Biot numbers with depth ratio $h = 0.03$; (b) marginal curves of pure Rayleigh instability for different Biot numbers with depth ratio $h = 0.12$.

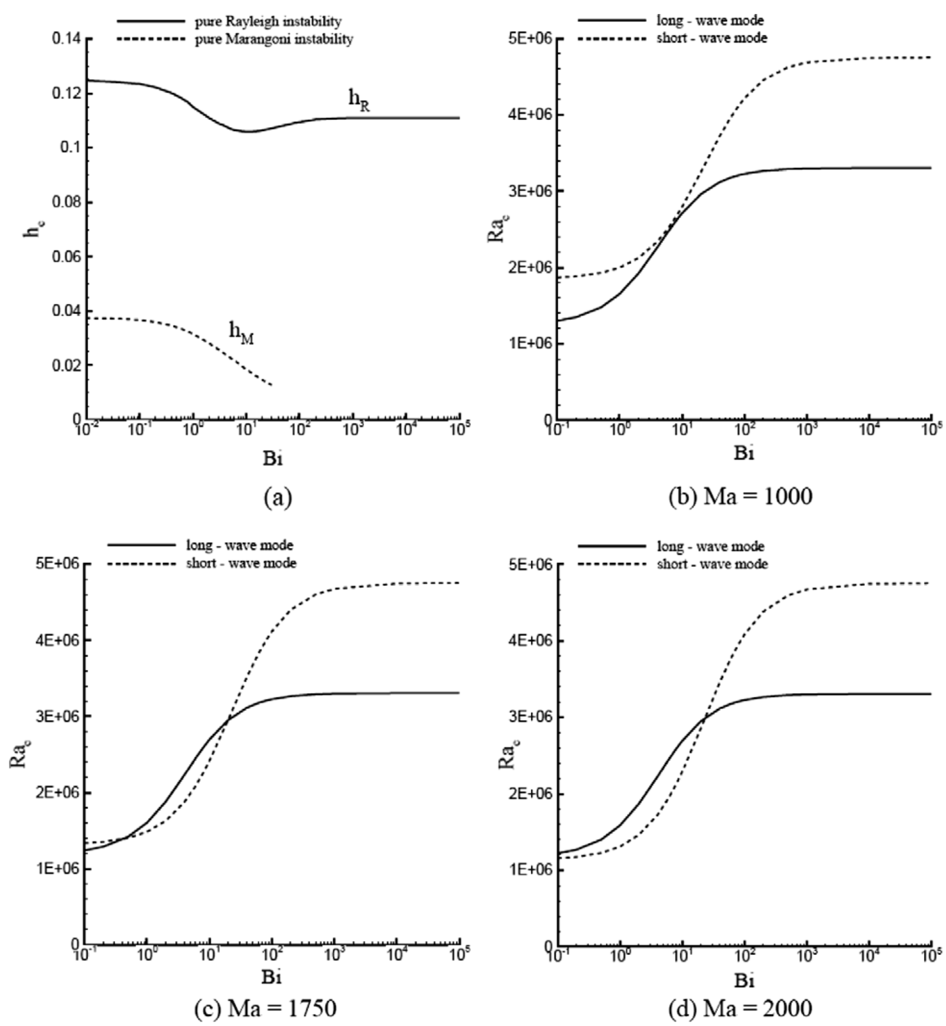

Fig. 2. Biot numbers versus critical Rayleigh numbers of different modes for various Marangoni numbers.

ciently forceful surface tension effect which enhances the flow in the liquid layer. Therefore, the coupling instability turns into the short-wave mode. When the Biot number becomes sufficiently large, the Marangoni effect is precluded obviously, and the instability is driven only by the buoyancy effect which forms the convection in the combined layer, so then the second mode transition occurs.

In Fig. 2(d), $Ma = 2000$, the Marangoni effect is forceful enough to form the coupling convection occurring the liquid layer even when $Bi = 0$. Therefore, its gradual preclusion by the increase of the Biot number makes the mode of coupling instability become long wave, and the mode transition takes place only once.

4. Conclusion

A linear analysis is proposed to study the convective instability in the fluid–porous system. The upper boundary is set to be free in order that the Marangoni effect can be taken into account. The influence of heat transfer condition at the upper surface, represented by *Bi*, on the instability mode of the system is investigated numerically. For pure Marangoni convection, the long-wave mode instability may disappear when *Bi* grows, and the surface tension effect can be precluded completely when the top surface becomes well thermal conductive. For pure Rayleigh instability, the variation of *Bi* does not change the bimodal characteristic of the marginal curve. For the coupled Rayleigh–Marangoni convection in the system with h locates in the open interval (h_M, h_R), the mode of combined instability is determined by both Marangoni number and Biot number. If the Marangoni effect is weak, the increase of the thermal effect at free surface cannot change the mode of convection appearing in the combined two layers. The mode transition of the instability only takes place evidently in a system with a forceful surface tension effect.

Acknowledgments

The authors are grateful for the financial support for this project from the National Nature Science Foundation of China (50890182 and 10772185) and the Knowledge Innovation Program of Chinese Academy of Sciences (KGCX-SW-409).

References

[1] H. Bénard, Rev. Gen. Sci. Pur. Appl. 11 (1900) 1261.
[2] L. Rayleigh, Philos. Mag. 32 (6) (1916) 529.
[3] C.W. Horton, F.T. Rogers, J. Appl. Phys. 16 (1945) 367.
[4] E.R. Lapwood, Proc. Camb. Philos. Soc. 44 (1948) 508.
[5] J.R.A. Pearson, J. Fluid Mech. 4 (1958) 489.
[6] M. Hennenberg, M.Z. Saghir, A. Rednikov, J.C. Legros, Transp. Porous Med. 27 (1997) 327.
[7] H.C. Brinkman, Appl. Sci. Res. A 1 (1947) 27.
[8] F. Chen, C.F. Chen, J. Heat Transfer 110 (1988) 403.
[9] G.S. Beavers, D.D. Joseph, J. Fluid Mech. 30 (1967) 197.
[10] D.A. Nield, Int. J. Heat Fluid Flow 15 (4) (1994) 337.
[11] B. Straughan, J. Comput. Phys. 170 (2001) 320.
[12] M.Z. Saghir, P. Mahendran, M. Hennenberg, Energy Sources 27 (2005) 151.
[13] I.S. Shivakumara, S.P. Suma, K.B. Chavaraddi, Arch. Mech. 58 (2) (2006) 71.
[14] R. Liu, Q.S. Liu, S.C. Zhao, Int. J. Heat Mass Transfer 51 (2008) 6328.
[15] Th. Desaive, G. Lebon, Phys. Rev. E 64 (2001) 066304.