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Characteristics of surface oscillation in thermocapillary convection

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ABSTRACT

The characteristics of surface oscillation in a rectangular pool of silicone oil have been investigated experimentally. The horizontal cross-section of the pool is 52 mm × 36 mm, and the depth of the silicone oil layer is in the range of 1.1–4.8 mm. The applied temperature difference between the two sidewalls leads to shear flow along the free surface from hot to cold and a back flow in the underlying layer. With the increase of the temperature difference, the original steady flow will become unstable to unsteady flow. A CCD laser displacement-sensor with high resolution is used to measure the position of the liquid surface dynamically. And the Hilbert–Huang transform is chosen to analyze the experiment data which is nonlinear and non-stationary. The characteristics of surface oscillation have been obtained. And the relationship of the characteristics with the temperature difference and liquid layer depth has been discussed in details.

1. Introduction

When a temperature gradient is imposed along a free liquid–gas interface, thermocapillary convection is driven by the surface tension gradient. Such convection has been of interest for it occurs in many processes such as thin-film coating and crystal growth from the melt. The instability of the convection has been paid more attention for it leads to oscillating convection, which produces poor crystal quality in the process of crystal growth [1]. Temperature oscillation and surface oscillation are the main characteristics of the instability.

A linear stability analysis of thermocapillary instability was given by Smith and Davis [2,3]. They considered a liquid layer of infinite horizontal extent bounded by a rigid adiabatic plane at the bottom and a free surface subjected to a constant temperature gradient at the top. When the free surface is assumed to be flat and non-deformable, the hydrothermal wave, which is traveling waves propagating in a lateral direction, is derived as the thermocapillary instability. The characteristics for the hydrothermal wave, such as the critical condition for onset and the propagating direction, depends on the Prantl number of the liquid and the Biot number of the interface. When the free surface is considered to be deformable, another thermocapillary instability was obtained, which is called surface wave. The surface wave is also traveling wave, and the characteristics of the instability are similar to that of the two-dimensional waves in an isothermal layer subjected by wind stresses analyzed by Smith and Davis [4]. So hydrothermal wave and surface wave are different thermocapillary instabilities with different characteristics, which were derived with different assumption.

Experiments have been conducted by Riley and Neitzel [5] to investigate the thermocapillary instability in a rectangular pool filled with silicone oil (Pr = 13.9). They have observed two kinds of instabilities which depend on the Bond number of the liquid layer. For small-Bo fluids, the fluid convection transits from steady unicellular flow to hydrothermal waves which are similar to the instability predicted by Smith and Davis [2]. For large-Bo fluids, it transits to steady multi-cellular flow, and then to oscillating multi-cellular flow, which is characterized with steady multi-cellular structures near the hot wall and a pair of oblique waves near the cold wall. Similar to Burguete et al. [6] has also observed the hydrothermal wave instability for thin liquid layer. However, for deep liquid layer, it is observed by him that the basic return flow transits to stationary rolls, which is characterized as a stationary pattern with a wave vector perpendicular to the applied temperature gradient. And the characteristics of thermocapillary waves such as frequency, amplitude and wave number have been studied in details.

Experiment research on thermocapillary instabilities of a thin liquid layer in the perspective of surface deformation was published by Schwabe et al. [7]. The shadowgraph technique was used in the experiment to observe the deformed liquid–gas interface dynamically. For $d < 1.4$ mm, short-wavelength instability was observed, which travels in the azimuthal direction with small surface deformation. While for $d > 1.4$ mm, long-wavelength instability was found, whose waves travel in radial at onset and in azimuthal directions later. The surface deformation of thermocapillary flow caused by temperature gradient along the liquid–gas interface has been investigated by Duan et al. [8]. A modified

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Nomenclature

h	depth of the liquid layer, m
t	time, s
v	dynamic viscosity, Pa s
γ	kinetic viscosity, $m^2\ s^{-1}$
L	stream wise domain length, m
g	gravitational acceleration, m s^{-2}
T	temperature of the interface, °C
$\triangle T$	applied temperature difference between the two sidewalls, °C
A	amplitude of surface oscillation, m
f_d	dominant frequency of surface oscillation, Hz

Greek symbols

β	thermal expansion coefficient, 1/°C
κ	thermal diffusivity, $m^2\ s^{-1}$
ρ	density, kg m^{-3}
σ	surface tension, N m^{-1}

Michelson interferometer was developed to measure the surface deformation quantitatively. The surface deformations are more declining when the temperature difference increases. It was found that the surface deformation also depends on the thickness of the liquid layer; the position of the surface near the cold end is higher than that near the hot end for a thin liquid layer, while it is opposite for a thick layer. The surface wave was observed by Duan et al. [9], which is superimposed in surface deformation. Besides these, there is no surface waves observed experimentally in the thermocapillary flow of a shallow liquid layer heated from the lateral. One reason for the absence of the experimental results of the surface waves may be the small magnitude of the oscillating amplitude leading to the difficulty to identify this instability and discuss it in details.

The details of surface oscillation need to be studied to explore the mechanism of thermocapillary instability. And the surface oscillation of thermocapillary convection reduces the quality of crystals in the process of crystal growth. The aim of the present research work is to observe the surface oscillation of thermocapillary convection and to analyze the characteristics of the oscillation. To achieve the goal, we used a CCD laser displacement-sensor with high resolution to record the position of the liquid surface dynamically. In order to obtain the characteristics of surface oscillation such as onset condition, oscillating frequency and amplitude, the Hilbert–Huang transform is used to analyze the experiment data. The advantage of the present experimental study is the high resolution of the displacement sensor which is suitable to identify the weak oscillating signal. Another advantage of the present study is the choice of the Hilbert–Huang transform which is applicable to analyze the nonlinear and non-stationary signal.

2. Experimental setup and procedure

In order to carry on the research of surface oscillation, we have established a buoyant–thermocapillary convection system (see Fig. 1). The rectangle pool consists of a right heater with the thickness of 6 mm made of copper heated by an electro-thermal film and a left copper end with the same thickness attached with a semiconductor cooling sheet. The semiconductor cooling sheet is used to keep the left wall at a constant temperature. The front, rear and bottom side of the pool is made of optical glass K9 with the thickness of 6 mm. In our experiment, the silicone oil of 100 cSt and 50 cSt is chosen because of their higher viscosity to avoid the outside interference. The horizontal temperature gradient in the fluid will be established through the two copper walls. A DC electrical source is controlled by a temperature controller to work the electro-thermal film and semiconductor cooling sheet. Two T-type thermocouples are used to measure the temperature of the two copper walls. With the increase of the applied temperature difference of the two copper walls, the flow of the fluid in the rectangle pool will transit from stable to unstable. In our experiment, the applied temperature difference is increased from 0 to about 65 °C with the rate of 1 °C per minute. The working fluid used is silicone oil 50 cSt and 100 cSt, whose physical properties are given in Table 1. Because the temperature difference is up to 65 °C in our experiment, the data of viscosity varying with the temperature is displayed in Table 2. And the critical Marangoni numbers are calculated by using the viscosity at the average temperature of the fluid layer.

To record the oscillating information on the fluid surface in a period of time, we used CCD laser displacement sensor, the measurement principle of which uses triangulation. The position of the reflected light on the CCD moves when the position of the measurement point changes. The displacement amount of the point is measured by detecting the change on the CCD. The resolution of the sensor is 0.01 μm, which is appropriate for the measurement of the surface oscillation. And the sampling speed of 50 kHz is adequate to supply the variation of the position of the measurement point versus time. In our experiment, the position for measurement is 20 mm away from the hot wall along the central line.

To choose the suitable method to process the measurement data, we should recognize the property of the data. The surface oscillation is an important characterization of the transition

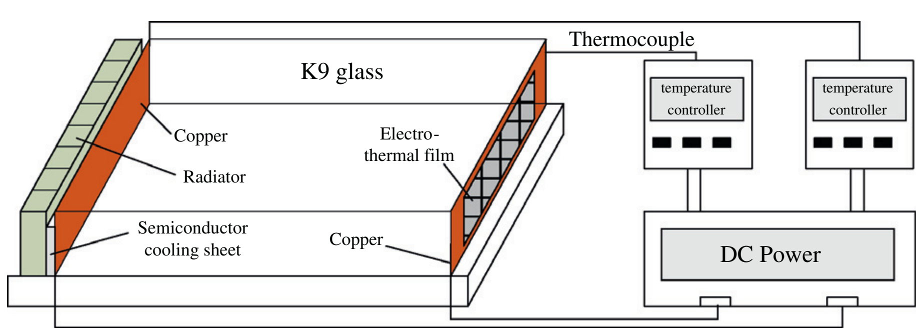

Fig. 1. Controlling system for buoyant–thermocapillary convection.

Table 1
Physical properties of silicone oil 50 cSt and 100 cSt at 25 °C.

Silicone oil (cSt)	γ ($m^2\ s^{-1}$)	ρ (kg m^{-3})	β (°C^{-1})	κ ($m^2\ s^{-1}$)	σ (N m^{-1})	$\partial\sigma/\partial T$ (N m^{-1} °C^{-1})	Pr = υ/κ
50	5.0e−5	960	9.6e−4	1.04e−7	2.08e−2	−6e−5	481
100	1.0e−4	965	9.5e−4	1.11e−7	2.09e−2	−6.11e−5	901

Table 2
Kinetic viscosity (mm^2/s) at various temperatures.

Silicone oil (cSt)	0 °C	25 °C	50 °C	100 °C
50	88.0	50.0	32.5	15.9
100	171	100	64.6	31.3

process for buoyant–thermocapillary convection from layer flow to turbulent. Therefore, the surface oscillation, as a route to turbulence, is a strongly non-linear system. On the other hand, the process is not stationary, because the surface oscillation appears only when $\triangle T$ reaches a certain value. And even after the onset, the amplitude of the oscillation changes a lot. In a word, the measurement data of the surface oscillation is of nonlinear and non-stationary.

Considering the property of the experiment data, we have chosen the Hilbert–Huang transform (HHT) as the data-processing tool. Though it is not full-developed, tests have showed that HHT is a superior method for time–frequency analysis of nonlinear and non-stationary data. Other than Fourier and wavelet analysis, the basis of HHT is adaptive and the frequency is defined by differentiation instead of convolution. Therefore, it can decompose the nonlinear and non-stationary signal into a series of physically meaningful components, and the time and frequency resolution is not limited by the uncertainty principle. So HHT is very applicable to be used for the time–frequency analysis of the data in our research, which has been pointed out to be nonlinear and non-stationary.

HHT is an empirically based data-analysis method proposed by Huang [10]. The widely used way to compute the instantaneous frequency is by using Hilbert transform, which can be described as

$$y(t) = H\{x(t)\} = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{x(\tau)}{\tau - t}d\tau,$$

in which $x(t)$ indicates an arbitrary real valued function of L^p space, and $y(t)$ is the complex conjugate of $x(t)$. Through the Hilbert transform, the analytic signal can be defined as

$$z(t) = x(t) + iy(t) = a(t)e^{i\theta(t)},$$

in which

$$a(t) = \sqrt{x^2 + y^2}, \quad \text{and} \quad \theta(t) = \arctan\frac{y}{x}.$$

Here, $a(t)$ is the instantaneous amplitude, $\theta(t)$ is the phase function, and the instantaneous frequency can be described as

$$\omega(t) = \frac{d\theta}{dt}.$$

However, the instantaneous frequency calculated through this way is not always meaningful for all signals. Sometimes, the calculating instantaneous frequency is even negative. Therefore, in order to obtain meaningful instantaneous frequency, Huang introduced the empirical mode decomposition method, in the process of which the original signal is decomposed into a serious of intrinsic mode functions. The time–frequency representation of the original signal can be obtained by combining the instantaneous frequencies and amplitudes of intrinsic mode functions.

3. Results and discussion

For small applied temperature difference $\triangle T$, a steady return flow is produced by thermocapillarity along the surface and pressure difference in the underlying layer. This steady state can be easily changed into unsteady flow by increasing $\triangle T$. Characteristics of surface oscillation, such as onset point, frequency and amplitude, can be obtained by investigating the periodic variation of vertical position of a single point on the surface.

A typical evolution of surface oscillation is shown in Fig. 2, which is measured by the displacement sensor. At the first two-thirds of time span, vertical displacement of measurement point increases monotonically with time, which is mainly due to thermal expansion. As $\triangle T$ increases, there appears oscillating signal in the latter approximate third of time span, which superimposes on the monotonic upward trend. The appearance of the surface oscillation corresponds to transformation of the flow state from steady to unsteady.

The time–frequency representation of the measurement signal can be obtained by the Hilbert–Huang transform, the result of which is displayed in the Hilbert spectrum (Fig. 3). In the Hilbert spectrum, as we can see, there are many bright colored points, which correspond to larger magnitude than the blue[1] colored background, distributing at the right side around one certain frequency value. That is to say, there is a component in the original signal, whose frequency is between 1.5 Hz and 2 Hz, and it becomes apparent after the approximate two-thirds point of time span. Therefore, this component corresponds to surface oscillation which is described in the last paragraph as shown in Fig. 2.

With the Hilbert spectrum results obtained above, we can also get the marginal spectrum by integrating the amplitude over the entire time span. The marginal spectrum provides respectively the total amplitude contribution of each frequency. Therefore, from the marginal spectrum, we can identify the dominant frequency easily. In the marginal spectrum in Fig. 4, the dominant frequency of the oscillation is 0.1706 Hz, which has the largest accumulated amplitude over the whole time span.

The moment is generally identified to be onset point, when the amplitude (or energy) of the signal with the specified frequency reaches a certain threshold. In order to get the onset point of the oscillation, we first intercept from the time–frequency representation the amplitude of signal whose frequency is in the certain range around the dominant frequency. And then, we obtain the amplitude variation of the oscillation with respect to time, which is shown in Fig. 5. Through an integrated consideration of different thickness and different Pr numbers, the value $A_0 = 0.023$ μm is selected to be the threshold to ensure that: (1) the amplitude of the onset point is larger than A_0; (2) most of amplitudes of the points after onset is larger than A_0; (3) A_0 is the lowest value which can meet the previous two requirements. Because of the limitation of the range of the dominant frequency and inevitable interference during the experiment, it is hard to require all the amplitude of the points after onset is larger than A_0. From the example of 2.1 mm shown in Fig. 5, we can see that this method can identify the onset point commendably.

[1] For interpretation of color in Fig. 3, the reader is referred to the web version of this article.

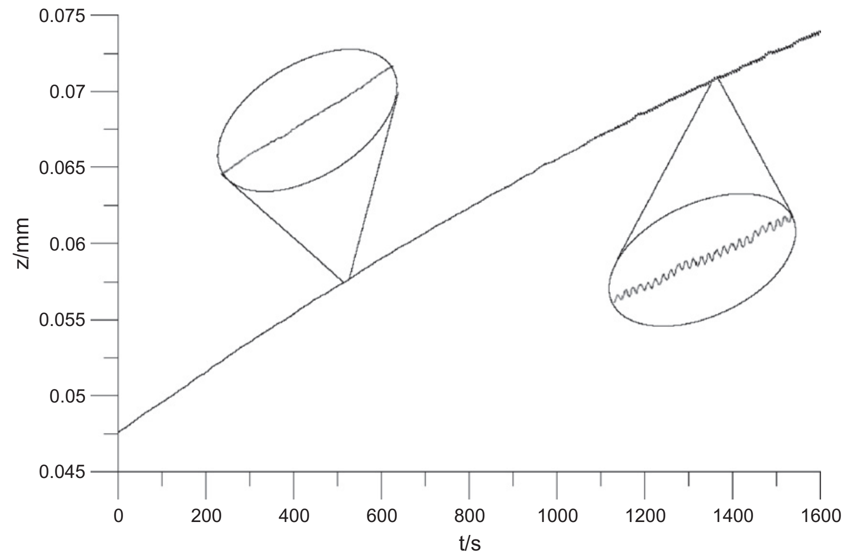

Fig. 2. The displacement sensor signals for $h = 2.1$ mm of silicone oil 100 cSt. The initial value of the displacement sensor is adjusted to 0 when $\triangle T = 0$.

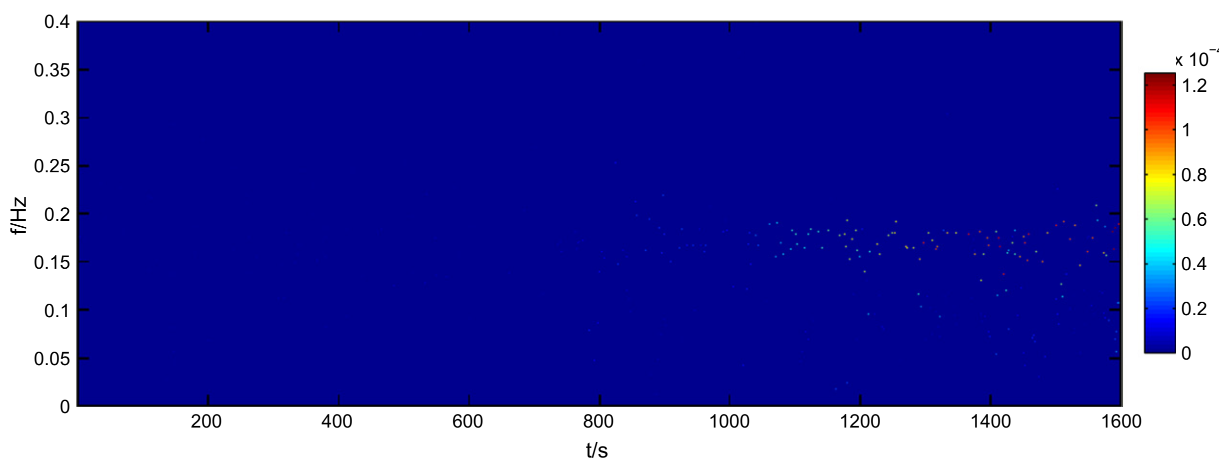

Fig. 3. The Hilbert spectrum of measurement signals for $h = 2.1$ mm of silicone oil 100 cSt. The brightness of each point represents the instantaneous amplitude of one certain frequency.

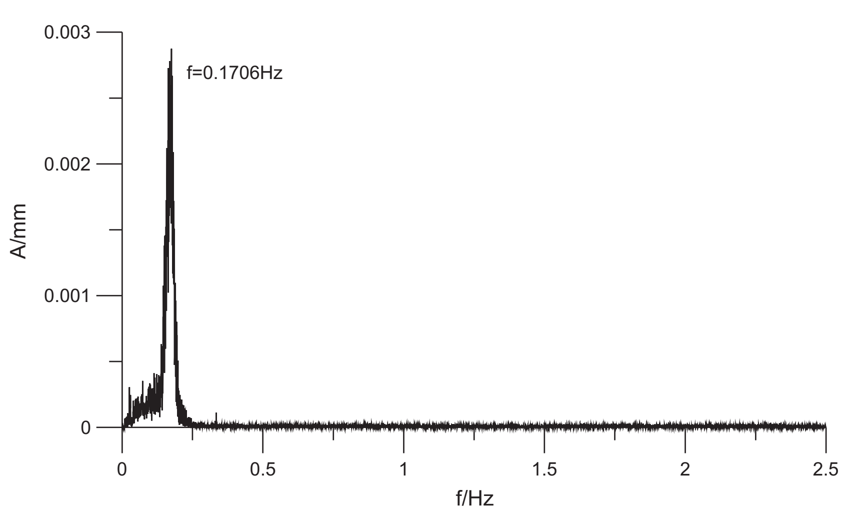

Fig. 4. The marginal spectrum for $h = 2.1$ mm of silicone oil 100 cSt.

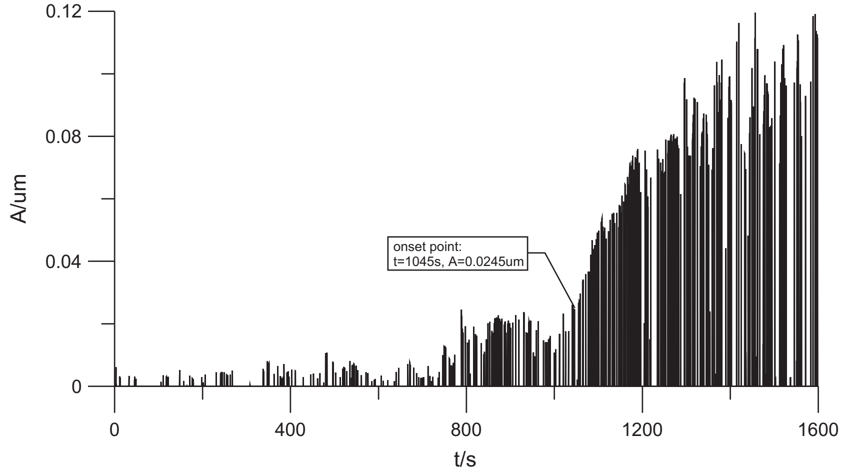

Fig. 5. The amplitude variation of the oscillation for h = 2.1 mm of silicone oil 100 cSt when $\triangle T$ increases with the rate of 1 °C per minute. The amplitude value is intercepted from the Hilbert spectrum for the frequency values in the range of 0.1706 ± 0.006 Hz.

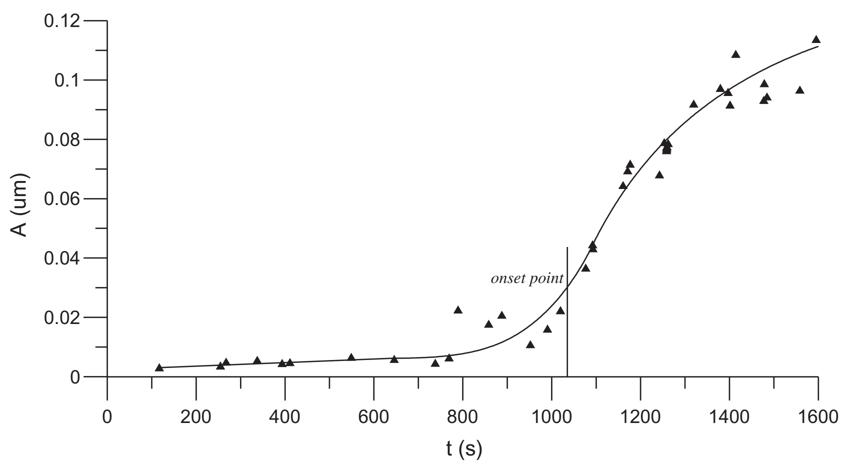

Fig. 6. Amplitude of surface oscillation versus time t for h = 2.1 mm of silicone oil 100 cSt. The fitted curve shows the trend of amplitude change. The vertical line points out the onset point judged through HHT analysis.

To make out the behavior of oscillating amplitude in the evolution of surface oscillation, Fig. 6 shows the amplitude variation of dominant frequency versus time t, which is extracted from the time–frequency representation shown in Fig. 3 for silicone oil 100 cSt with liquid depth 2.1 mm. Before the onset point, the amplitude of the dominant frequency is less than 0.01 μm. When it approaches to the onset point, the amplitude increases slowly. After the onset point, the amplitude increases quickly from around 0.03 μm to 0.1 μm.

As the results of 100 cSt silicone oil with the depth of 2.1 mm, the surface-oscillation characteristics of silicone oil with different Pr numbers and different depths can be obtained in the same way. Now we are going to show the comparisons of the results, summarize the characteristics of surface oscillation, and discuss the influence factors to the thermocapillary instability.

The surface oscillation is observed for liquid depths 1.1 mm < h < 4.8 mm, when liquid layer of silicone oil 100 cSt is imposed by temperature difference from 0 to 65 °C. Fig. 7 shows the critical applied temperature difference $\triangle T_c$ for different depths, which is obtained through the same threshold. For h < 1.5 mm, the value of $\triangle T_c$ reduces significantly with the increase of liquid depth. Meanwhile, above the liquid depth of 1.5 mm, $\triangle T_c$ decreases on the whole with increasing the liquid depth, though the value changes irregularly locally. In addition, we have also observed surface oscillation for silicone oil of 50 cSt; the comparison of $\triangle T_c$ between 50 cSt and 100 cSt is shown in Table 3. Through comparison, it is easy to find that, for the same liquid depth surface oscillation for 50 cSt has a smaller $\triangle T_c$ than that of 100 cSt.

The thermocapillary instability is basically driven by the tension gradient along the surface. So the onset of the instability can be indicated by the critical Marangoni number Ma, defined as

$$Ma = (\partial\sigma/\partial T)\Delta T h^2/(\rho\nu L\kappa)$$

For the flow coupling buoyant convection and thermocapillary convection, the relative strength of buoyant forces to thermocapillary forces is measured by the Bond number Bo, defined as

$$Bo = \rho g\beta h^2/(\partial\sigma/\partial T)$$

In Fig. 8 the critical Marangoni numbers for liquid layer of 50 cSt and 100 cSt are shown. For the same kind of silicone oil, the critical Marangoni number Ma_c is in direct proportion to the Bo number. Since the increase of the Bo number indicates the enhancement of buoyancy convection, we can conclude that the enhancement of buoyancy convection can stabilize the whole flow. This conclusion is in agreement with the experimental results measured by

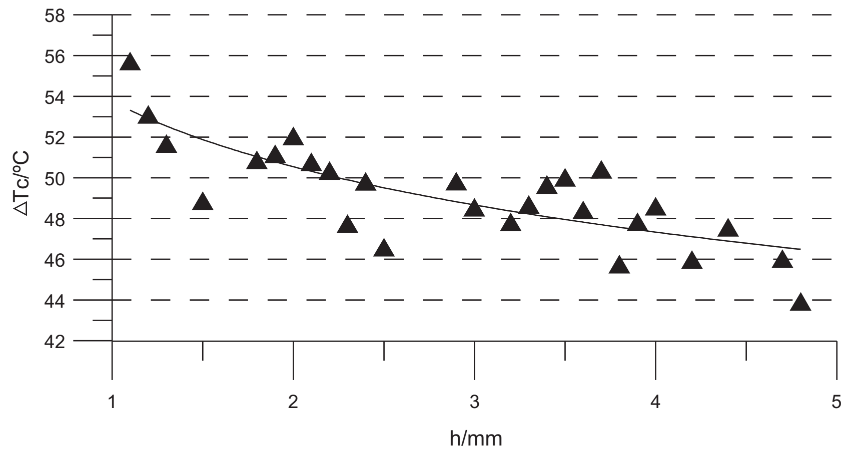

Fig. 7. Critical applied temperature difference $\triangle T$ for various liquid depths.

Table 3
The comparison of critical applied temperature difference $\triangle T_c$ between silicone oil 50 cSt and 100 cSt for various liquid depths.

h/mm	$\triangle T_c$/K	
	50 cSt	100 cSt
1.8	42.98	50.86
2.2	30.16	50.34
2.3	38.1	47.73
3.4	40.54	49.65

Schwabe and Scharmann [11], that thermocapillary instability in microgravity has a lower Ma_c than that on ground. On the other hand, for the same thickness, the silicone oil of 50 cSt has a larger Ma_c than that of 100 cSt; the decrease of the viscosity makes the flow more stable.

The dominant frequency of the surface oscillation can be obtained from marginal spectrum of original signal, which is the peak value in marginal spectrum. Fig. 9 displays the dominant frequencies of silicone oil 100 cSt for different depths. As we can see, the dominant frequencies are in the range of 0.17 ± 0.005 Hz, whose relative deviation from 0.17 Hz is less than 3%. Therefore, we can conclude that the dominant frequency of surface oscillation changes little with the variation of liquid depths, and that it can be considered to be a constant as 0.17 Hz for silicone oil 100 cSt. Likewise, the surface oscillation of silicone oil 50 cSt has a constant dominant frequency of 0.10 Hz.

4. Conclusions

The surface oscillation of thermocapillary convection for different conditions is studied experimentally in the present work. The experiment results show that with the increase of liquid depth, the critical $\triangle T$ reduces significantly for $h < 1.5$ mm and decreases slowly on the whole for $h > 1.5$ mm, while it changes irregularly locally. The frequency of the surface oscillation is related to the Prantl number of the liquid layer and changes little with the variation of liquid depth. The oscillating amplitude increases gradually with the augment of applied temperature gradient. The analysis of non-dimensional parameters reveals that the enhancement of buoyant convection and reduce of the liquid viscosity can stabilize the flow driven by temperature gradient.

The experiment results of the surface oscillation will be compared to the temperature oscillation which is measured in the same condition to explore the mechanism of the thermocapillary instability. The phase relationship of thermocapillary oscillation between different point is also of interest in our later study, to compare to theoretical results [2,3].

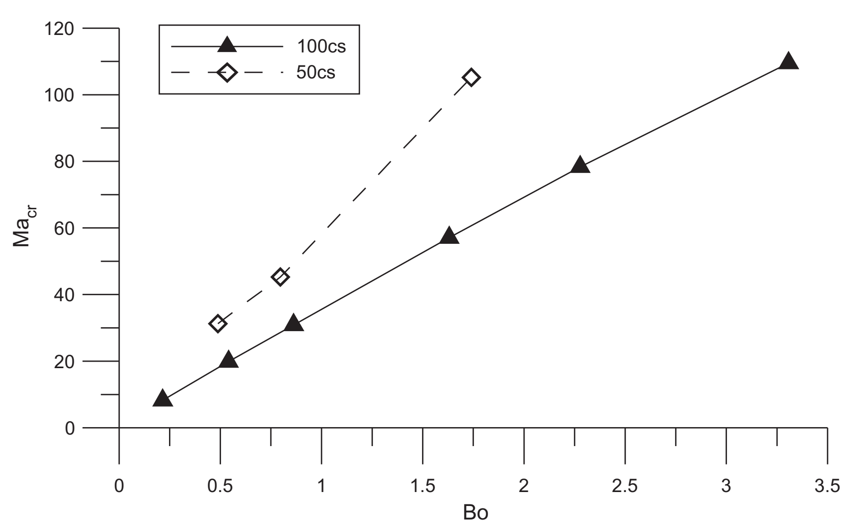

Fig. 8. The critical Marangoni number versus the Bond number for silicone oil 50 cSt and 100 cSt.

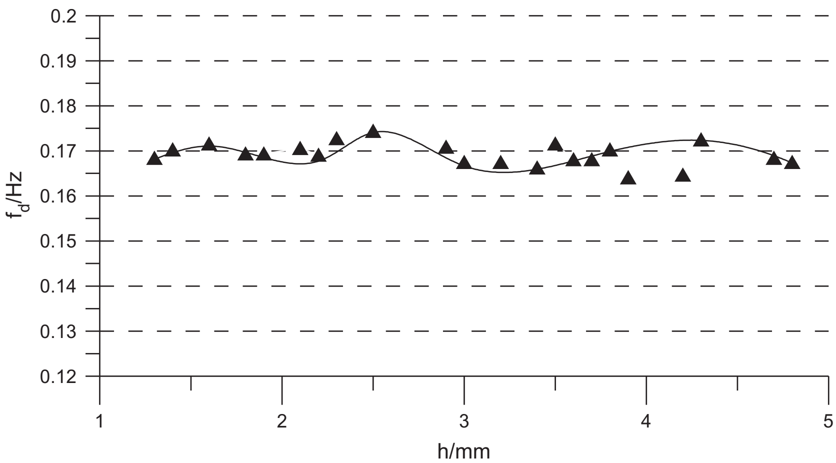

Fig. 9. Dominant frequency of surface oscillation of silicone oil 100 cSt for various liquid depths.

Acknowledgments

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