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Fracture of a slope by percolating

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ABSTRACT: A theoretical analysis based on a pseudo-three-phase media is presented, and the instability of a slope causing the debris flow is studied. The occurrence of fractures (or water films) in saturated soils in two cases is analyzed. The first case is that the water is forced to percolate through the soil upwards. A perturbation method is used in this case to obtain an approximate solution to explain the formation mechanism of cracks. The second case is about the cracks in liquefied soils. Numerical analysis and theoretical analysis are processed for this case. The formation of fractures in saturated grain accumulation under gravity is due to the blocking by redeposit of finer grains. The formation of fractures presents a sliding surface for landslides and debris flow.

Keywords: fracture, slope, percolation, debris flow

1 INTRODUCTION

It is occurred often in the mountain area that grain deposit or grain flow translates to debris flow which is usually related with the strong seepage induced by earthquake or raining (Cui, 1992; Cui et al., 2009, Hu et al., 2009). When saturated grain deposit stays or slide along a slope, it may become instability and the cracks or fragments occur and develop. As a result, debris flow happens. The fracture in saturated grain deposit is a water gap which forms when the permeability is nonuniform and therefore the pore water is trapped by relatively low permeable layers. The grains do not support one another and are therefore suspended in the condition of zero effective stresses. The grains in suspension eventually settle because they are heavier than water. The rate of such settlement is restricted by the fact that water must flow upward around the grains. If liquefiable deposits are overlain by less permeable soils in a stratified or layered deposit, the overlaying deposit can restrict the pore water to pass through. If there is no downward drainage through the deposit, this relative flow at the interface, by continuity, must be equal to the velocity of settlement at the upper liquefied grain surface. Thus an accumulation of water in the form of a water gap at the interface forms (Lu et al., 2006). Seed (1987) was the first to suggest that the existence of "water film" in sand bed is the reason of slope failures in earthquakes. Later, some researchers (Fiegel and Kutter, 1994; Kobusho, 1999; Kokusho, 2000; Kokusho and Kojima, 2002; Zhang et al., 1999) performed some experiments to investigate the formation of "water film" in layered sand or in sand containing a seam of non-plastic silt. Kokusho (1999) performed shake table tests using sand samples containing a seam of nonplastic silt and showed that water films were formed beneath the silt layer. In this case the column was subjected to horizontal dynamic loadings to simulate earthquakes. Experimental observations on the formation of water films in vertical columns of saturated sand contained in circular cylinders have also been reported by Zhang et al. (1999) and Peng et al. (2000). In both cases care is taken in preparing the sample by feeding wetted uniform sand continuously into a column of water to avoid intentional stratification. However, small inhomogenneity still exits due to uneven settling velocity.

However, the mechanism of the formation of fractures or "water film" in grain deposit is not clear, which is cute to forecast grain deposit translates to debris flow. As debris flow may move a much more long distance than grain flow because of the lower obstruction, debris flow may cause heavier damage. In the viewpoints above, a primary analysis is presented in this paper.

2 FORMULATION OF THE PROBLEM

It is considered that a grain accumulation stays or slides along a slope. The accumulation is water saturated and the water is not over the surface of the accumulation. The surface of the accumulation is free while the bottom is sliding. The fine grains are assumed to be eroded from the skeleton, the eroding relation is assumed to be the following type (Lu et al., 2006). The x axis is upward and parallel to the slope.

$$\frac{1}{\rho_s}\left(\frac{\partial Q}{\partial t}+u_s\frac{\partial Q}{\partial x}\right)=\frac{\lambda}{T}\left(\frac{u-u_s-u_c}{u^*}-q\right)\quad \text{if } -\varepsilon(x,0)\le\frac{Q}{\rho_s}\le\frac{Q_c(x)}{\rho_s} \tag{1}$$

$$\frac{1}{\rho_s}\left(\frac{\partial Q}{\partial t}+u_s\frac{\partial Q}{\partial x}\right)\le 0\quad \text{otherwise} \tag{2}$$

in which the first term on the right side of the first equation shows how fine grains are being transferred to water, the second term describing deposition places a limit on the amount of grains that can be carried in the field, Q the mass of grains eroded per unit volume of the grain/water mixture, ρ_s the density of the grains, u and u_s the velocities of the percolating fluid containing fine grains and the coarse grains, q the volume fraction of fine grains carried in the percolating fluid, T and u^* physical parameters, λ a small dimensionless parameter used to arrive at a perturbation solution, $\varepsilon(x, t)$ the porosity, $Q_c(x)$ the maximum Q, u_c is the critical velocity to cause erosion.

A pseudo-three-phase model is presented. Mass conservation equations are:

$$\frac{\partial(\varepsilon-q)\rho}{\partial t}+\frac{\partial(\varepsilon-q)\rho u}{\partial x}=0 \tag{3}$$

$$\frac{\partial q\rho_s}{\partial t}+\frac{\partial q\rho_s u}{\partial x}=\frac{\partial Q}{\partial t}+u_s\frac{\partial Q}{\partial x} \tag{4}$$

$$\frac{\partial(1-\varepsilon)\rho_s}{\partial t}+\frac{\partial(1-\varepsilon)\rho_s u_s}{\partial x}=-\frac{\partial Q}{\partial t}-u_s\frac{\partial Q}{\partial x} \tag{5}$$

in which ρ is the density of water. A general equation which denotes the volume conservation may be obtained from these three equations, which is

$$\varepsilon u+(1-\varepsilon)u_s=U(t) \tag{6}$$

The momentum equations are written as follows

$$\left[(\varepsilon-q)\rho+q\rho_s\right]\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}\right)=-\varepsilon\frac{\partial p}{\partial x}-\frac{\varepsilon^2(u-u_s)}{k(\varepsilon,q)}-\left[(\varepsilon-q)\rho+q\rho_s\right]g\sin\theta \tag{7}$$

$$
\begin{aligned}
&[(\varepsilon-q)\rho+q\rho_s]\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}\right)+(1-\varepsilon)\rho_s\left(\frac{\partial u_s}{\partial t}+u_s\frac{\partial u_s}{\partial x}\right)=-\frac{\partial p}{\partial x}-\frac{\partial \sigma_e}{\partial x}\\
&-[(\varepsilon-q)\rho+q\rho_s]g\sin\theta-(1-\varepsilon)\rho_s g\sin\theta-\frac{\partial \tau}{\partial x}-\left(\frac{\partial Q}{\partial t}+u_s\frac{\partial Q}{\partial x}\right)(u-u_s)
\end{aligned}
\tag{8}
$$

in which eqn. (7) denotes the momentum conservation of grains, eqn. (8) denotes the total momentum conservation, the last term on the right hand side of eqn. (8) denotes the momentum caused by the eroded fine grains, P is the pore pressure, k the permeability, σ_e the effective stress, θ the slope, τ the shear stress on the bed. Here k assumed to be a function of ε and q in the following form

$$k(\varepsilon, q) = k_0\,(-\alpha q + \beta\varepsilon) \tag{9}$$

in which α, β are parameters and $1 < \beta << \alpha$, we choose to let α much greater than β, so that changes in q overweighs that of ε.

3 ANALYSIS AND RESULTS

Based on the model presented above, we will analyze the occurrence of the fracture in saturated soils in two cases. The first case is water flow is forced to percolate through the grain soils (Peng et al., 2000) and the movement of the skeleton may be neglected relative to that of the water. A perturbation method is used here to obtain an approximate solution to explain the fracture mechanism. The second case is about the fractures in a liquefied grain layer ($\sigma_e = 0$, $\tau \approx 0$) where the grains sink while the water is pressed to move upward just like the consolidation. Here, the slope is assumed to be long enough to neglect the boundary effects for the convenience to obtain the solutions.

(1) In the first case, a perturbation solution will be sought using λ as the small parameter. All solutions will be expanded in the following form, e.g.,

$$u(x,t)=\sum_{0}^{\infty}\lambda^n u_n(x,\ t) \tag{10}$$

The initial porosity distribution $\varepsilon_0(x)$ is taken as

$$\varepsilon_0(x)=a-b\sin\left(\frac{cx-dL}{L}\pi\right) \tag{11}$$

in which L is the length of the grain deposit on the slope, a, b, c, d are coefficients.

It is assumed that the skeleton is static relative to percolating fluid so that

$$\varepsilon u = U(t) \tag{12}$$

$U(t)$ denotes the flow rate per unit cross sectional area of the grain column. The initial and boundary conditions are given as follows

$$\varepsilon(0,\ t)u(0,\ t) = U(t),\ q(0,\ t) = 0 \tag{13}$$

Using perturbation method, eq. (1) ~ eq (5) yields

$$u=\frac{U(t)}{\varepsilon_0(x)}-\lambda\frac{U(t)}{Tu^*\varepsilon_0^3(x)}\int_0^t U(\tau)d\tau+O(\lambda^2) \tag{14}$$

$$\varepsilon = \varepsilon_0(x) + \frac{\lambda}{Tu^*\varepsilon_0(x)}\int_0^t U(\tau)\,d\tau + O(\lambda^2) \tag{15}$$

A numerical example about porosity ε is given in Figure 1. It is shown that the porosity will develop periodic and the frequency becomes higher if it is periodic initially. The higher frequency indicates that the porosity develops nonlinearly.

The equilibrium equations yield an expression of the effective stress as follows

$$\sigma_e = \int_x^H \frac{g\sin\theta(1-\varepsilon)(\rho_s-\rho)(\varepsilon-q)}{\varepsilon} + \frac{\partial\tau}{\partial x}dx - U(t)\int_x^H \frac{dx}{k} \tag{16}$$

Liquefaction or location will occur and develop when $\sigma_e = 0$ and $\frac{\partial\sigma_e}{\partial t} < 0$ is satisfied. Which leads to the following equations:

$$\int_{x_c,t_c}^H \frac{g\sin\theta\,(1-\varepsilon)(\rho_s-\rho)(\varepsilon-q)}{\varepsilon} + \frac{\partial\tau}{\partial x}dx - U(t)\int_{x_c,t_c}^H \frac{dx}{k} = 0 \tag{17}$$

$$\left.\frac{\partial\sigma}{\partial t}\right|_{x_c,t_c} = -\frac{\alpha\lambda U^2(t)H}{Tu^*}\int_{\xi(0,t)}^1 \frac{d\xi}{k}\left(1-\frac{Tu^*}{H}\frac{\partial(q/\varepsilon_0(\xi))}{\partial\xi}\right) \tag{18}$$

($T = 0.1$, $\lambda = 0.01$, $u^* = 1\times10^{-4}\ m/s$, $U = 0.01$, $a = 0.4$, $b = 0.1$, $c = 0.1$, $d = 0.5$).

It is shown in Fig. 2 that the effective stress will develop periodic if the initial porosity is periodic and at some points in the grain column it becomes zero some time, e.g., liquefaction.

(2) In the second case, the grains are not static, then eqn. (6) becomes as follows if the flow rate per unit cross sectional area of the grain column $U(t) = 0$, which means, when the grains sink downwards under gravity while the water flows upwards just like the consolidation.

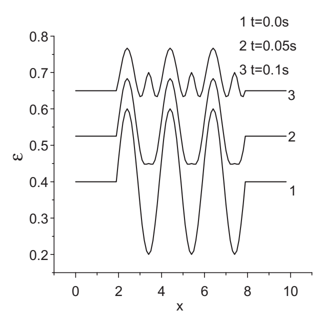


Figure 1. Changes of porosity with time.

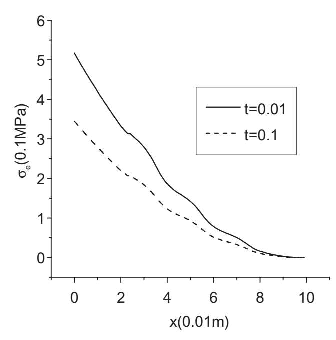


Figure 2. Development of effective stress with depth.

$$\varepsilon u + (1-\varepsilon)u_s = 0 \tag{19}$$

For the convenient of analysis, Let's take T in (1) as the characteristic time, u_t denote the characteristic velocity and L the characteristic length of the problem. We shall make them more specific in the following discussion. For the time being we use them to make eqn. (3), (4), (7) and (8) non-dimensional. Letting

$$\bar{u} = \frac{u}{u_t},\ \tau = \frac{t}{T},\ \xi = \frac{x}{L} \tag{20}$$

eqn. (3) and (4) become

$$\begin{cases} \dfrac{\partial\varepsilon}{\partial\tau} + \dfrac{Tu_t}{L}\dfrac{\partial\varepsilon\bar{u}}{\partial\xi} = (\bar{u}-\bar{u}_c)\dfrac{u_t}{u^*(1-\varepsilon)} - q \\ \dfrac{\partial q}{\partial\tau} + \dfrac{Tu_t}{L}\dfrac{\partial q\bar{u}}{\partial\xi} = (\bar{u}-\bar{u}_c)\dfrac{u_t}{u^*(1-\varepsilon)} - q \end{cases} \tag{21}$$

Equations (7) and (8) can be further simplified by the observation that fracture formation is a slow and late stage process during which the inertia effect is very small (Lu et al., 2010). Instituting eqn. (7) into eqn. (8) and by using eqn. (20) and (9), we can obtain the following equation

$$\bar{u} = \left(\frac{1-\varepsilon}{\varepsilon}\right)^2 (\varepsilon - q) f(q,\varepsilon)\frac{k_0\rho_s g(1-\rho/\rho_s)}{u_t} + u_c = \left(\frac{1-\varepsilon}{\varepsilon}\right)^2 (\varepsilon - q) f(q,\varepsilon + u_c) \tag{22}$$

when u_t is taken to be

$$u_t = k_0\rho_s g(1-\rho/\rho_s) \tag{23}$$

and

$$f(q,\varepsilon) = -\alpha q + \beta\varepsilon \tag{24}$$

Aside from a constant factor on the order of one, $u_t/(1-\varepsilon)$ is the settling velocity of grains in a uniform column of grains with constant diffusion coefficient k_0.

In order to de-couple the problem from the complication arising from the effect of consolidation wave initiated from the bottom of the grain column, we assume that the grain column is very tall so that fracture would develop before the consolidation wave arrives.

Let us now examine the magnitude of the parameter u_tT/L. In the experiments of zhang et al. (1999) and Peng et al. (2000), T and u_t are typically on the order of 20 sec and $10^{-4} \sim 10^{-5}$ m/s respectively, while L is on the order of 0.06 m. Hence u_tT/L is on the order of 0.03~0.003. Therefore, when the initial non-uniformity of the grain column is small ($\delta << 1$), the second terms in eq. (21) can be neglected until such time that the non-uniformity becomes sufficiently large and concentrated at certain locations. Consequently for limited time eq. (21) can be further simplified to

$$\begin{aligned} \frac{\partial\varepsilon}{\partial\tau} &= (\bar{u}-\bar{u}_c)\frac{u_t}{u^*(1-\varepsilon)} - q \\ \frac{\partial q}{\partial\tau} &= (\bar{u}-\bar{u}_c)\frac{u_t}{u^*(1-\varepsilon)} - q \end{aligned} \tag{25}$$

which shows that ε and τ are periodic in ς when $\varepsilon_0(\eta)$ is. Quadrature eq. (25) yields

$$\varepsilon = \varepsilon_0(\varsigma) + q \tag{26}$$

$$\tau = \int_0^q \frac{dq}{\frac{u_t}{u_*}\frac{1-\varepsilon_0(\varsigma)-q}{[\varepsilon_0(\varsigma)+q]^2}\varepsilon_0(\varsigma)(\alpha q + \alpha\varepsilon_0(\varsigma) - \beta q) - q} \tag{27}$$

This solution indicates that fractures are likely to develop at such values of when q reaches the largest value at the smallest time. It provides an answer to where and when fractures develop. In particular, it shows that fractures would develop at equal intervals of x if the initial porosity distribution is periodic in x. However this solution is not sufficiently accurate in describing how fracture eventually develops because the non-linear terms in eq. (21) are then no longer negligible.

Fractures do not take place under all circumstances. In our formulation, aside from the initial porosity distribution there are a number of constants. In a non-dimensional form they are α, β, λ. We now show under what constraints must be placed on these constants to ensure that fracture do develop. To do so, we make the problem more specific by considering the case where the initial porosity at distances far greater than Tu_t from $\varsigma = 0$ is constant so that

$$\varepsilon_0(\varsigma) \to \varepsilon_0^+ \text{ as } \varsigma \to \infty \text{ and } \varepsilon_0(\varsigma) \to \varepsilon_0^- \text{ as } \varsigma \to -\infty$$

We want to know under what conditions the two parts of the grain column will eventually be moving apart or closer. Fracture will clearly develop between them if they are moving apart. This leads us to examine the grain velocities u_s at large $|\varsigma|$ as $\tau \to \infty$. Now, according to eq. (26) $\tau \to \infty$ clearly requires the denominator in the integrand to be zero. This enables us to solve for the porosity ε^+ and ε^- at $\varsigma \to \pm\infty$ from the following equations as τ approaches infinity asymptotically

$$\varepsilon_0^{\pm}\left[1 + \lambda\frac{1-\varepsilon^{\pm}}{(\varepsilon^{\pm})^2}(\alpha\varepsilon^{\pm} + \beta\varepsilon_0^{\pm} - \beta\varepsilon^{\pm})\right] - \varepsilon^{\pm} = 0 \tag{34}$$

Defining the corresponding grain velocity by $u_s^{\pm}$, we obtain

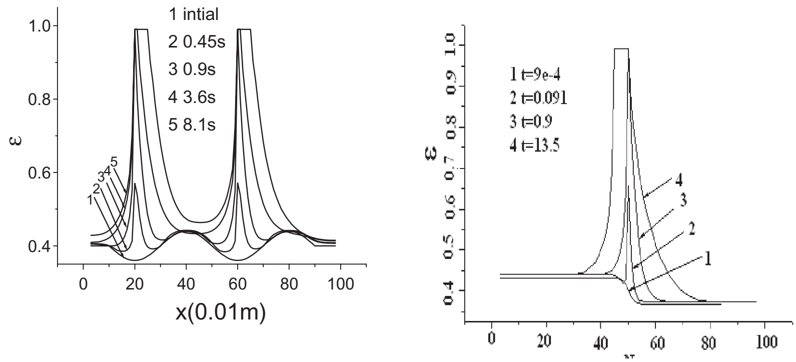
$$u_s^{\pm} = \frac{\varepsilon_0^{\pm}}{\varepsilon^{\pm}}(1-\varepsilon^{\pm})[(\beta-\alpha)(\varepsilon^{\pm} - \varepsilon_0^{\pm}) - \alpha\varepsilon_0^{\pm}] \tag{35}$$

This condition places the necessary condition on values of α, β, λ for given ε_0^+ and ε_0^-. If $u_{s0}^+ - u_{s0}^- > 0$, the fracture will expand gradually, e.f., the faster sinking of grains at the lower layer leads fractures develop gradually when the upper layer is dense while the lower layer is loose. As an example, we take $\varepsilon_0^- = 0.408$, $\varepsilon_0^+ = 0.392$, $\alpha = 1$, $\beta = 56$ and $\lambda = 14$. This yields $u_s^+ - u_s^- = 3.56\times10^{-3}$ as $\tau \to \infty$ with $u_{s0}^+ - u_{s0}^- = 3.2\times10^{-3}$. This results clearly demonstrates that fracture will eventually develop near $\varsigma = 0$.

NND difference method[9] is used here to solve eqn. (25). It is shown that if the initial permeability is periodic or non-periodic, then the grain column will rupture periodic or non-periodic (Figs. 3 a,b). The reason is that the grain above the blocked position will be prevented to drop across the blocked position and so the porosity becomes smaller and smaller, while the grain below the positions will settle gradually and cause the fracture initials and grows gradually[10]. Non-uniformity of grain size distribution along the depth of the grain column is an essential precondition for fracture to initiate and grow. The transport of fine grains by percolation tends to aggravate this non-uniformity. The porosity of the upper part must be smaller than that of the lower part if fractures form. Liquefaction is a necessary condition for the formation of fractures.

Assuming that the grain column is always jammed once the permeability k of a point reaches zero. The parameters in our numerical computing are adopted as:

If the coordinate x is less than 0.05 m or greater than 0.45 m, then ε_0 equals 0.4. Otherwise, ε_0 is distributed as eqn. (11). And $\beta = 80.0$, $a = 0.4$, $b = 0.04$, $c = 0.025$, $d = 0.5$, $u_* = 0.04$, $k_0 = 4 \times 10^{-5}$ m/s, $h = 0.5$ m.



(a) there are two places with initial nonuniformity (b) There is one place with initial nonuniformity

Figure 3. Development of fractures.

4 CONCLUSIONS

It is shown that the occurrence of fractures in saturated grain accumulation under gravity is due to the jamming of the finer component by percolation. A theory of pseudo-three-phase media is presented to explain the basic mechanism of the formation of such fractures. The primary theoretical analysis and calculations indicate the theory does catch the main features. If the distribution of initial porosity is periodic, the fractures will occur periodic also. After enough fractures occur and each block becomes fragment, debris flow will form.

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