Slip-line field solution for ultimate bearing capacity of a pipeline on clayey soils

Fuping Gao,[1,a)] and Bo Zhao[1,2]

[1)] *Institute of Mechanics, Chinese Academy of Sciences, Beijing 100190, China*

[2)] *Engineering Design Institute, CNPC Offshore Engineering Company Limited, Beijing 100028, China*

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Abstract A slip-line field solution is presented for the ultimate bearing capacity of the pipeline on a purely-cohesive clay soil, taking into account the circular configuration of the pipe, the pipe embedment, and the pipe-soil interfacial cohesion. The derived bearing capacity factors for a smooth rigid pipe limit to those for the conventional rectangular strip footing while the pipe embedment approaches zero. Parametric studies indicate that, the pipe-soil interfacial properties have much influence on the bearing capacity for the pipe foundation on clayedy soils. [doi:10.1063/2.1205104]

Keywords collapse loads, slip-line field theory, submarine pipeline, shallow foundation, clayedy soils

The ultimate bearing capacity of a submarine pipeline on the seabed is the pressure causing shear failure of the supporting soil immediately below and adjacent to the pipe foundation. In the recently-issued Det Norske Veritas (DNV) Recommended Practice,[1] the vertical stability of pipelines on and in soils has been specially documented, along with the lateral stability. It is highly desired to efficiently evaluate the bearing capacity of pipeline foundations. When laid on the seabed, the submarine pipeline settles into the soil with certain embedment under the action of its submerged weight (shown as Fig. 1). During the laying process or the operating period, additional vertical loads due to cyclic movements of the catenary riser can also be created at the touchdown zones. The bearing capacity of soft clayey sediments is one of the main geotechnical concerns for the vertical and the lateral on-bottom stability, in particular in deep water to lateral buckling of the pipelines.

Unlike the conventional rectangular strip footing, a pipeline holds a circular cross-section. As such, the effective bearing width of the pipe-soil interface is a function of pipeline embedment, and the existing formulas for the ultimate bearing capacity of conventional footing could not be efficiently employed for evaluating the ultimate load for the pipeline foundations. A proper determination of the ultimate bearing capacity is crucial for evaluation of the on-bottom stability of submarine pipelines in ocean currents and/or waves.[2]

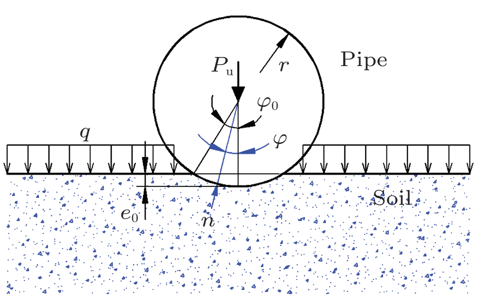

Fig. 1. Sketch map of pipeline embedment in soil (note: $q = 0$ for the case of $e_0/r \leqslant 1$).

[a)] Corresponding author. Email: fpgao@imech.ac.cn.

The settlement and bearing capacity of the pipeline have received much attention in the past few decades. Conventional bearing capacity theories are mainly for the footings with plane bottom. In the theoretical analyses, the soil is absolutely divided into the plastic yield zone and the outer elastic deformation zone. Small et al.[3] treated the pipeline with certain submerged weight as an equivalent uniform distributed pressure upon a rectangular footing, and proposed empirical formulas for the bearing capacity factors by modifying the solutions for a conventional strip footing. Their treatment obviously could not take into account the effects of the circular section of the pipeline. Karal[4] applied the upper bound theorems of classical plasticity theory to develop a prediction of pipe penetration, idealizing the pipe as a rigid wedge indenter. The approximation of pipeline with wedge indenter might be reasonable at small embedment but error becomes significant with increasing embedment. Upper and lower bound solutions to penetration of a pipe into cohesive soil were presented by Murff et al.[5] Finite element method was further adopted by Aubeny et al.[6] for the plane-strain calculation of collapse loads of the pipeline foundation for the soil profiles with the shear strength varying linearly with depth. Hodder and Cassidy[7] proposed a plasticity model for predicting the undrained behaviour of a rigid pipe in clay soils when subjected to combined vertical and horizontal loading.

In this study, the bearing capacity of the pipeline on Tresca soils is analyzed theoretically by employing the slip-line field theory. The bearing capacity for a submarine pipeline laid upon the horizontally flat seabed can be treated as a plane-strain problem (shown as Fig. 1). The clayey seabed is regarded as a rigid-perfectly plastic material. The Tresca yielding criterion is adopted for the saturated soft clay under undrained conditions.

There exists an embedment (e_0) of the pipe with radius of r. For the case of $e_0/r \leqslant 1$, the uniform overburden load at the two sides of the pipe $q = 0$. For the case of $e_0/r > 1$, the pipe-soil contact condition can be treated as that for $e_0/r = 1$, the weight of soil above the pipe center is replaced by an equivalent uniform surcharge pressure $q = (e_0 - r)\,\gamma'$, where γ' is the effective (buoyant) unit weight of soil. The pipe-soil contact friction is taken into account. Following the assumption by Randolph and Houlsby,[8] the adhesion at the rough pipe-soil interface is taken as a constant factor of the soil cohesion $a = \alpha c$ $(0 \leqslant \alpha \leqslant 1)$, where α is the pipe-soil interfacial adhesion coefficient, c is the soil shear strength (cohesion). Thus, for a certain point E at the pipe-soil interface (shown as Fig. 2), the direction for the slip-line is $\theta_E = \pi/4 - \varphi + \Delta/2$, in which $\Delta = \arcsin \alpha$.

According to the well-known slip-line field theory, the coordinates of the slip-lines can be obtained by solving the characteristic for slip-lines under certain boundary conditions using finite-differential method, then the mean stress σ (note: $\sigma = (\sigma_1 + \sigma_3)/2 = (\sigma_x + \sigma_y)/2$) at a certain point in the slip-line field, and the angle θ between the tangent line and the x-axis can be calculated from the Hencky stress equations.

As shown in Fig. 2, the boundaries CG and CEB are the Riemann conditions for determining the uniform field CFG and the extrusion field CBD, respectively, The boundaries CF and CD are the regressive Riemann conditions for determining the transition region CDF. Based on the stress analysis, on the line CG the minimum stress can be determined with the magnitude of q and its direction is vertical. On the line CEB, the maximum stress is located, whose direction is perpendicular to the line CEB, and whose magnitude is to be determined. Lines CF and CD are the boundary for the field CFD, whose solution can be determined from the results of the uniform field CFG and those of the extrusion field CBD. By employing the finite-differential method, the slip-line fields for the pipeline foundations can be constructed. Figure 2 gives the slip-line fields for the smooth pipeline ($\alpha = 0$) and the rough pipeline ($\alpha = 0.5$), respectively. As indicated in this figure, the whole slip-line field can be divided into three regions, i.e., the uniform region CFG, the extrusion region CBD, and the transition region CDF. The magnitude of the slip-line field for the case of the rough pipelines is larger than that of the smooth pipes.

Based on the aforementioned basic assumptions and the constructed slip-line fields, the ultimate load for pipeline foundations can be further derived as follows. The ultimate bearing load P_{u} is expressed in the integral form as

$$P_{\mathrm{u}} = 2\int_0^{\varphi_0} r\sigma_{E,y}\,\mathrm{d}\,\varphi, \tag{1}$$

where $\sigma_{E,y}$ is the vertical component of the pipe-soil contact force, and φ_0 is the embedment angle $\angle BOC$ (shown as Fig. 2), $\varphi_0 = \arccos(1 - e_0/r)$. As shown in Fig. 2, the points A and E are along the same α line,

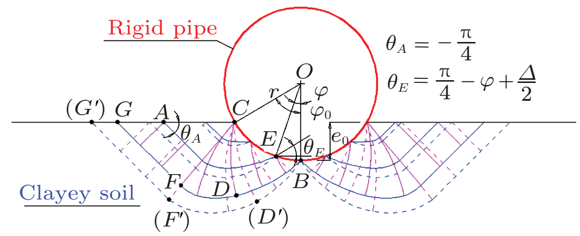

Fig. 2. Slip-line fields of the pipeline foundation on the clayey soil obeying Tresca yield criterion (solid lines is for smooth pipe ($\alpha = 0$); dash lines is for rough pipe ($\alpha = 0.5$)).

and let $\angle BOC = \varphi$. Submitting the values of σ and θ at points A and E into the Hencky stress equations, one has

$$\sigma_A - 2c\theta_A = \sigma_E - 2c\theta_E. \tag{2}$$

That is

$$\sigma_E = \sigma_A + 2c(\theta_E - \theta_A) = \sigma_A + 2c\left(\frac{\pi}{2} + \frac{\Delta}{2} - \varphi\right). \tag{3}$$

As $\sigma_E = \sigma_{E,1} - c$ ($\sigma_{E,1}$ is the first principal stress at point E along the pipe-soil contact arc), then

$$\sigma_{E,1} = \sigma_E + c = \sigma_A + 2c\left(\frac{\pi}{2} + \frac{\Delta}{2} - \varphi\right) + c, \tag{4}$$

in which $\sigma_A = q + c$. At the point E along the pipe-soil contact arc, the vertical component of the pipe-soil contact force $\sigma_{E,y}$ can be expressed as

$$\sigma_{E,y} = \sigma_{E,1}\cos(\varphi - \Delta/2). \tag{5}$$

Submitting Eqs. (4) and (5) into Eq. (1), the ultimate bearing load P_{u} can be derived as

$$\begin{aligned} P_{\mathrm{u}} = 2\int_0^{\varphi_0} [\,c\,(\pi + \Delta + 2 - 2\varphi) + q]\,\cdot \\ \cos\left(\varphi - \frac{\Delta}{2}\right) r\,\mathrm{d}\,\varphi = 2[cr(2 + \pi + \Delta) + qr]\,\cdot \\ \left[\sin\left(\varphi_0 - \frac{\Delta}{2}\right) + \sin\left(\frac{\Delta}{2}\right)\right] - \\ 4cr\left[\varphi_0 \sin\left(\varphi_0 - \frac{\Delta}{2}\right) + \right. \\ \left. \cos\left(\varphi_0 - \frac{\Delta}{2}\right) - \cos\left(\frac{\Delta}{2}\right)\right] \end{aligned} \tag{6}$$

Referring to the formula of the bearing capacity for conventional strip footings, the bearing capacity for pipeline foundations may be expressed in the following form

$$\frac{P_{\mathrm{u}}}{2r\sin\varphi_0} = cN_c + qN_q, \tag{7}$$

where $2r\sin\varphi_0$ is the width of the pipe-soil interface.

Submitting Eq. (6) into Eq. (7), the bearing capacity factor for cohesion (N_c) and the bearing capacity factor for distributed load (N_q) can thereby be obtained

$$N_{\rm c} = \frac{1}{\sin\varphi_0}(2+\pi+\Delta)\left[\sin\left(\varphi_0-\frac{\Delta}{2}\right)+\sin\left(\frac{\Delta}{2}\right)\right]-\frac{2}{\sin\varphi_0}\left[\varphi_0\sin\left(\varphi_0-\frac{\Delta}{2}\right)+\cos\left(\varphi_0-\frac{\Delta}{2}\right)-\cos\left(\frac{\Delta}{2}\right)\right] \tag{8a}$$

$$N_q=\frac{\sin\left(\varphi_0-\frac{\Delta}{2}\right)+\sin\left(\frac{\Delta}{2}\right)}{\sin\varphi_0}. \tag{8b}$$

In the analysis on the general shear failure mechanism of a conventional rectangular-shaped strip footing, e.g., Prandtl-Reissner solution,[8] the smooth strip footing carries a uniform pressure on the surface of a mass of homogeneous, isotropic soil; the shear strength parameters for the soil are c and ϕ; a surcharge pressure q acting on the soil surface has been taken into account. The following exact solution has been widely used for the ultimate bearing capacity of a rectangular-shaped strip footing on the surface of a weightless soil

$$\frac{P_{\rm u}}{b} = cN_c + qN_q \quad \text{(for a rectangular strip footing)}, \tag{9}$$

where b is the width of the conventional strip footing (note that, for the pipeline foundation, $b = 2r\sin\varphi_0$ (shown as Fig. 2)), N_c and N_q are the bearing capacity factors, i.e.

$$N_c = (q-1)\cot\phi, \tag{10a}$$

$$N_q = {\rm e}^{\pi\tan\phi}\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right), \tag{10b}$$

in which ϕ is the internal angle of soils. For the rectangular strip footing on a pure cohesive soil (i.e., $\phi = 0$), the bearing capacity factors are $N_c = \pi + 2$, $N_q = 1$.

For the case of the partially-embedded pipeline on Tresca soils, if the pipeline surface is fully-smooth ($\Delta = 0$), then the bearing capacity factors Eqs. (8a) and (8b) are simplified as

$$N_c = (\pi+2) + 2\left(\frac{1-\cos\varphi_0}{\sin\varphi_0}-\varphi_0\right), \tag{11a}$$

$$N_q = 1. \tag{11b}$$

Now to examine the two extrema of N_c (shown as Eq. (11a)), one has

$$\lim_{\varphi_0\to 0} N_c = (\pi+2) + 2\left(\frac{1-\cos\varphi_0}{\cos\varphi_0}-\varphi_0\right) = \pi+2, \tag{12a}$$

$$\lim_{\varphi_0\to \pi/2} N_c = 4. \tag{12b}$$

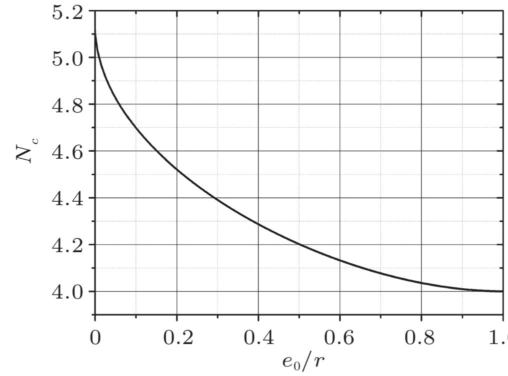

Fig. 3. Variation of N_c with e_0/r for smooth pipes.

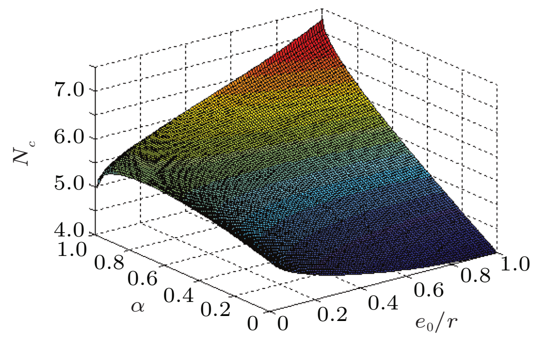

Fig. 4. Variation of N_c with α and e_0/r.

Figure 3 gives the variation of N_c with e_0/r for smooth pipes. When $\varphi_0 \to 0$ (i.e., the pipeline just touches the soil surface $e_0/r = 0$), the bearing capacity factor N_c for pipeline foundations (shown as Eq. (12a)) matches that for the conventional strip footings. This indicates that, while the pipeline embedment is approaching zero, the formulae for the bearing capacity of pipeline foundations degenerate into those for th nventional rectangular-shaped strip footings.

With the increase of the pipeline embedment, the value of N_c decreases gradually and finally reaches 4.0 when the pipeline is half buried (shown as Fig. 3). Therefore, if pipeline foundations are directly simplified as conventional strip footings, the bearing capacity

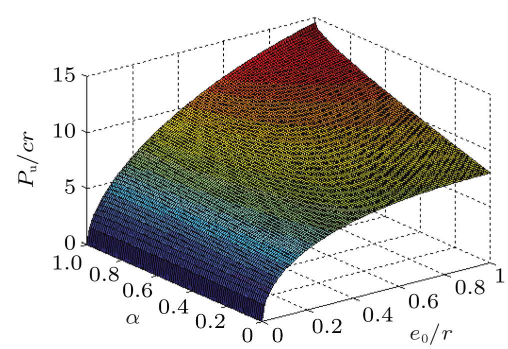

Fig. 5. Variation of $P_{\rm u}/cr$ with α and e_0/r ($q = 0$).

factor N_c would be over evaluated, whose error may be up to 28.5%. Based on the derived formulae for the bearing capacity of the partially embedded pipeline on Tresca soils, i.e., Eqs. (8a) and (8b), the relationship between the bearing capacity factors (N_c, N_q) and the non-dimensional pipeline embedment (e_0/r), and the pipe-soil interfacial cohesion coefficient (α) can be established.

Figure 4 gives the variation of N_c with the parameters e_0/r and α. As shown in Fig. 4, when $e_0/r < 0.6$, the values of N_c initially increases to a maximum value, then decreases continuously with the increase of α; when $e_0/r > 0.6$, the values of N_c increases with increasing α. The effect of α on N_c gets more significant with the increase of pipeline embedment (e_0/r). The maximum value of N_c emerges ($N_c = 7.30$) under the condition of the fully-bonding ($\alpha = 1$) and half-burial ($e_0/r = 1$).

For better understanding the bearing capacity of pipeline foundations, the dimensionless ultimate bearing load P_u/cr is introduced. Eq. (7) is thereby rewritten as

$$P_\mathrm{u}/cr = 2\left(N_c + \frac{q}{c}N_q\right)\sin\varphi_0, \tag{13}$$

in which, the bearing capacity factors N_c and N_q are calculated with Eqs. (8a) and (8b). Figure 5 gives the variation of P_u/cr with the dimensionless pipeline embedment (e_0/r) and the pipe-soil interfacial cohesion coefficient (α), under the condition that the embedment is less than the pipeline radius ($q = 0$). For the fixed value of α, P_u/cr increases with increasing e_0/r. For the fixed values of e_0/r, P_u/cr increases with increasing α; the effects of α on P_u/cr are higher for larger values of e_0/r. When $\alpha = 1$ and $e_0/r = 1$, P_u/cr reaches its maximum value.

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