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THE INFLUENCES OF RESIDUAL SURFACE STRESS ON THE THERMO-ELASTIC BENDING OF SIMPLY SUPPORTED NANOPLATES

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ABSTRACT

When thickness of plates is in nanoscale, the surface energy effect will become prominent, which renders the effective mechanical behaviors of plates to be size-dependent. In this paper, the classical plate theory is modified, by taking the surface energy effect into account, to investigate the size-dependent thermal bending of simply supported nanoplates. Results show that not only the surface elasticity but also the surface residual stress has significant effects on the thermal bending deformations.

Key Words: residual surface stress; thermal bending; nanoplates;

INTRODUCTION

Nanoplates have been widely used as the building blocks for ultra-sensitive and ultrafine resolution applications in the field of nanoelectromechanical systems (NEMS), due to their potentially remarkable thermal-mechanical properties at nano scales, which deviate from macroscopic counterparts and depend on their characteristic size [1]. By taking into account surface effects, the size-dependent elastic properties of nanoplates are investigated [2-4]; however, few of theoretical studies investigated the mechanical responses of nanoplates with temperature effect [5]. Recently, in the framework of continuum thermodynamics, the thermo-hyperelastic models for both the surface and the bulk of nanostructured materials are developed, in which the residual stresses are taken into account [6, 7].

In this paper, analytical model is developed for the size-dependent thermal bending of simply supported nanoplates by modifying Kirchhoff plate theory, in which surface elasticity and surface residual stresses are taking into account. First, the linear thermo-elastic constitutive relations for both the surface and the bulk are presented in the component forms. Then, based on the variational method of energy

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functional, the governing equations and the boundary conditions for thermal bending of nanoplates are obtained. It can be found that not only the surface elasticity but also the surface residual stress can modify the corresponding equations for the classical plate thermal bending deformation.

CONSTITUTIVE EQUATIONS

Under the Kirchhoff hypothesis, the linear filaments of the plate in the reference configuration perpendicular to the middle surface remain straight and perpendicular to the deformed middle surface during stretching and bending. The displacement components of a point with coordinates (x_1, x_2, x_3) in the reference configuration can be denoted by

$$u_\alpha = u_\alpha^0 - x_3 u_{3,\alpha}, \quad u_3 = u_3^0, \tag{1}$$

where $u_i^0(x_1, x_2)$ $(i = 1,2,3)$ is the displacement components of a point on the middle neutral surface S_m. Then, strains for stretching and bending of plates are

$$\varepsilon_{\alpha\beta} = \varepsilon_{\alpha\beta}^0 - x_3 u_{3,\alpha\beta}, \tag{2}$$

where

$$\varepsilon_{\alpha\beta}^0 = \frac{1}{2}\left(u_{\alpha,\beta}^0 + u_{\beta,\alpha}^0\right) \tag{3}$$

are the strain components of the middle surface.

The stress-strain relations for the surface and the bulk of the nano plates are given in the following.

Surface thermoelasticity

Assuming the surface to be linear isotropic thermo-elastic, the component form of surface stress can be written as [4, 7]

$$S_{\alpha\beta}^s = \gamma_0^* \delta_{\alpha\beta} + \left(\gamma_0^* + \gamma_1^*\right)\varepsilon_{\kappa\kappa}^s \delta_{\alpha\beta} + \gamma_0^* u_{\beta,\alpha}^s + \left(\gamma_1 - 2\gamma_0^*\right)\varepsilon_{\alpha\beta}^s + \beta_s \theta_s \delta_{\alpha\beta}, \quad S_{3\alpha}^s = \gamma_0^* u_{3,\alpha}^s \tag{4}$$

where $S_{i\beta}^s$ is the first kind Piola-Kirchhoff stress of the surface, $\delta_{\alpha\beta}$ designates the Kronecker delta; the constants γ_0^*, γ_1^* and γ_1 are the surface residual stress and the surface Lame moduli; β_s is the surface thermal expansion coefficient, and θ_s is the change of the surface temperature; $\varepsilon_{\alpha\beta}^s$ is surface strain, and a subscript preceded by a comma indicates differentiation with respect to the corresponding coordinate, α, β and κ range over the integers 1 and 2, summation convention is used.

Thermoelasticity with Residual Stresses for Bulk Materials

In the absence of external mechanical or thermal loading, surface residual stress will induce residual stresses in the bulk of nano plates [4]

$$\hat{T}_{x_1} = \hat{T}_{x_2} = -\frac{2\gamma_0^*}{h}, \tag{5}$$

where h is the thickness of nanoplates. Thus, the bulk materials will deform elastically from a residual stressed state. According to the plate theory, we formulate the thermoelastic stress-strain relations for the bulk materials with in-plane residual stress, as [4, 7]

$$S_{\alpha\beta} = \hat{T}_{\alpha\beta} + u_{\alpha,\kappa}\hat{T}_{\kappa\beta} + \frac{Y}{1-\nu^2}\left[(1-\nu)\varepsilon_{\alpha\beta} + \nu\varepsilon_{\kappa\kappa}\delta_{\alpha\beta}\right] + \beta_b\theta_b\delta_{\alpha\beta}, \quad S_{3\alpha} = u_{3,\kappa}\hat{T}_{\kappa\alpha}. \quad (6)$$

in which $S_{i\beta}$ is the first Piola-Kirchhoff stress, $\hat{T}_{\alpha\beta}$ is the residual stress induced by surface residual stress in the reference configuration, $\varepsilon_{\alpha\beta}$ are the strain components, Y and ν designate the Young's modulus and the Poisson's ratio of the bulk, β_b is the bulk thermal expansion coefficient, θ_b is the change of bulk temperature.

THERMAL BENDING OF NANOPLATES

Under the action of thermal loadings, nanoplates will deform. In the following, we investigate the thermal bending of simply supported nanoplates.

Governing Equations

Assume that there is no external lateral load acting on the simply supported nanoplates during thermal deformation process. Then, the governing equations for thermal bending of nanoplates can be obtained from the principle of potential energy, which is written for the present problem as follows [4]

$$\delta U = \iint_{S_m}\left[\int_{-h/2}^{h/2} S_{i\alpha}\delta u_{i,\alpha} dx_3\right]dx_1 dx_2 + \iint_{S} S^s_{i\alpha}\delta u^s_{i,\alpha} dx_1 dx_2, \quad (7)$$

where s denote the upper and the lower surfaces of nanoplates, which are defined by $x_3 = \pm\frac{h}{2}$, respectively.

Substituting Eqs. (1), (4) and (6) into (7), we can find that δU includes the stretching part $\delta U_{stretching}$ and the bending part $\delta U_{bending}$. In the present paper, we pay attention to the latter one. For simplicity, assume that $\theta_b = \tau(x,y)z$ and $\theta_b|_{\pm\frac{h}{2}} = \theta_s^{\pm}$. Through the use of integrations by parts and the Gaussian theorem, we can get the minimum condition of the bending part potential energy

$$\begin{aligned}&\iint_{S_m}\left\{\left[D+\left(\gamma_1^* + \gamma_1 - \frac{1}{3}\gamma_1^*\right)\frac{h^2}{2}\right]u^0_{3,\alpha\alpha\beta\beta} - \beta_e\frac{h^3}{12}\tau_{,\alpha\alpha}\right\}\delta u^0_3 dx_1 dx_2 + \\ &\oint_C\left\{\left[D\nu + (\gamma_0^* + \gamma_1^*)\frac{h^2}{2}\right]u^0_{3,\alpha\alpha} + \left[D(1-\nu) + \left(\gamma_1 - \frac{4}{3}\gamma_0^*\right)\frac{h^2}{2}\right]\eta_1 - \beta_e\frac{h^3}{12}\tau\right\}\frac{\partial(\delta u^0_3)}{\partial n}ds = 0,\end{aligned} \quad (8)$$

where $D = Yh^3/\left[12(1-\nu^2)\right]$ is flexural stiffness, $\beta_e = \beta_b + \frac{6\beta_s}{h}$ is the effective thermal expansion coefficient and

$$\eta_1 = u^0_{3,11}\cos^2\theta + 2u^0_{3,12}\sin\theta\cos\theta + u^0_{3,22}\sin^2\theta. \quad (9)$$

Since δu^0_3 are arbitrarily on surface S_m, we obtain the equilibrium equation for thermal bending

$$D_e u^0_{3,\alpha\alpha\beta\beta} - \beta_e \frac{h^3}{12} \tau_{,\alpha\alpha} = 0, \tag{10}$$

in which $D_e = D\left[1 + 6\left(1-\nu^2\right)\frac{\gamma_1^* + \gamma_1 - \frac{1}{3}\gamma_0^*}{Yh}\right]$.

For simply supported nanoplates, $\delta u_3^0 = 0$ and $\partial\left(\delta u_3^0\right)/\partial n \neq 0$ on the boundary C. Therefore, the coefficient of the contour integral in Eq. (8) must be zero. For the cases that $x_1 = \pm a$ and $\theta = 0$, it becomes

$$D_e u^0_{3,11} - \beta_e \frac{h^3}{12} \tau = 0. \tag{11}$$

Eq.(10) together with boundary condition (11) can be solved.

From Eqs. (10) and (11), it can be found that the surface residual stress is included in the governing equations and boundary conditions. Furthermore, when surface parameters vanish or the thickness becomes large, the corresponding equations will reduce to the classical ones.

CONCLUSIONS

By taking the surface effects into accounts, the size-dependent thermal bending of simply supported nanoplates is discussed in the present paper. Results show that not only the surface elasticity but also the surface residual stress will effect the thermal bending of nanoplates.

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