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Chaos synchronization between fractional-order unified chaotic system and Rossler chaotic system

Wu Xian-yong [1,a], Cheng Yi-long[1,b], Liu Kai[2] , Yu Xin-liang[2], Wu Xian-qian[3]

[1.] College of Electrical Engineering & Renewable Energy, China Three Gorges University, Yichang, 443002, People's Republic of China;

[2.] College of Mechanical & Material Engineering , China Three Gorges University, Yichang, 443002, People's Republic of China;

[3.] Key Laboratory for Hydrodynamics and Ocean Engineering, Institute of Mechanics, Chinese Academy of Sciences, Beijing, 100190, People's Republic of China.
[a.] email: wu_xianyong@163.com; [b.] email cscyl1984@126.com.

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Abstract: The chaotic dynamics of the unified chaotic system and the Rossler system with different fractional-order are studied in this paper. The research shows that the chaotic attractors can be found in the two systems while the orders of the systems are less than three. Asymptotic synchronization of response and drive systems is realized by active control through designing proper controller when system parameters are known. Theoretical analysis and simulation results demonstrate the effective of this method.

Introduction

Chaos and its application is a hot issue in the nonlinear science in the last few years, and chaotic system exhibits complicated dynamics behavior. In 1898, chaotic control was first introduced by Hubler[1]. In 1990, Ott, Grebogi and Yorke proposed OGY method[2]. In the same year, Pecora and Carroll introduced the chaotic synchronization theory[3]. Later a wide variety of approaches have been proposed for the synchronization of chaotic systems which include drive-response synchronization[4], linear and nonlinear feedback control[5], adaptive control method[6], coupling synchronization[7], active control[8], pulse synchronization[9] etc. In this paper, we apply active control theory which has advantage of not calculating the Lyapunov index, and achieve global asymptotic synchronization of two systems through designing proper controller.

At present, the designs of synchronization scheme are mostly related to chaotic systems with two identical structures, but the practical applications are mostly about synchronization of chaotic systems with different structures, such as it is difficult to assume two identical systems in secure communications. Reference[10] is about the synchronization with different structures between integral-order Rossler system and unified chaotic system. Numerous researches indicate that it will still occur chaotic phenomena even with fractional order in chaotic system. Moreover, it can reflect the physical phenomena of system in engineering preferably. In this paper, we study the chaotic dynamics behavior of fractional-order Rossler system and fractional-order unified system. It is found that two systems have chaotic attractors when the order is less than three. We accomplish different structures synchronization between fractional-order Rossler system and fractional-order unified system by using active control technique. Theoretical analysis and numerical simulation demonstrate the effectiveness of this method.

1. Fractional-order differential and approximate calculation

Fractional-order differential has various definitions[11-12], fractional-order differential defined by Riemann-Liouville and by Caputo are used regularly. In this paper, we select Riemann-Liouville (R-L) definition which is used widely in application study. R-L differential is defined by:

$$\frac{d^{\alpha} f(t)}{dt^{\alpha}} = \frac{1}{\Gamma(n-\alpha)} \frac{d^{n}}{dt^{n}} \int_0^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau \,, \tag{1}$$

Where n is integer, and $\alpha > 0$, $n-1 \le \alpha < n$, $\Gamma(\cdot)$ is Gamma function. Laplace transform of R-L fractional-order differential is:

$$L\left\{\frac{d^{\alpha} f(t)}{dt^{\alpha}}\right\} = s^{\alpha} L\{f(t)\} - \sum_{k=0}^{n-1} s^{k} \left[\frac{d^{\alpha-1-k} f(t)}{dt^{\alpha-1-k}}\right]_{t=0} , \tag{2}$$

Where n is integer, and $\alpha > 0$, $n-1 \le \alpha < n$[13]. Considering that initial condition is zero, it can be simplified as follows:

$$L\left\{\frac{d^{\alpha} f(t)}{dt^{\alpha}}\right\} = s^{\alpha} L\{f(t)\} \tag{3}$$

Thus α order fractional-order integral operator can be transformed into frequency domain calculation, the transfer function is: $F(s) = 1/s^{\alpha}$.

At present, adopting the way of fitting integral-order into fractional-order and using the method of solving integral-order to calculate are the main solutions of implementing the operating of fractional-order calculus. Time and frequency domain transformation method[13] is most commonly employed in engineering, afterwards piecewise linear approximate method is carried out to compute. In this article, we adopt time and frequency domain transformation method to fit the fractional-order differential equation into integral-order differential equation and analyze the numerical simulation.

2. Chaos synchronization of two different systems

Consider the following nonlinear fractional dynamical system:

$$\begin{cases} \dfrac{d^{\alpha} x}{dt^{\alpha}} = f(t,x) \\ \dfrac{d^{\alpha} y}{dt^{\alpha}} = g(t,y) + u(t,x,y) \end{cases} \tag{4}$$

Where $x, y \in R^n$, $f, g \in R \times R^n \to n^n$ are differentiable functions, $u(t,x,y)$ is a control input in equations (4). The upper equation is a drive system and the nether one is a response system. Defining $e=y-x$ as the synchronization errors, we devise a controller u to make the trajectory of the response system whose initial condition is y_0 gradually tend to the drive system whose initial condition is x_0, and eventually we achieve the synchronization between the two systems, as followed:

$$\lim_{t\to\infty} \|e\| = \lim_{t\to\infty} \|y(t,y_0) - x(t,x_0)\| = 0$$

Where: $\|\cdot\|$ is Euclidean distance.

Assume that the fractional-order unified chaotic system and Rossler system are drive system and response system respectively:

$$\begin{cases} \dfrac{d^{\alpha} x_1}{dt^{\alpha}} = (25k+10)(x_2 - x_1) \\ \dfrac{d^{\alpha} x_2}{dt^{\alpha}} = (28-35k)x_1 + (29k-1)x_2 - x_1 x_3 \\ \dfrac{d^{\alpha} x_3}{dt^{\alpha}} = x_1 x_2 - \dfrac{(k+8)}{3} x_3 \end{cases} \tag{5}$$

$$\begin{cases} \dfrac{d^{\alpha} y_1}{dt^{\alpha}} = -y_2 - y_3 + u_1(t) \\ \dfrac{d^{\alpha} y_2}{dt^{\alpha}} = y_1 + a y_2 + u_2(t) \\ \dfrac{d^{\alpha} y_3}{dt^{\alpha}} = b + y_3(y_1 - c) + u_3(t) \end{cases} \tag{6}$$

We have introduced three control functions $u_1(t)$, $u_2(t)$, $u_3(t)$ in (6). These functions are to be determined. Let us define the errors signals as $e_1=y_1-x_1$, $e_2=y_2-x_2$, $e_3=y_3-x_3$.

We obtain the error system by abstracting (5) from (6):

$$\begin{cases} \dfrac{d^\alpha e_1}{dt^\alpha} = -(e_2+e_3)-(25k+10)e_1-(25k+11)x_2-x_3+(25k+10)y_1+u_1(t) \\ \dfrac{d^\alpha e_2}{dt^\alpha} = e_1+ae_2-(27-35k)x_1-(29k-1-a)x_2+x_1x_3+u_2(t) \\ \dfrac{d^\alpha e_3}{dt^\alpha} = -ce_3+\dfrac{(k-3c+8)}{3}x_3-x_1x_2+y_1y_3+b+u_3(t) \end{cases} \tag{7}$$

Control functions are chosen as :

$$\begin{cases} u_1(t)=V_1(t)+(25k+11)x_2+x_3-(25k+10)y_1 \\ u_2(t)=V_2(t)+(27-35k)x_1+(29k-1-a)x_2-x_1x_3 \\ u_3(t)=V_3(t)-\dfrac{(k-3c+8)}{3}x_3+x_1x_2-y_1y_2-b \end{cases} \tag{8}$$

Hence the error system becomes :

$$\begin{cases} \dfrac{d^\alpha e_1}{dt^\alpha} = V_1(t)-(e_2+e_3)-(25k+10)e_1 \\ \dfrac{d^\alpha e_2}{dt\alpha} = V_2(t)-e_1+ae_2 \\ \dfrac{d^\alpha e_3}{dt^\alpha} = V_3(t)-ce_3 \end{cases} \tag{9}$$

The control inputs $V_1(t)$, $V_2(t)$ and $V_3(t)$ are functions of the error states e_1, e_2 and e_3 and are chosen as:

$$\begin{bmatrix} V_1(t) \\ V_2(t) \\ V_3(t) \end{bmatrix} = \begin{bmatrix} 25k-9 & 1 & 1 \\ -1 & -a-1 & 0 \\ 0 & 0 & c-1 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} \tag{10}$$

In this particular choice, the error system (9) to be controlled has the eigenvalues -1, -1, -1. This choice will lead to the error states e_1, e_2 and e_3 converge to zero as time t tends to infinity. Consequently, the different structures synchronization between fractional-order unified chaotic system and Rossler chaotic system is achieved.

3. Numerical simulation

Synchronization between 2.7 order unified chaotic system and Rossler system when α=0.9 is simulated in this paper. Due to 2.7 order unified chaotic system is a family system, the condition k=0.8 is taken as a representative in the simulation.

We Select the parameters of 2.7 order Rossler system as α=0.9, a=0.2, b=0.2, c=5.7 and the parameters of 2.7 unified chaotic system as α=0.9, k=0.8, so that each of the drive system and response system exhibits a chaotic behavior. The initial vales of two systems are $x_1(0)$=0, $x_2(0)$=2, $x_3(0)$=20 and $y_1(0)$=8, $y_2(0)$=4, $y_3(0)$=3 respectively. Synchronization error curves are shown in Fig. 1-4, it demonstrates that the response system can quickly track the drive system and reach an agreement.

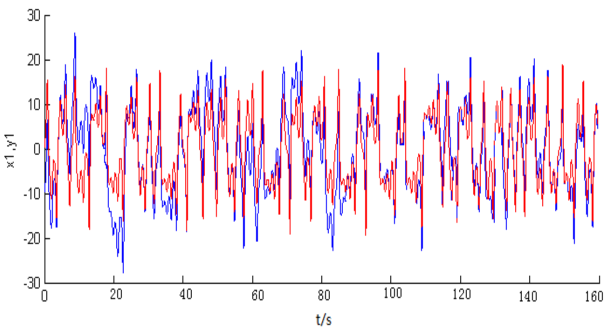

Fig. 1 (a) shows the signal x_1,y_1

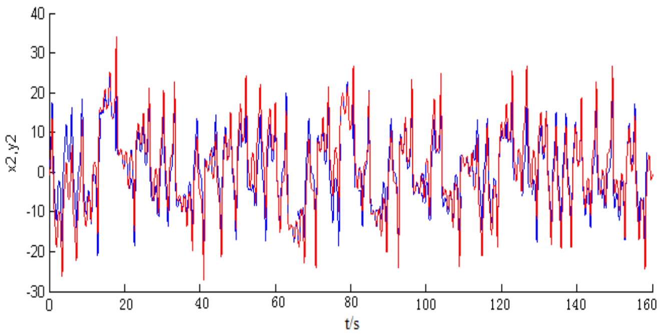

Fig. 2 (b) shows the signal x_2,y_2

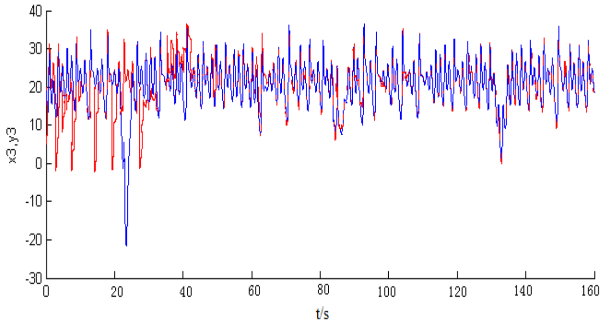

Fig. 3 (c)shows the signal x_3,y_3

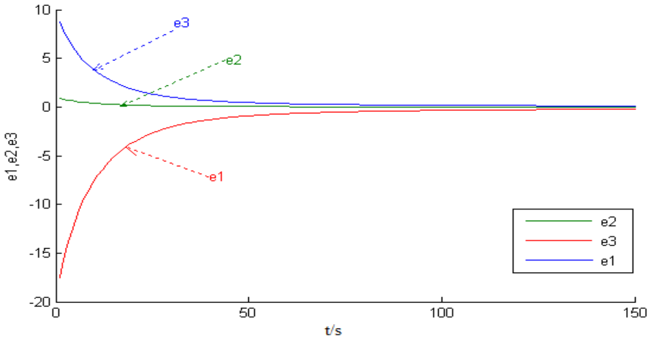

Fig. 4 (d)shows the errors states e_1, e_2 and e_3

4. Summary

In this paper, the dynamics behavior between fractional-order unified chaotic system and Rossler chaotic system is presented, and different structure synchronization between two fractional-order systems is realized using active control. The theory and simulation results validate the effectiveness of this method.

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