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Progressive failure constitutive model of fracture plane in geomaterial based on strain strength distribution

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ABSTRACT

Progressive failure constitutive model of fracture plane in geomaterial based on strain strength distribution is proposed. The basic assumption is that strain strength of geomaterial comply with a certain distribution law in space. Failure of tensile fracture plane and shear fracture plane in representative volume element (RVE) with iso-strain are discussed, and generalized failure constitutive model of fracture plane in RVE is established considering combined effect of tension and shear. Fracture plane consists of elastic microplanes and fractured microplanes. Elastic microplanes are intact parts of the fracture plane, and fractured microplanes are the rest parts of the fracture plane whose strain have ever exceeded their strain strength. Interaction mode on elastic microplanes maintains linear elasticity, while on fractured microplanes it turns into contact and complies with Coulomb's friction law. Intact factor and fracture factor are defined to describe damage state of the fracture plane which can be easily expressed with cumulative integration of distribution density function of strain strength. Strong nonlinear macroscopic behavior such as yielding and strain softening can be naturally obtained through statistical microstructural damage of fracture plane due to distribution of strain strength. Elastic–brittle fracture model and ideal elastoplastic model are special cases of this model when upper and lower limit of distribution interval are equal.

1. Introduction

Geomaterial is usually discontinuous with joints and pre-existing cracks (Harrison and Hudson, 2000; Jing, 2003; Selvadurai and Yu, 2005; Brzakala, 2011). Even for the intact geomaterial sample, it is also heterogeneous due to multi-components, internal micro-cracks or micro-defects in mesoscopic view (Schulz and Evans, 2000; Yue et al., 2003; Lindqvist et al., 2007; Xua et al., 2008). Nature of failure process of geomaterial is the growth and coalescence of inner micro cracks and micro defects until macro structural failure takes place under external loads (Patton, 1996; Tanga and Kou, 1998; Grasselli and Egger, 2000; Yang and Chiang, 2000). In order to describe this physical process, plastic model based on continuum mechanics was founded with nonlinear constitutive relation established by macro experimental parameters (Lade and Kim, 1995; Ketan, 1997; Cekerevaca et al., 2006). Plasticity is a phenomenological model, which translates the complex geometrical problem into complicated physical problem. Plastic models such as ideal elastoplastic model, brittle fracture model, strain softening model etc. can only be used to the situation that crack length is much less than research scale and macro homogeneity is applicable. Under this circumstance, internal crack and fracture state in material cannot be described clearly. Fracture mechanics model and damage mechanics model are proposed to analysis bearing capacity and property of geomaterial with macro cracks or micro defects (Palaniswamy and Knauss, 1972; Misra et al., 2002; Anderson, 2005). However, it is not easy to use fracture mechanics model to analysis global structural failure of material since it concentrates on single-crack property and the result strongly depends on size of element mesh when used in computational mechanics (Marjia et al., 2006); Damage constitutive models for geomaterial based on elastoplasticity (Salari et al., 2004; Fang et al., 2011), energy (Swoboda1 and Yang, 1999), and statistic models considering statistics micro cracks and defects (Suvorov and Selvadurai, 2011), or distributive material parameters (Tang, 1997; Cao and Fang, 1998; Wang et al., 2007; Jiang et al., 2009; Deng and Gu, 2011) have also been established to get progressive failure macroscopic behavior. However, in these models, statistic damage of material was expressed as reduction of elastic modulus. Internal state such as mesoscopic discontinuity have not been fully described, and damage and failure caused by combined effect of tension and shear is hardly mentioned.

The objective of this paper is to propose a new progressive failure constitutive model for fracture plane in geomaterial based on strain strength distribution. Fracture plane here refers to interface, plane with plane crack or any section of RVEs in geomaterial. Three types of failure mode are discussed which include tensile fracture,

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shear fracture and fracture with combined effect of tension and shear. Strain strength is used as statistic and nonlinear macroscopic behaviors such as yielding and strain softening of geomaterial are naturally obtained through fracture of microplanes.

2. Statement of the problem and basic assumptions

Geomaterial is highly heterogeneous and discontinuous. Constant material parameters are inadequate to describe its complicated internal characteristics. Reasonable descriptions of the discontinuity are necessary for a theoretical model to achieve realistic mechanical behavior of geomaterial. In the theory of progressive failure constitutive model proposed in this paper, strength is distributed and fractured microplanes are defined to describe the internal fracture state of material. The geometric concept and physical meaning of the model is shown in Fig. 1. Research scale is restricted to the size of representative volume element with iso-strain. That is also the characteristic length of damage or crack. Then singularity of stress and strain needs not to be considered. Strain is used as the measurement index of failure and assumption of strain strength distribution in material is the major characteristics of this model. Mechanical behavior on elastic area and fractured area of fracture plane are described separately and dependently. Effective stress on fracture plane is the weighted summation of the two parts.

There are four basic assumptions:

Assumption 1. This model is applied to fracture plane of representative volume elements with iso-strain. Iso-strain means strain is equal everywhere in space. Research and computational scale is restricted to representative volume element whose size must be defined small enough that iso-strain condition is applicable. So strength criterion can be used to describe fracture irrespective of stress singularity at the crack tip.

Assumption 2. The strain strength complies with a certain distribution law in representative volume element. Strain is used as the measure of strength. Considering discreteness of strength in material, strain strength is not uniform, but defined to comply with a certain distribution law in space.

Assumption 3. Any section or plane in representative volume element can be composed of elastic microplanes and fractured microplanes. Since representative volume element is iso-strain, any section or plane in it is also iso-strain. But strain strength is distributed in space. Thus, fractured microplanes are formed where strain exceeded strain strength, and the intact parts where strain are lower than strain strength are defined as elastic microplanes.

Assumption 4. Interactions on elastic microplanes remains elasticity, while on fractured microplanes it turns into contact. Elastic microplanes are continuous and maintain elastic behavior, while fractured microplanes become discontinuous and can be described with contact theory. Normal tress on fractured microplanes is zero in tensile and keeps the same value as the elastic microplanes on fracture plane in compression. Shear stress on fractured microplanes comply with Coulomb's friction law.

3. Progressive tensile failure model for tensile plane based on distribution of strain strength

Just consider tensile failure behavior at this part. According to the assumptions above, strain on tensile plane of representative volume element is even, but tensile strain strength is distributed. Under this condition, a tensile plane may be composed of intact part and fractured part. Intact part consists of elastic microplanes

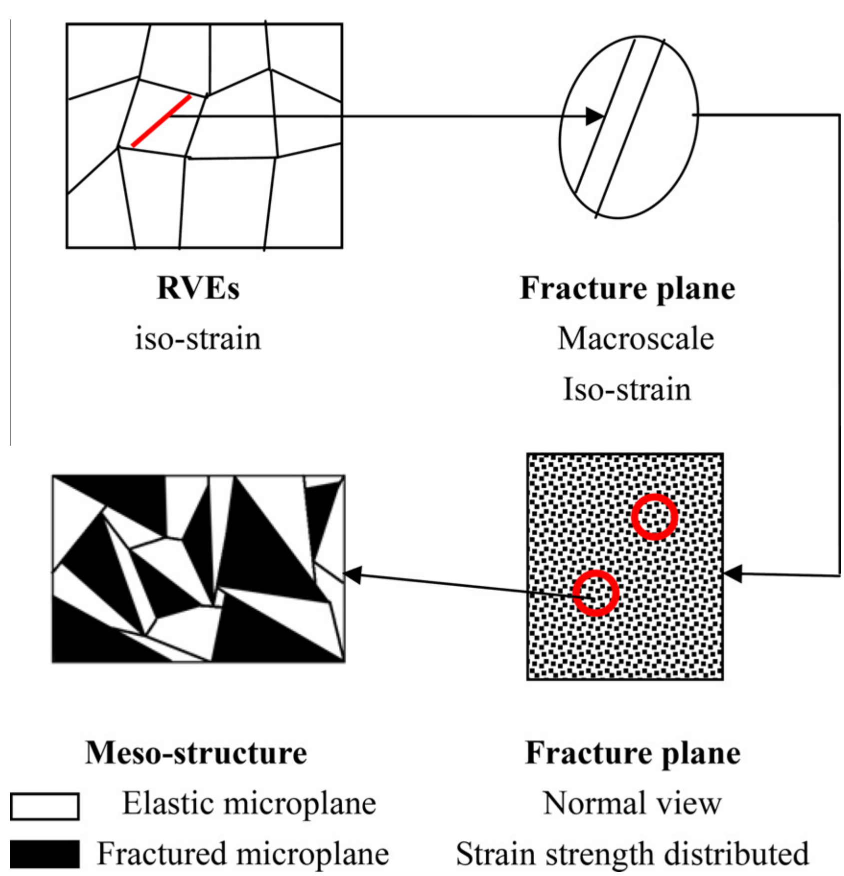

Fig. 1. Geometric concept of fracture plane.

whose tensile strain are always below their tensile strain strength and fractured part consists of fractured microplanes whose tensile strain have ever exceeded their tensile strain strength. Elastic microplanes remain linear elasticity, but fractured microplanes could not bear tension any more.

3.1. Intact factor and fracture factor on tensile fracture plane

In order to represent damage state of tensile fracture plane quantitatively, intact factor α_I and fracture factor α_D are defined. Intact factor α_I represents the ratio of elastic microplanes on the tensile plane and fracture factor α_D represents the ratio of fractured microplanes on the tensile plane. The value of α_I and α_D are both between 0 and 1, and satisfy that

$$\alpha_I + \alpha_D = 1 \tag{1}$$

Intact factor and fracture factor are key parameters of this model to describe fracture degree or damage state of any fracture plane in representative volume element. If distribution law of strain strength and maximum value of strain history is known, intact factor α_I and fracture factor α_D can be calculated in statistic way. α_I is expressed as

$$\alpha_I = \begin{cases} 1 & \bar{\varepsilon} \leqslant \varepsilon_{\min} \\ F_I(\bar{\varepsilon}) = \int_{\bar{\varepsilon}}^{\varepsilon_{\max}} f(\xi) d\xi & \varepsilon_{\min} < \bar{\varepsilon} < \varepsilon_{\max} \\ 0 & \bar{\varepsilon} \geqslant \varepsilon_{\max} \end{cases} \tag{2}$$

where $\bar{\varepsilon}$ is maximum tensile strain in history. $\varepsilon_{\min}$, $\varepsilon_{\max}$ are the lower and upper limits of tensile strain strength. ξ is integral variable. $f(\xi)$ and $F_I(\bar{\varepsilon})$ are distribution density function and cumulative distribution function of tensile strain strength respectively.

In this model, $\varepsilon_{\min}$ refers to linear limit strain and $\varepsilon_{\max}$ represents failure strain corresponding to experimental stress–strain curve. Intact factor α_I on tensile fracture plane is defined as a piecewise function of maximum tensile strain in history and depends on the distribution law of tensile strain strength. The value of intact factor α_I is between 0 and 1, which represents the weight of elastic area remaining to total area of the plane in tension. If $\bar{\varepsilon}$ is less than minimum value of tensile strain strength $\varepsilon_{\min}$, α_I is equal to 1, that means the plane is intact. If $\bar{\varepsilon}$ is larger than maximum value of tensile strain strength $\varepsilon_{\max}$, the tensile plane is totally fractured, α_I is equal to 0. Otherwise, the tensile plane is partly fractured, and α_I can be expressed as the integration of distribution density function of tensile strain strength from $\bar{\varepsilon}$ to $\varepsilon_{\max}$. α_D is the proportion of tensile fractured area to total area, equals to 1 - α_I, which represents fracture degree caused by tension and can be express as

$$\alpha_D = 1 - \alpha_I = \begin{cases} 0 & \bar{\varepsilon} \leqslant \varepsilon_{\min} \\ F_D(\bar{\varepsilon}) = \int_{\varepsilon_{\min}}^{\bar{\varepsilon}} f(\xi) d\xi & \varepsilon_{\min} < \bar{\varepsilon} < \varepsilon_{\max} \\ 1 & \bar{\varepsilon} \geqslant \varepsilon_{\max} \end{cases} \tag{3}$$

where $F_D(\bar{\varepsilon})$ is the cumulative distribution function of tensile strain strength.

Different distribution law of tensile strain strength could be adopted according to different material characteristics, and it will directly determine the change law of intact factor and fracture factor, and influence the fracture process of tensile plane. For example, if tensile strain strength complies with uniform distribution law, $F_I(\bar{\varepsilon})$ and $F_D(\bar{\varepsilon})$ could be written as

$$F_I(\bar{\varepsilon}) = \int_{\bar{\varepsilon}}^{\varepsilon_{\max}} f(\xi) d\xi = \int_{\bar{\varepsilon}}^{\varepsilon_{\max}} \frac{1}{\varepsilon_{\max} - \varepsilon_{\min}} d\xi = \frac{\varepsilon_{\max} - \bar{\varepsilon}}{\varepsilon_{\max} - \varepsilon_{\min}} \tag{4}$$

$$F_D(\bar{\varepsilon}) = \int_{\varepsilon_{\min}}^{\bar{\varepsilon}} f(\xi) d\xi = \int_{\varepsilon_{\min}}^{\bar{\varepsilon}} \frac{1}{\varepsilon_{\max} - \varepsilon_{\min}} d\xi = \frac{\bar{\varepsilon} - \varepsilon_{\min}}{\varepsilon_{\max} - \varepsilon_{\min}} \tag{5}$$

α_I and α_D depend on two strength parameters $\varepsilon_{\max}$ and $\varepsilon_{\min}$.

Weibull distribution (Weibull et al., 1951) is the most popular among empirical distributions due to its wide applicability (Wang et al., 2007), probability density function of Weibull is

$$f(x) = (m/n)(x/n)^{m-1} e^{-(x/n)^m} \tag{6}$$

where m is shape parameter, n is scale parameter, x is independent variable. However, distributive interval of Weibull distribution is between zero and positive infinity. In order to restrict the interval to be between $\varepsilon_{\min}$ and $\varepsilon_{\max}$, the original probability density function should be modified. Integration of original probability density function from $\varepsilon_{\min}$ to $\varepsilon_{\max}$ is

$$\begin{aligned} P &= \int_{\varepsilon_{\min}}^{\varepsilon_{\max}} f(\xi) d\xi = \int_{\varepsilon_{\min}}^{\varepsilon_{\max}} (m/n)(\xi/n)^{m-1} e^{-(\xi/n)^m} d\xi \\ &= e^{-(\varepsilon_{\min}/n)^m} - e^{-(\varepsilon_{\max}/n)^m} \end{aligned} \tag{7}$$

One can use P as a adjustment factor and the new probability density function could be written as

$$f(x) = \frac{(m/n)(x/n)^{m-1} e^{-(x/n)^m}}{e^{-(\varepsilon_{\min}/n)^m} - e^{-(\varepsilon_{\max}/n)^m}} \tag{8}$$

Then, the cumulative distribution function $F_I(\bar{\varepsilon})$ and $F_D(\bar{\varepsilon})$ could be written as

$$\begin{aligned} F_I(\bar{\varepsilon}) &= \int_{\bar{\varepsilon}}^{\varepsilon_{\max}} f(\xi) d\xi = \int_{\bar{\varepsilon}}^{\varepsilon_{\max}} \frac{(m/n)(\xi/n)^{m-1} e^{-(\xi/n)^m}}{e^{-(\varepsilon_{\min}/n)^m} - e^{-(\varepsilon_{\max}/n)^m}} d\xi \\ &= \frac{e^{-(\bar{\varepsilon}/n)^m} - e^{-(\varepsilon_{\max}/n)^m}}{e^{-(\varepsilon_{\min}/n)^m} - e^{-(\varepsilon_{\max}/n)^m}} \end{aligned} \tag{9}$$

$$\begin{aligned} F_D(\bar{\varepsilon}) &= \int_{\varepsilon_{\min}}^{\bar{\varepsilon}} f(\xi) d\xi = \int_{\varepsilon_{\min}}^{\bar{\varepsilon}} \frac{(m/n)(\xi/n)^{m-1} e^{-(\xi/n)^m}}{e^{-(\varepsilon_{\min}/n)^m} - e^{-(\varepsilon_{\max}/n)^m}} d\xi \\ &= \frac{e^{-(\varepsilon_{\min}/n)^m} - e^{-(\bar{\varepsilon}/n)^m}}{e^{-(\varepsilon_{\min}/n)^m} - e^{-(\varepsilon_{\max}/n)^m}} \end{aligned} \tag{10}$$

It guarantees that the integration over $\varepsilon_{\min}$ to $\varepsilon_{\max}$ equals to 1. Using Weibull distribution, α_I and α_D are controlled by four parameters $\varepsilon_{\max}$, $\varepsilon_{\min}$, m and n. For Weibull distribution, relationship between intact factor α_I and tensile strain is shown in Fig. 2.

Intact factor displays different change law with variation of m and n as shown in Fig. 2. Shape of the curve is mainly controlled by m and the scale that the curve covered mainly depends on parameter n.

3.2. Stress–strain relationship on tensile plane

With intact factor α_I and fracture factor α_D defined above, the stress–strain relationship on tensile plane can be written as

$$\sigma_n = \alpha_I E\varepsilon + \begin{cases} (1-\alpha_I)E\varepsilon & \varepsilon < 0 \\ 0 & \varepsilon \geqslant 0 \end{cases} \tag{11}$$

where E is young's modulus, σ_n is effective normal stress on tensile plane, $E\varepsilon$ is an abbreviated expression of linear elastic stress obtained from elastic constitutive equation, the complete form can be written as $E\varepsilon = \lambda e + 2\mu\varepsilon_n$, where λ and μ are Lame constant, e is strain invariant, ε_n is normal strain of the tensile plane. ε is called nominal normal strain here after.

There are two parts in the equation. The first part represents stress on elastic microplanes which maintains linear elastic with the weight of α_I. The second part represents stress on fractured microplanes with the weight of 1 - α_I, which equals to α_D. Piecewise functions are used to express the stress on fractured microplanes. Normal stress on fractured microplanes is zero in tension, but keeps the same value as elastic microplanes on the plane in compression.

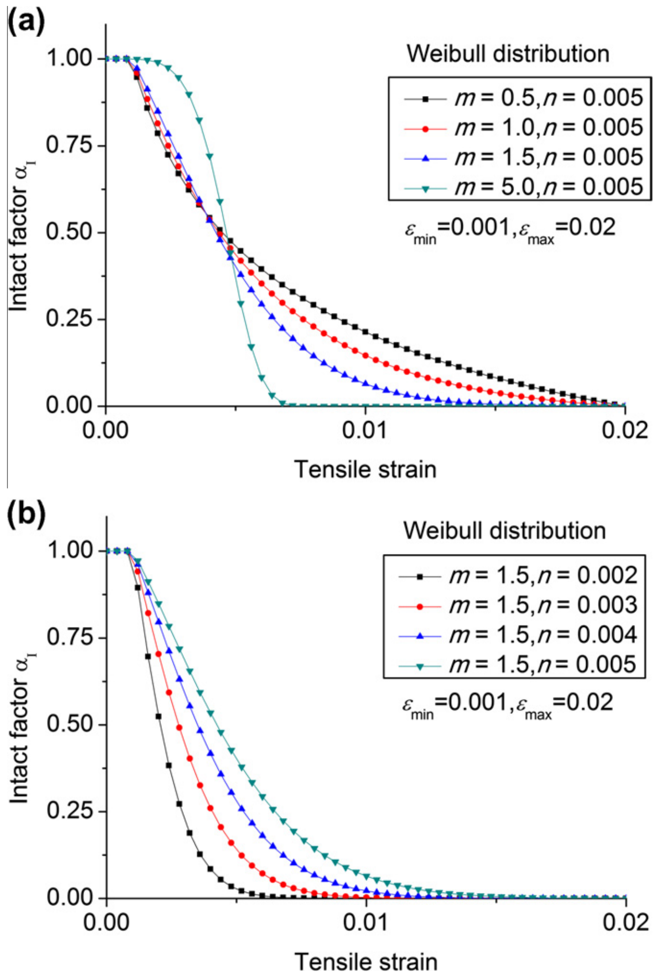

Fig. 2. Relationship between intact factor and tensile strain in Weibull distribution law: (a) Influence of *m*; (b) Influence of *n*.

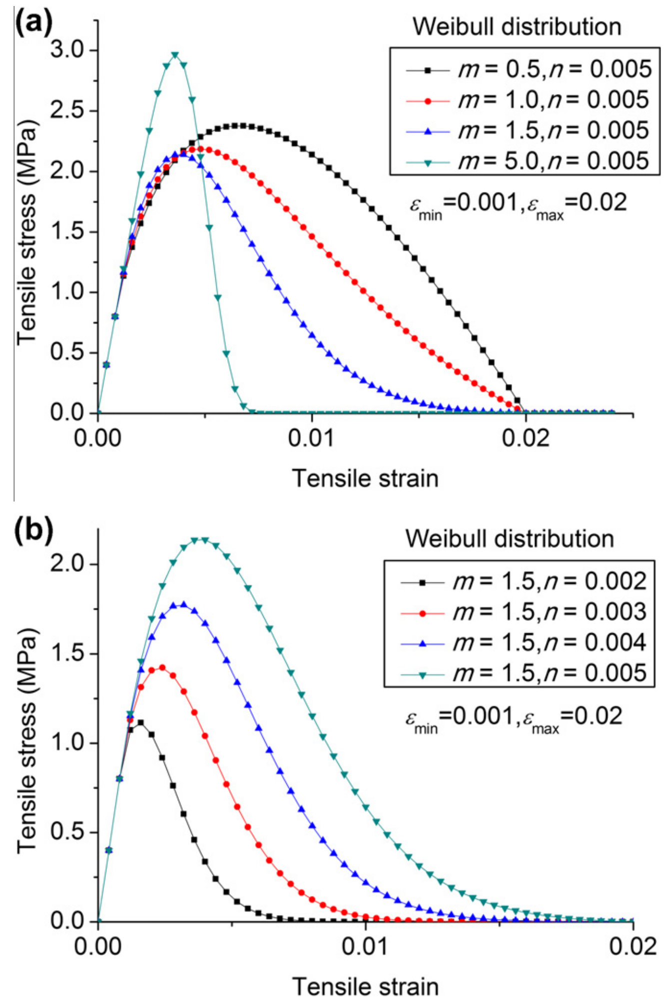

Fig. 3. Failure process of tensile plane under Weibull distribution law of tensile strain strength: (a) Influence of *m*; (b) Influence of *n*.

Tensile stress–strain curve on tensile plane obtained from Eq. (11) is shown in Fig. 3. With progressive failure constitutive model based on strain strength distribution, the stress–strain curve displays well accordance to practical stress strain relation. Nonlinearity and strain softening is naturally obtained with this model. Through variation of m and n, different form of stress–strain curve can be obtained. Shape of stress–strain curve is influenced by shape parameter m, which would determine the form of strain softening. Scale parameter n determines the scale of the curve. Once the experimental tensile curve is given, the upper and lower limits of tensile strain strength can be determined. $\varepsilon_{\max}$ is corresponding to failure strain when tensile stress is zero and $\varepsilon_{\min}$ is corresponding to elastic proportional limit. With variation of m and n, different distribution form would be adopted to Weibull distribution and peak value of tensile stress as well as the strain softening mode could be determined.

4. Progressive shear failure model for shear plane based on distribution of strain strength

Only shear failure is considered at this part. Given distribution law of shear strain strength, the shear plane will fail progressively through growth of shear strain. Elastic microplanes whose shear strain haven't yet reach their shear strain strength will keep linear elasticity. While fractured microplanes whose shear strain have already or have ever exceeded their shear strain strength should comply with the coulomb's law of friction.

4.1. Intact factor and fracture factor on shear fracture plane

The same as the definition of tensile intact factor and fracture factor, intact factor β_I and β_D on shear fracture plane can be written as

$$\beta_I = \begin{cases} 1 & \bar{\gamma} \leqslant \gamma_{\min} \\ F_I(\bar{\gamma}) = \int_{\bar{\gamma}}^{\gamma_{\max}} f(\xi) d\xi & \gamma_{\min} < \bar{\gamma} < \gamma_{\max} \\ 0 & \bar{\gamma} \geqslant \gamma_{\max} \end{cases} \quad (12)$$

$$\beta_D = 1 - \beta_I = \begin{cases} 0 & \bar{\gamma} \leqslant \gamma_{\min} \\ F_D(\bar{\gamma}) = \int_{\gamma_{\min}}^{\bar{\gamma}} f(\xi) d\xi & \gamma_{\min} < \bar{\gamma} < \gamma_{\max} \\ 1 & \bar{\gamma} \geqslant \gamma_{\max} \end{cases} \quad (13)$$

where $\bar{\gamma}$ is maximum shear strain in history. $\gamma_{\min}$ and $\gamma_{\max}$ are the lower and upper limits of shear strain strength. ξ is integral variable. $f(\xi)$ is distribution density function. $F_I(\bar{\gamma})$ and $F_D(\bar{\gamma})$ are and cumulative distribution function of shear strain strength.

Intact factor β_I represents the weight of elastic microplanes remaining on shear plane. Fracture factor β_D is the ratio of shear fractured microplanes to total area, equals to 1 - β_I, which represents fracture degree caused by shear.

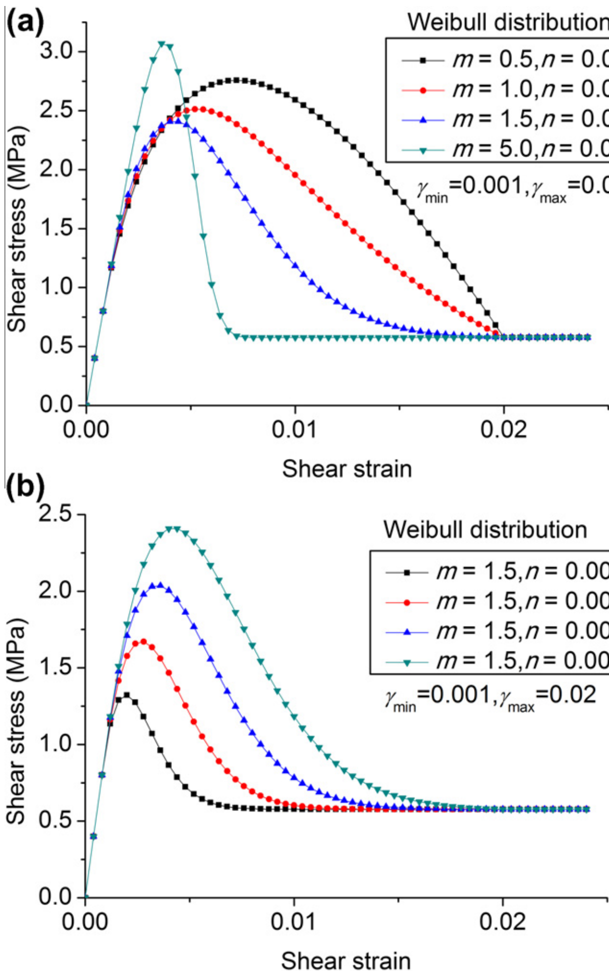

Fig. 4. Failure process of shear plane under Weibull distribution law of shear strain strength: (a) Influence of m; (b) Influence of n.

4.2. Stress–strain relationship on shear plane

Effective shear stress on shear plane could be expressed as the weighted summation of elastic stress on elastic microplanes and friction stress on fractured microplanes with shear intact factor. Stress–strain relationship can be expressed as

$$\tau = \beta_I G\gamma + \begin{cases} (1-\beta_I)G\gamma & G\gamma < |\sigma_n| tg\varphi, \varepsilon < 0 \\ (1-\beta_I)|\sigma_n| tg\varphi & G\gamma \geqslant |\sigma_n| tg\varphi, \varepsilon < 0 \\ 0 & \varepsilon \geqslant 0 \end{cases} \quad (14)$$

where G is shear modulus, φ is internal friction angle, τ is effective shear stress on shear plane, γ is shear strain, ε is nominal normal strain on shear plane with the same meaning as introduced in the previous section, $\sigma_n = E\varepsilon$ is elastic compressive normal stress on shear plane.

The first part of the equation on the right side of equal sign shows the elastic shear stress on elastic microplanes with weight of β_I. The second part expresses the discontinuous contact stress on fractured microplanes with weight of 1 - β_I, which equals to β_D. There are three types of stress on fractured microplanes. If the fractured microplanes are in tension, shear stress on them is zero. If the fractured microplanes are in compression and elastic shear stress is below the maximum static friction stress, friction stress is equal to elastic shear stress. Otherwise, if the fractured microplanes are in compression and elastic shear stress is above the maximum static friction stress, the friction stress is equal to the maximum static friction stress.

Shear stress–strain curve got form Eq. (14) on shear plane is shown in Fig. 4. $\gamma_{\max}$ and $\gamma_{\min}$ can be determined corresponding to yield limit and proportional limit respectively on experimental curve of direct shear. Through modification of m and n, peak stress and strain soften mode could also be determined. When exceeding the proportional limit, which means all area on the shear plane is fractured, only friction remained on this plane. So shear stress keeps a constant value that equals to the friction stress at last.

5. Joint failure model based on strain strength distribution

Under many circumstances in reality, fracture and damage are the combined result of tension and shear. Especially on tension–shear plane, mutual influence of tension and shear should be considered.

5.1. Joint intact factor and joint fracture factor

Assume that distribution law of tensile strain strength and shear strain strength are independent, and then joint intact factor can be expressed as $\alpha_I\beta_I$, which represents the proportion of elastic microplanes remaining when tension and shear act simultaneously on a plane. Joint fracture factor equals to 1 - $\alpha_I\beta_I$, representing the ratio of fractured microplanes on the plane.

Fig. 5 shows the contour map of joint intact factor in tension and shear. With combined effect of tension and shear, value of

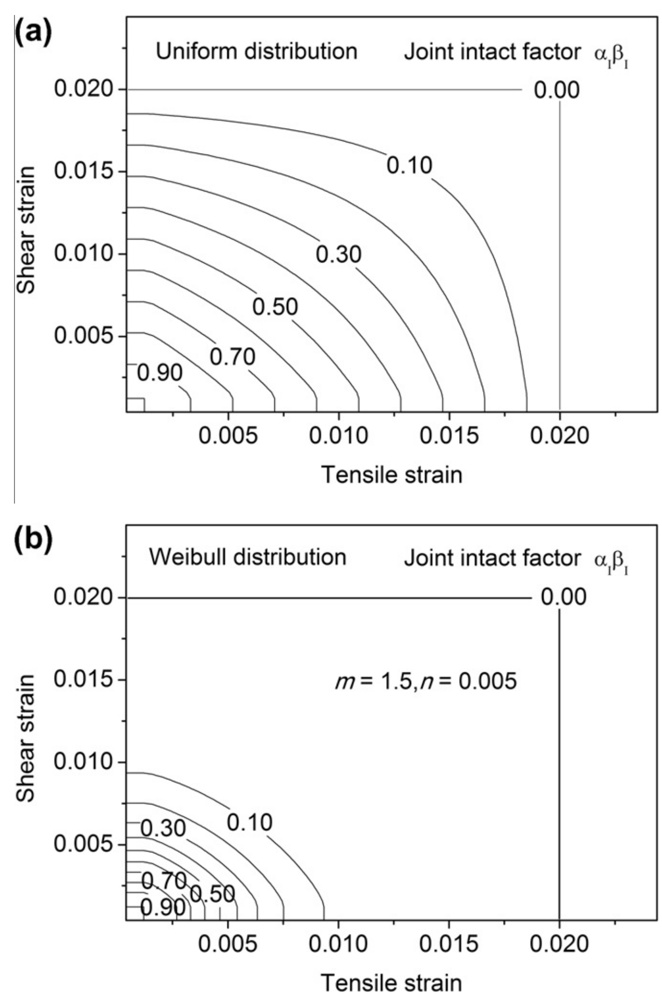

Fig. 5. Joint intact factor in different distribution law with $\varepsilon_{\min}$ = 0.001, $\varepsilon_{\max}$ = 0.02, $\gamma_{\min}$ = 0.001, $\gamma_{\max}$ = 0.02: (a) Uniform distribution; (b) Weibull distribution.

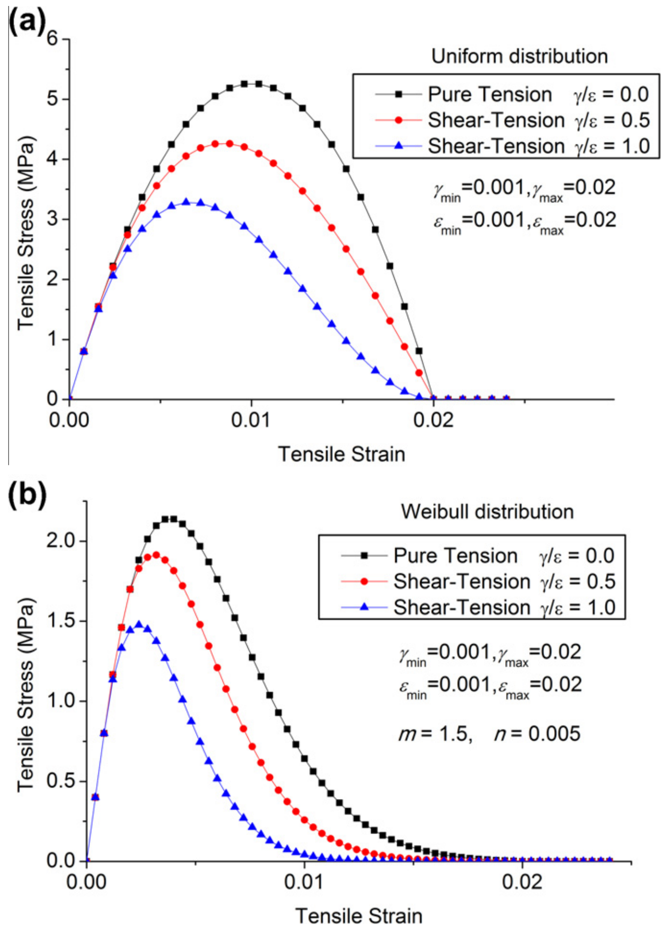

Fig. 6. Stress–strain curve on tension–shear plane in different distribution law of strain strength, $\varepsilon_{min} = 0.001$, $\varepsilon_{max} = 0.02$, $\gamma_{min} = 0.001$, $\gamma_{max} = 0.02$: (a) Uniform distribution; (b) Weibull distribution.

intact factor becomes smaller under the same value of strain compared to the previous single effect cases. That means material is more likely to fail under tension–shear load.

5.2. Stress–strain relationship on any plane in representative volume element

With joint intact factor and joint fracture factor defined above, given any plane in the representative volume element, stress–strain relationship can be written as

$$\sigma_n = \alpha_I\beta_I E\varepsilon + \begin{cases} (1-\alpha_I\beta_I)E\varepsilon & \varepsilon < 0 \\ 0 & \varepsilon \geqslant 0 \end{cases}$$
$$\tau = \alpha_I\beta_I G\gamma + \begin{cases} (1-\alpha_I\beta_I)G\gamma & G\gamma < |\sigma_n|tg\varphi, \varepsilon < 0 \\ (1-\alpha_I\beta_I)|\sigma_n|tg\varphi & G\gamma \geqslant |\sigma_n|tg\varphi, \varepsilon < 0 \\ 0 & \varepsilon \geqslant 0 \end{cases} \tag{15}$$

where σ_n is normal stress, τ is shear stress, E is young's modulus, G is shear modulus, φ is internal friction angle, γ is shear strain, ε is nominal normal strain on shear plane with the same meaning as introduced in the previous section.

It is a generalized failure model on fracture plane which has considered the mutual influence of tension and shear, and it will naturally reduce to one of the previous two models under certain circumstances.

Stress–strain curves on tension–shear plane are calculated with uniform distribution and Weibull distribution according to Eq. (15). In this case, shear and tensile strain grow simultaneously with a special ratio, stress–strain relationship is shown in Fig. 6. Comparing to failure behavior of pure tension, material appears to be more vulnerable and much easier to fail under combined effect of tension and shear in both uniform distribution (Fig. 6(a)) and Weibull distribution (Fig. 6(b)).

6. Degenerated form of the constitutive model

When upper and lower limit of strain strength is equal, which means strain strength in the material is identical, it degenerates to classical elastic–brittle model and ideal elastoplastic model naturally. The degenerated stress–strain relationship on shear plane is shown in Fig. 7. Under this condition, if maximum elastic shear stress is larger than the maximum static friction stress, brittle failure phenomenon takes place. Otherwise, if maximum elastic shear stress is less than the maximum static friction stress, ideal elastoplastic behavior takes place.

7. Applications and verification

Progressive failure constitutive model of fracture plane can make full use of the experimental data. Strain strength parameters and distributive law of fracture plane can be obtained through tensile test and direct shear test. For tensile strength, ε_{max} is corresponding to failure strain when tensile stress is zero and ε_{min} is corresponding to elastic proportional limit on tensile test curve.

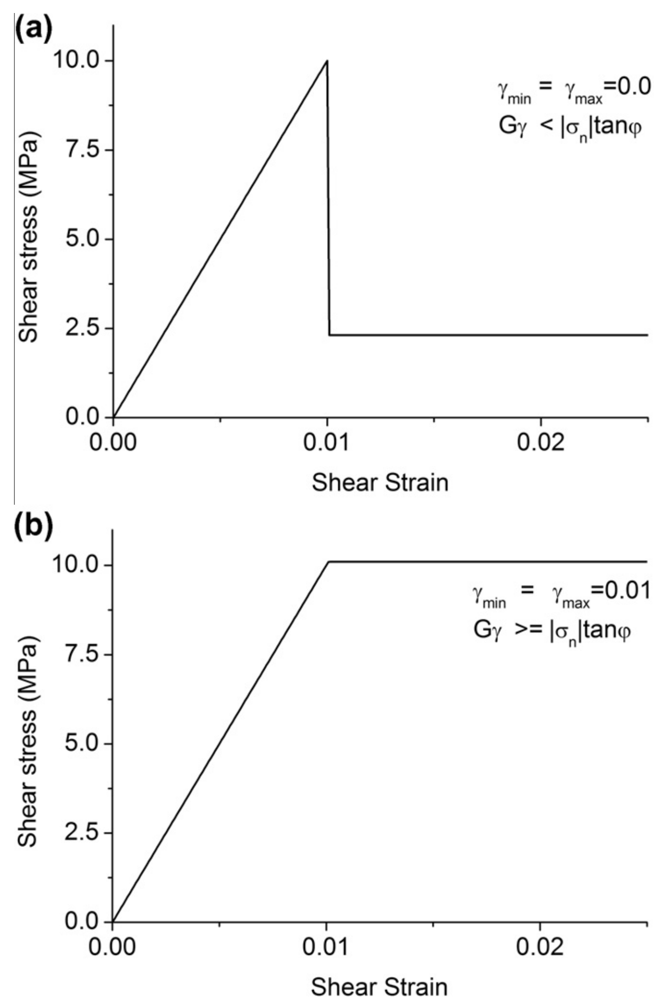

Fig. 7. Failure process of shear plane in degenerated form when $\gamma_{min} = \gamma_{max}$: (a) Elastic–brittle failure; (b) Ideal elastoplastic failure.

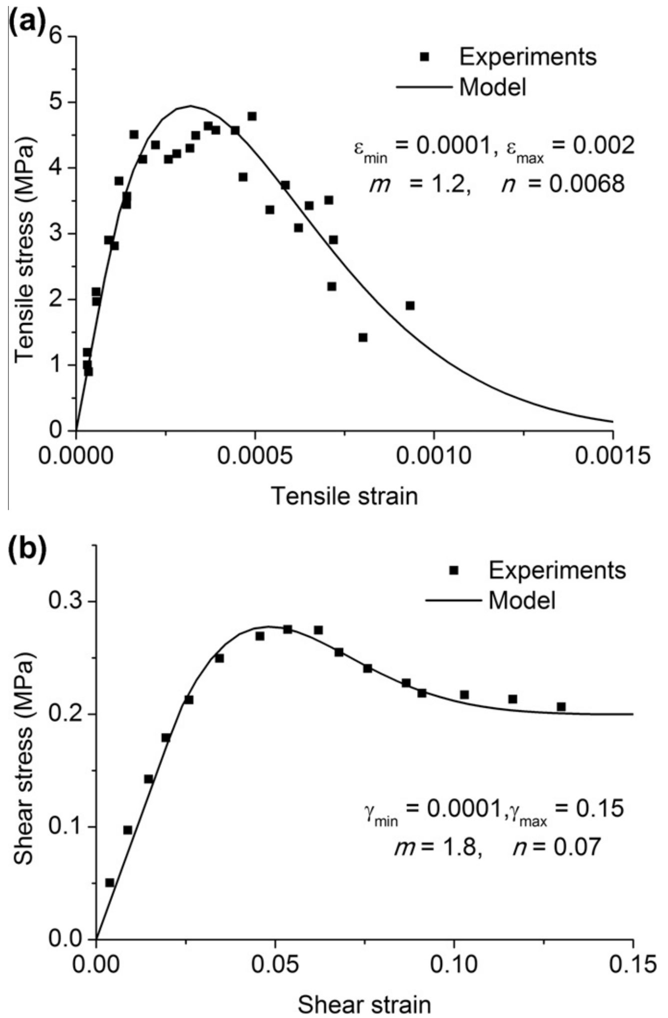

Fig. 8. Simulation of tension and shear test with Weibull distribution of strain strength: (a) Tension PIED uniaxial test; (b) Direct shear test.

For shear strength, $\gamma_{\max}$ and $\gamma_{\min}$ can be determined corresponding to break limit and elastic proportional limit respectively on experimental curve of direct shear, and internal friction angle can be calculated through friction stress after shear failure. Distributive law can be determined by the shape of the experimental curve.

Given experimental test data of tension or shear, strain strength and distribution law should be obtained in accordance with the above method. Different materials have different distribution characteristics of strain strength. As examples, assuming that strain strength complies with Weibull distribution law, distribution of strain strength is controlled by shape parameter m, scale parameter n and strength interval. Since strength interval can be directly determined corresponding to elastic proportional limit and break limit, m and n should be adjusted to make sure that stress–strain behavior calculated by the theoretical model is in good agreement with the experimental data. Simulations of tension PIED uniaxial test of Bažant and Pijaudier-Cabot (1989) and direct shear test of Bagherzadeh-Khalkhali and Mirghasemi (2009) are shown in Fig. 8.

8. Conclusions

Progressive failure constitutive model of fracture plane in geomaterial based on strain strength distribution is introduced in this paper. Strain is used as measure of strength and strain strength is assumed to be distributed in material. Tensile fracture, shear fracture and joint fracture in tension and shear are discussed. Intact factor and fracture factor are proposed to describe the fracture state on fracture plane of representative volume element in geomaterial. With definition of elastic microplanes and fractured microplanes, more detailed description of mesoscopic mechanics behavior in geomaterial is conducted. Interactions in elastic microplanes are linear elastic, while on fractured microplanes are contact and friction.

Stress–strain relationship in tension, shear and combined effect show that this model is applicative to describe nonlinearity and strain softening of geomaterial. Through variation of distribution parameters or distribution law of strain strength, one can get different types of stress–strain relationships corresponding to different strain softening forms. Once the tensile and shear experimental data is given, distribution characteristics of strain strength can be obtained. Failure under combined effect of tension and shear is discussed and result shows that material is more vulnerable and much easier to fail under combined effect of tension and shear. Progressive failure constitutive model can reduce to elastic–brittle fracture model and ideal elastic–plastic model when upper and lower limit of strain strength is equal.

The model proposed here concentrates on the behavior of progressive failure for fracture plane in geomaterial. Complicated macroscopic mechanical behavior of fracture plane in geomaterial is naturally obtained through mesoscopic fracture and mechanism of interactions on mesoscopic discontinuity. Fracture and friction are quantitatively determined in statistic way and with the definition of fractured microplanes. It provides a new way to understand and describe nature of progressive failure mode and process of geomaterial in detail. Thus future work may extend this model to other materials and to include, for example, scale dependent model considering relationship of scale between microplanes and macroscopic fracture plane in representative volume element, or energy dissipation model considering dissipation of fracture energy and frictional energy. Since fracture is directional, relationship between stress on fracture plane and homogeneous stress tensor of representative volume element during the progressive failure process also deserves further study.

Acknowledgments

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