

大挠度简支矩形薄板受热磁耦合作用分岔与混沌

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摘要: 研究大挠度四边不可移简支薄板在机械载荷、电磁场与温度场耦合作用下的混沌运动。在板壳与磁弹性力学理论上,考虑温度场影响,推导出在横向稳恒磁场与机械载荷共同作用下薄板的非线性磁弹性耦合振动方程。利用 Melnikov 函数法,求出该动力系统 Smale 马蹄变换意义下出现混沌运动条件,并对该系统振动方程进行数值模拟。通过算例得到系统分岔图、位移波形图、相平面轨迹及庞加莱截面图。讨论机械载荷、磁场及温度场参数对系统混沌运动影响。仿真结果表明,通过变化机械载荷、磁场及温度场参数,可控制系统的振动特性。

关键词: 大挠度; 热磁弹性; 耦合场; Melnikov 函数法; 分岔; 混沌

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Bifurcation and chaos of a thin rectangular plate simply supported with large deflection in a coupled environment of heating, force and magnetic field

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Abstract: Here, the chaotic motion of a thin rectangular plate simply supported with large deflection and four unmovable sides in a coupled environment of mechanical load, magnetic field and temperature field was investigated. Based on the theories of plates, shells and magnetic elasticity and considering the effect of temperature field, the nonlinear magnetic-elastic coupled vibration equations of the rectangular plate in the coupled environment of a transverse steady magnetic field and mechanical load were derived. Using Melnikov function method, the chaotic motion condition of the dynamic system under the meaning of Smale horseshoe transformation was obtained. The numerical simulations were performed with the vibration equations of the system. Through a specific example, the bifurcation diagram, the displacement wave diagram, the phase diagram and Poincare section diagram of this system were shown here. The influences of parameter variations including mechanical load, magnetic field and temperature field on the chaotic motion of this system were discussed. The simulation results showed that the vibration characters of this system can be controlled by changing parameters of mechanical load, magnetic field and temperature field.

Key words: large deflection; thermal-magneto elasticity; coupling field; melnikov function method; bifurcation; chaos

弹性物体中耦合场理论的创立与发展,对处在高温、高压及强电磁场作用下结构元件强度和可靠性分析具有重要意义^[1]。随着电磁结构在高新技术领域中的应用日益增多,针对工程中许多构件工作在温度变化环境中,对电磁耦合热弹性现象的研究有非常强的工程背景与理论价值^[2-3]。热弹耦合振动以热弹耦合与振动理论为基础发展而来的新兴研究方向。Trajkor-

ski 等^[4]分析了给定边界条件下圆板热弹耦合振动特性。戴宏亮等^[5]研究了厚壁圆筒在热、磁耦合作用下的动态响应。树学锋等^[6-7]建立圆板横向热弹耦合振动方程,并对常厚度及变厚度圆板非线性热弹性耦合振动进行分析。Yeh^[8]研究大变形简支正交异性矩形薄板的热弹耦合振动。

混沌动力学的不断发展与完善极大丰富了非线性科学的研究领域,为许多古老研究课题注入新的活力,为力学学科发展提供新的动力。目前,各种形式构件热弹性耦合振动的混沌运动研究,也取得一些成果。韩强等^[9]考虑几何非线性效应与温度效应影响,建立

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椭圆板周期激励作用下热弹耦合的非线性动力方程, 利用 Melnikov 函数法给出系统发生混沌运动的临界条件。吴晓等^[10]研究了矩形板非线性热振动分岔。Yeh 等^[11]对热弹耦合矩形板分岔与混沌进行研究。李映辉等^[12]研究了变温环境下粘弹性梁的混沌运动。

综上所述, 由目前研究成果看, 对非铁磁类材料大挠度矩形薄板在热、力、磁耦合共同作用下热磁性耦合振动的混沌运动研究尚少。因此本文针对大挠度矩形薄板, 考虑温度场影响, 研究在横向稳恒磁场与机械载荷共同作用下的非线性热磁弹性耦合振动问题。结果表明, 薄板在多场耦合作用下, 具有丰富的非线性动力行为。通过改变机械载荷、电磁场及温度场参数, 可控制系统的振动特性。

1 运动方程

将矩形薄板置于横向稳恒磁场 $B(0, 0, B_z)$ 中, 同时受分布载荷 $P(0, 0, q)$ 作用。在笛卡儿直角坐标系 $oxyz$ 中, ρ_{xy} 为薄板中面 z 为法向坐标, 不考虑极化、磁化影响, 将洛伦兹力^[2] $\rho f_x, \rho f_y, \rho f_z$ 作为质动力载荷加入板运动方程, 得薄板热、力、磁耦合运动方程为:

$$\left. \begin{aligned} \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} + \rho f_x &= \rho h \frac{\partial^2 u}{\partial t^2} \\ \frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} + \rho f_y &= \rho h \frac{\partial^2 v}{\partial t^2} \\ \frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + q + \rho f_z &= \rho h \frac{\partial^2 w}{\partial t^2} \\ \frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - (N_x \theta_x + N_{xy} \theta_y) &- \\ Q_x + m_x &= \frac{\rho h^3}{12} \frac{\partial^2 \theta_x}{\partial t^2} \\ \frac{\partial M_y}{\partial y} + \frac{\partial M_{xy}}{\partial x} - (N_y \theta_y + N_{xy} \theta_x) &- \\ Q_y + m_y &= \frac{\rho h^3}{12} \frac{\partial^2 \theta_y}{\partial t^2} \end{aligned} \right\} \quad (1)$$

式中: h 为板厚; q 为法向机械载荷分量; $N_x, N_{xy}, N_y; Q_x, Q_y; M_x, M_{xy}, M_y$ 为内力分量; θ_x, θ_y 为角位移分量; ρ 为质量密度; $\rho f_x, \rho f_y, \rho f_z$ 分别为对应 x, y, z 的洛伦兹力分量; m_x, m_y 为洛伦兹力矩。

洛伦兹力与洛伦兹力矩为^[2]:

$$\left. \begin{aligned} \rho f_x &= \sigma h B_z \left(e_y - \frac{\partial u}{\partial t} B_z \right) \\ \rho f_y &= \sigma h B_z \left(-e_x - \frac{\partial v}{\partial t} B_z \right) \\ \rho f_z &= 0 \\ m_x &= \frac{\sigma h^3}{12} B_z^2 \frac{\partial^2 w}{\partial t \partial x} \\ m_y &= \frac{\sigma h^3}{12} B_z^2 \frac{\partial^2 w}{\partial t \partial y} \end{aligned} \right\} \quad (2)$$

式中: u, v, w 分别为中面内点 x, y, z 向位移; t 为时间; σ 为电导率; e_x, e_y 为磁感应电场强度分量。

矩形板在热状态下的内力方程为^[13]:

$$\left. \begin{aligned} M_x &= -D \left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} \right) - M_T \\ M_y &= -D \left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2} \right) - M_T \\ M_{xy} &= -D(1-\nu) \frac{\partial^2 w}{\partial x \partial y} \end{aligned} \right\} \quad (3)$$

式中: $D = \frac{Eh^3}{12(1-\nu^2)}$ 为抗弯刚度; E 为弹性模量; w 为中面内点 z 向位移; ν 为泊松比; M_T 为热矩。

中面内力及挠度表示的相容方程为^[14]:

$$\begin{aligned} \frac{\partial^2 N_x}{\partial y^2} + \frac{\partial^2 N_y}{\partial x^2} - \nu \left(\frac{\partial^2 N_x}{\partial x^2} + \frac{\partial^2 N_y}{\partial y^2} \right) - \\ 2(1+\nu) \frac{\partial^2 N_{xy}}{\partial x \partial y} + \nabla^2 N_T = \\ Eh \left[\left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 - \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} \right] \end{aligned} \quad (4)$$

式中: N_T 为热力。热力、热矩的表达式为^[14]:

$$(N_T, M_T) = \frac{E\alpha_0}{1-\nu} \int_{-h/2}^{h/2} (1, z) T(x, y, z) dz \quad (5)$$

式中: α_0 为热膨胀系数。

引入应力函数 $\varphi(x, y)$:

$$[N_x, N_y, N_{xy}] = \left[h \frac{\partial^2 \varphi}{\partial y^2}, h \frac{\partial^2 \varphi}{\partial x^2}, -h \frac{\partial^2 \varphi}{\partial x \partial y} \right] \quad (6)$$

令机械载荷 $q = q_0 \cos(\omega t)$, 其中 q_0 为机械力幅值, ω 为激振频率。式(1)后三个式子中去掉惯性力项, 由式(1)~式(4)及式(6)可得矩形板在热状态下非线性控制方程为:

$$\left. \begin{aligned} D \nabla^4 w + \nabla^2 M_T - \frac{\sigma h^3}{12} B_z^2 \frac{\partial(\nabla^2 w)}{\partial t} + \\ \rho h \frac{\partial^2 w}{\partial t^2} - hL(\varphi, w) - q_0 \cos(\omega t) &= 0 \\ h \nabla^4 \varphi + \nabla^2 N_T &= -\frac{Eh}{2} L(w, w) \end{aligned} \right\} \quad (7)$$

式中: $L(w, w) = 2 \left[\frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 \right]$,

$$\begin{aligned} L(\varphi, w) &= \frac{\partial^2 \varphi}{\partial x^2} \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 \varphi}{\partial y^2} \frac{\partial^2 w}{\partial x^2} - 2 \frac{\partial^2 \varphi}{\partial x \partial y} \frac{\partial^2 w}{\partial x \partial y}, \\ \nabla^4 &= \frac{\partial^4}{\partial x^4} + 2 \frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4}, \quad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}. \end{aligned}$$

矩形薄板四边为不可移简支, a, b 为板边长, 边界条件为:

$$\left. \begin{aligned} x=0, a; w = \frac{\partial^2 w}{\partial x^2} = 0, N_{xy} = 0, \Delta_x = 0 \\ y=0, b; w = \frac{\partial^2 w}{\partial y^2} = 0, N_{xy} = 0, \Delta_y = 0 \end{aligned} \right\} \quad (8)$$

板端部表达式为:

$$\left. \begin{aligned} \Delta_x &= \frac{1}{b} \int_0^a \int_0^b \left[\frac{1}{Eh} \left(\frac{\partial^2 \varphi}{\partial y^2} - \nu \frac{\partial^2 \varphi}{\partial x^2} \right) - \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 + \frac{1-\nu}{Eh} N_T \right] dx dy \\ \Delta_y &= \frac{1}{a} \int_0^a \int_0^b \left[\frac{1}{Eh} \left(\frac{\partial^2 \varphi}{\partial x^2} - \nu \frac{\partial^2 \varphi}{\partial y^2} \right) - \frac{1}{2} \left(\frac{\partial w}{\partial y} \right)^2 + \frac{1-\nu}{Eh} N_T \right] dx dy \end{aligned} \right\} \quad (9)$$

设温度函数为:

$$T(x, y, z) = T_0(1 + z + z^2) \quad (10)$$

式中: T_0 为常数。由于工程实际中对最低阶振型感兴趣, 故设:

$$w(x, y, t) = \Theta(t) \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} \quad (11)$$

由式(5)、式(7)及式(11)得:

$$\nabla^4 \varphi = \frac{E\pi^4}{2a^2b^2} \Theta^2(t) \left(\cos \frac{2\pi x}{a} + \cos \frac{2\pi y}{b} \right)$$

利用待定系数法^[16], 设应力函数为:

$$\varphi = A \cos \frac{2\pi x}{a} + B \cos \frac{2\pi y}{b} + \frac{1}{2} P_x y^2 + \frac{1}{2} P_y x^2$$

将其代入上式, 两边比较系数, 得:

$$A = \frac{Ea^2}{32b^2} \Theta^2(t), B = \frac{Eb^2}{32a^2} \Theta^2(t)$$

则应力函数为:

$$\varphi = \frac{E}{32} \Theta^2(t) \left(\frac{a^2}{b^2} \cos \frac{2\pi x}{a} + \frac{b^2}{a^2} \cos \frac{2\pi y}{b} \right) + \frac{1}{2} P_x y^2 + \frac{1}{2} P_y x^2 \quad (12)$$

式中: P_x, P_y 由式(9)求得(令: $\Delta_x = \Delta_y = 0$)

$$\left. \begin{aligned} P_x &= \frac{Eh\pi^2(b^2 + \nu a^2)}{8a^2b^2(1-\nu^2)} \Theta^2 - \frac{Eh\alpha_0 T_0}{1-\nu} \left(1 + \frac{h^2}{12} \right) \\ P_y &= \frac{Eh\pi^2(a^2 + \nu b^2)}{8a^2b^2(1-\nu^2)} \Theta^2 - \frac{Eh\alpha_0 T_0}{1-\nu} \left(1 + \frac{h^2}{12} \right) \end{aligned} \right\} \quad (13)$$

对式(7)中第一式利用 Galerkin 原理有:

$$\int_0^a \int_0^b \left[D \nabla^4 w + \nabla^2 M_T - \frac{\sigma h^3 B_z^2}{12} \frac{\partial(\nabla^2 w)}{\partial t} + \rho h \frac{\partial^2 w}{\partial t^2} - hL(\varphi, w) - q_0 \cos(\omega t) \right] \cdot \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} dx dy = 0$$

将式(5)、(6)、(11)、(13)代入上式, 计算得:

$$\ddot{\Theta}(t) + \gamma \dot{\Theta}(t) + \alpha \Theta(t) + \beta \Theta^3(t) - F \cos(\omega t) = 0 \quad (14)$$

式中:

$$\alpha = \frac{E\alpha_0 h \lambda_1}{\rho(1-\nu)} \left[\frac{h \lambda_1}{12(1+\nu)\alpha_0} - T_0 \left(1 + \frac{h^2}{12} \right) \right]$$

$$\beta = \frac{E\pi^4}{16\rho} \left[\lambda_2 + \frac{2h(a^4 + b^4 + 2\nu a^2 b^2)}{a^4 b^4 (1-\nu^2)} \right]$$

$$F = \frac{16q_0}{\rho h \pi^2}, \gamma = \frac{\sigma h^2 B_z^2}{12\rho} \lambda_1$$

$$\lambda_1 = \frac{\pi^2}{a^2} + \frac{\pi^2}{b^2}, \lambda_2 = \frac{1}{a^4} + \frac{1}{b^4}$$

由 β 表达式看出 $\beta > 0$, 令 $\eta = |\alpha|$, 则式(14)变为:

$$\ddot{\Theta}(t) + \gamma \dot{\Theta}(t) + \frac{\alpha}{|\alpha|} \eta \Theta(t) + \beta \Theta^3(t) - F \cos(\omega t) = 0 \quad (15)$$

采用无量纲化, 令: $\Theta = \sqrt{\frac{\eta}{\beta}} x, t = \frac{\tau}{\sqrt{\eta}}, \omega = \sqrt{\eta} \omega_0$,

$\gamma = \sqrt{\eta} \mu, F = \frac{\eta \sqrt{\eta}}{\sqrt{\beta}} f$, 最终将式(15)简化为:

$$\ddot{x} + \mu \dot{x} + \frac{\alpha}{|\alpha|} x + x^3 = f \cos(\omega_0 \tau) \quad (16)$$

令 $\dot{x} = y, f = \varepsilon f_1, \mu = \varepsilon \delta_1$, 则式(16)的等价系统为:

$$\left. \begin{aligned} \dot{x} &= y \\ \dot{y} &= -\frac{\alpha}{|\alpha|} x - x^3 + \varepsilon [-\delta_1 y + f_1 \cos(\omega_0 \tau)] \end{aligned} \right\} \quad (17)$$

当 $\varepsilon = 0$ 时, 式(17)为未扰动的 Hamilton 系统, 即:

$$\left. \begin{aligned} \dot{x} &= y \\ \dot{y} &= -\frac{\alpha}{|\alpha|} x - x^3 \end{aligned} \right\} \quad (18)$$

讨论 $\alpha < 0$ 情况, 即式(16)为:

$$\ddot{x} + \mu \dot{x} - x + x^3 = f \cos(\omega_0 \tau)$$

当 $\alpha < 0$ 时, 未扰动的 Hamilton 系统有三个奇点, 两个鞍点($\pm 1, 0$) 和一个中心点($0, 0$)。

由 Melnikov 理论^[15], 当条件: $\left| f_1 \omega_0 \operatorname{sech} \left(\frac{\pi \omega_0}{2} \right) \right| >$

$$\frac{4|\delta_1|}{3\sqrt{2}\pi} \text{ 即 } \left| \frac{f}{\mu} \right| = \left| \frac{f_1}{\delta_1} \right| > \frac{4}{3\sqrt{2}\pi \omega_0 \left| \operatorname{sech} \left(\frac{\pi \omega_0}{2} \right) \right|} \text{ 成立时,}$$

Melnikov 函数 $M(t_0)$ 有简单零点。说明在上面条件下, 扰动系统具有 Smale 马蹄变换意义下的混沌。

2 数值解及算例分析

设薄板为铝质材料。板厚 $h = 2 \times 10^{-2}$ m, 密度 $\rho = 2.67 \times 10^3$ kg/m³, 弹性模量 $E = 71$ GPa, 泊松比 $\nu = 0.34$, 电导率 $\sigma = 3.6 \times 10^7$ (Ωm)⁻¹, 热膨胀系数 $\alpha_0 = 23 \times 10^{-6}$ ($^\circ\text{C}$)⁻¹。为便于计算, 且 $\alpha < 0$, 取 $a = 3\pi$ m, $b = 2\pi$ m, $\omega_0 = 1$ rad/s。将其代入方程, 用四阶 R-K 方法编程求解。计算中采用积分容忍误差为 10^{-6} 。为模拟矩形薄板的混沌运动, 采样周期为 120, 在积分所得时间响应中反复 4 000 次后开始取资料, 以去掉足够的瞬态, 保证系统动态响应已达到稳定状态。资料包括

位移、加速度、时间等,用于产生分岔图、位移波形图、相平面轨迹图、及庞加莱截面图。

2.1 机械载荷影响

当 $\omega_0 = 1 \text{ rad/s}$, $B_z = 1 \text{ T}$, $T_0 = 20^\circ\text{C}$ 时,通过改变机械载荷幅值 q_0 大小,使 f 在 $0.01 \sim 10 \text{ N/m}^2$ 之间变化,得系统随 f 变化时的分岔图(图1)。从图中看出,以机械载荷 f 为分岔参数,随着机械载荷幅值 q_0 的不断增大,系统的混沌与周期区交替出现。在 $f = 0.2 \text{ N/m}^2$ 附近,系统由周期1运动经由阵发性进入瞬间混沌运动,转而又开始周期1运动,当 f 在 $1 \sim 1.6 \text{ N/m}^2$ 之间系统有大面积混沌区,期间也存在一些零散的周期窗口, $f = 1.6 \text{ N/m}^2$ 时系统离开混沌运动状态进入周期3运动,在 $f = 1.8 \text{ N/m}^2$ 附近转而又开始周期6运动, f 在 $2 \sim 2.3 \text{ N/m}^2$ 之间系统处于周期2运动状态,之后进入混沌区,在 $f = 3.1 \text{ N/m}^2$ 附近系统离开混沌进入周期1运动,后经由倍周期分岔,在 $f = 4.8 \text{ N/m}^2$ 附近进入混沌运动。后面空白处为周期窗口,之后再进入混沌运动状态,如此反复。图2~图3给出在不同 f 值下系统位移波形图、相平面轨迹图及庞加莱截面图。当 $f = 0.5 \text{ N/m}^2$ 时,系统处于单周期运动状态; $f = 1.6 \text{ N/m}^2$ 时,系统处于3周期运动状态; $f = 4.5 \text{ N/m}^2$ 时,系统处于2周期运动状态; $f = 6.5 \text{ N/m}^2$ 时,系统处于混沌运动状态。

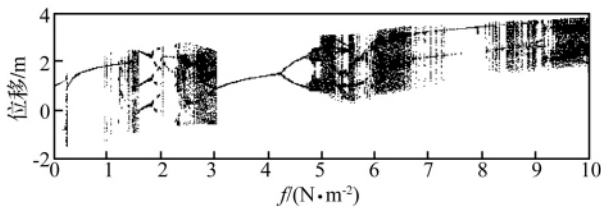


图1 $\omega_0 = 1 \text{ rad/s}$, $B_z = 1 \text{ T}$, $T_0 = 20^\circ\text{C}$ 时系统随 f 变化时的分岔图

Fig.1 Bifurcation diagram when f is variable ($\omega_0 = 1 \text{ rad/s}$, $B_z = 1 \text{ T}$, $T_0 = 20^\circ\text{C}$ 时)

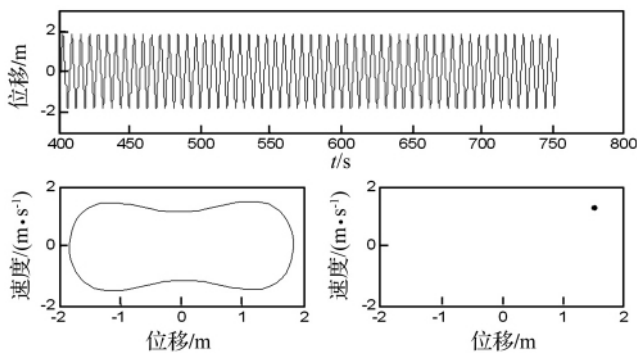


图2 $f = 0.5 \text{ N/m}^2$ 时的位移波形图,相平面轨迹图及庞加莱截面图

Fig.2 Wave diagram of displacement, Phase diagram and Poincare map ($f = 0.5 \text{ N/m}^2$)

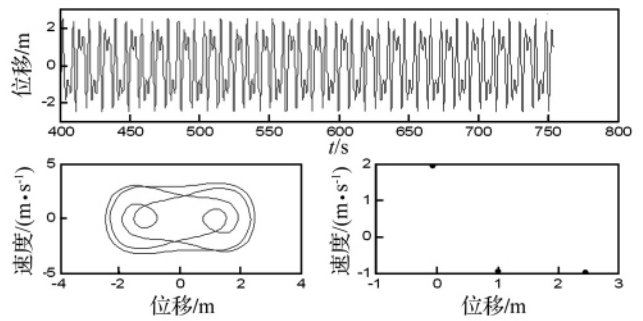


图3 $f = 1.6 \text{ N/m}^2$ 时的位移波形图,相平面轨迹图及庞加莱截面图

Fig.3 Wave diagram of displacement, Phase diagram and Poincare map ($f = 1.6 \text{ N/m}^2$)

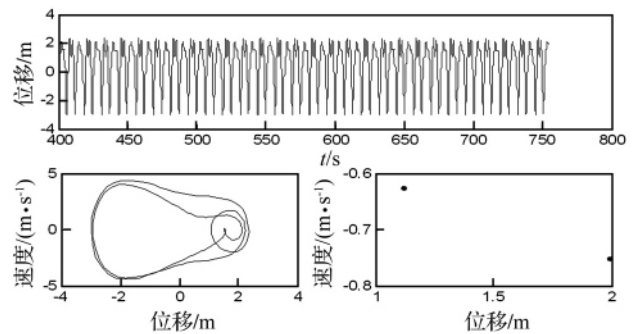


图4 $f = 4.5 \text{ N/m}^2$ 时的位移波形图,相平面轨迹图及庞加莱截面图

Fig.4 Wave diagram of displacement, Phase diagram and Poincare map ($f = 4.5 \text{ N/m}^2$)

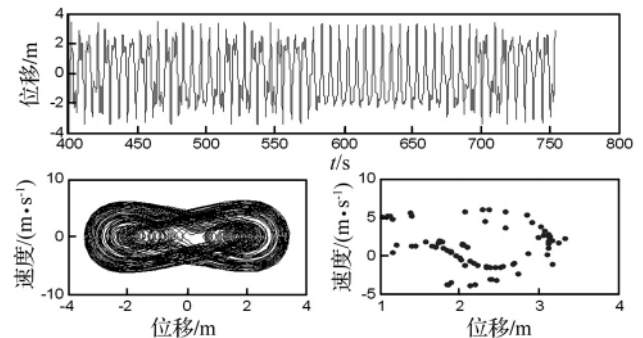


图5 $f = 6.5 \text{ N/m}^2$ 时的位移波形图,相平面轨迹图及庞加莱截面图

Fig.5 Wave diagram of displacement, Phase diagram and Poincare map ($f = 6.5 \text{ N/m}^2$)

2.2 电磁场影响

当 $\omega_0 = 1 \text{ rad/s}$, $T_0 = 20^\circ\text{C}$, $f = 1.5 \text{ N/m}^2$ ($q_0 = 0.7669 \text{ N/m}^2$) 时,通过改变磁感应强度大小, B_z 取 $0.01 \sim 2 \text{ T}$,得系统分岔图如图6所示。由图中看出,随着磁感应强度 B_z 的不断增大,系统运动状态有较明显变化。当 B_z 小于 0.45 T ,系统一直处于混沌运动状态;在 $B_z = 0.45 \text{ T}$ 附近进入3周期运动状态;在 $B_z = 0.78 \text{ T}$ 附近离开3周期运动状态进入混沌运动状态;在 $B_z = 0.78 \sim 1.28 \text{ T}$ 范围内,系统基本处于混沌运动状态;在

$B_z = 1.2T$ 附近空白处有微小周期窗口; 在 $B_z = 1.28 \sim 2T$ 范围内系统基本处于单周期运动状态; 只是在 $B_z = 1.45T$ 左右有短暂的 3 周期运动。图 7 ~ 图 9 为不同 B_z 值下系统位移波形图、相平面轨迹图及庞伽莱截面图。 $B_z = 0.2T$ 时, 系统处于混沌运动状态; $B_z = 0.6T$ 时, 系统处于 3 周期运动状态; $B_z = 1.6T$ 时, 系统处于单周期运动状态。

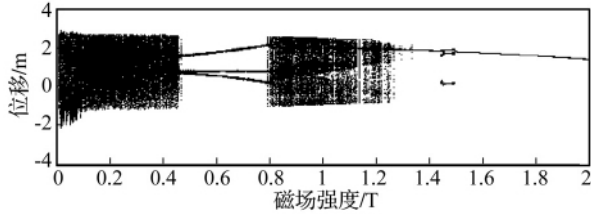


图 6 $\omega_0 = 1 \text{ rad/s}$, $T_0 = 20^\circ\text{C}$, $f = 1.5 \text{ N/m}^2$ 时系统随 B_z 变化时的分岔图

Fig. 6 Bifurcation diagram when B_z is variable ($\omega_0 = 1 \text{ rad/s}$, $T_0 = 20^\circ\text{C}$, $f = 1.5 \text{ N/m}^2$)

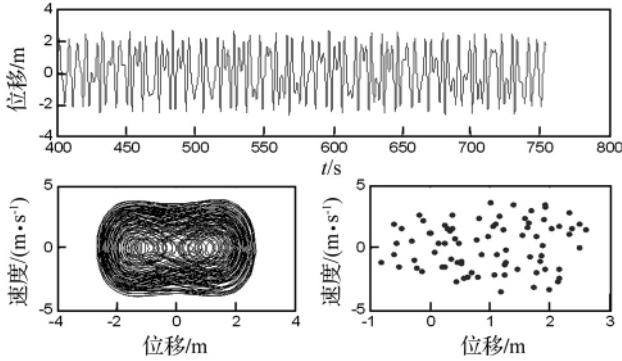


图 7 $B_z = 0.2T$ 时的位移波形图, 相平面轨迹图及庞伽莱截面图

Fig. 7 Wave diagram of displacement Phase diagram and Poincare map ($B_z = 0.2 T$)

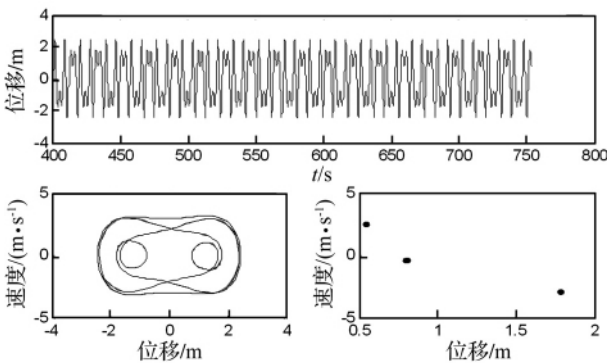


图 8 $B_z = 0.6T$ 时的位移波形图, 相平面轨迹图及庞伽莱截面图

Fig. 8 Wave diagram of displacement Phase diagram and Poincare map ($B_z = 0.6 T$)

2.3 温度影响

当 $\omega_0 = 1 \text{ rad/s}$, $B_z = 1T$, $q_0 = 70 \text{ N/m}^2$ 时, 通过改变温度 T_0 的大小 ($20 \sim 40^\circ\text{C}$), 得系统分岔图如图 10 所示。由图中看出, 系统在 $T = 22^\circ\text{C}$ 和 $T = 25^\circ\text{C}$ 附近有短暂单周期区; 在 $T = 27^\circ\text{C}$ 和 $T = 29^\circ\text{C}$ 附近及 $T = 30$

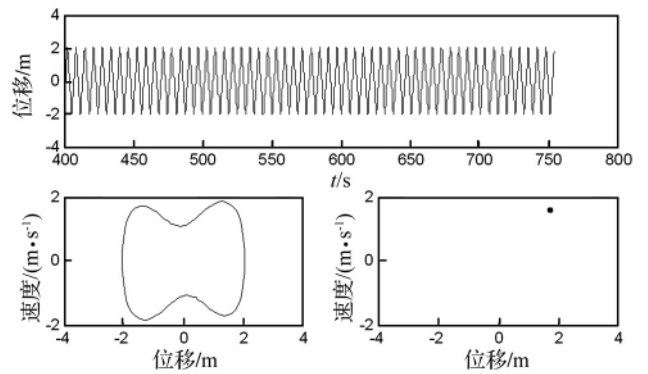


图 9 $B_z = 1.6T$ 时的位移波形图, 相平面轨迹图及庞伽莱截面图

Fig. 9 Wave diagram of displacement Phase diagram and Poincare map ($B_z = 1.6 T$)

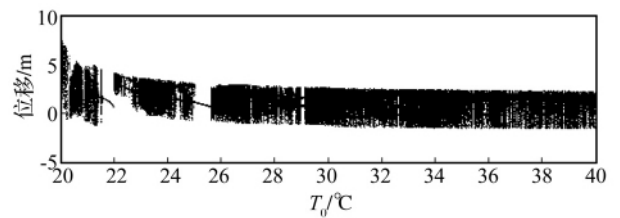


图 10 $\omega_0 = 1 \text{ rad/s}$, $B = 1T$, $q_0 = 70 \text{ N/m}^2$ 时系统随 T_0 变化时的分岔图

Fig. 10 Bifurcation diagram when T_0 is variable ($\omega_0 = 1 \text{ rad/s}$, $B = 1T$, $q_0 = 70 \text{ N/m}^2$)

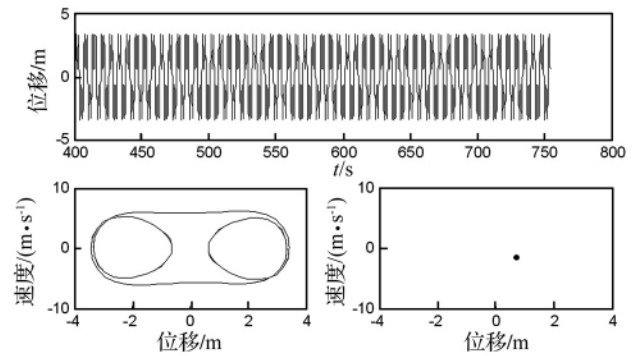


图 11 $T_0 = 22^\circ\text{C}$ 时的位移波形图, 相平面轨迹图及庞伽莱截面图

Fig. 11 Wave diagram of displacement Phase diagram and Poincare map ($T_0 = 22^\circ\text{C}$)

$\sim 40^\circ\text{C}$ 范围内的空隙处存在微窄大周期区。图 11 ~ 图 14 为在不同温度 T_0 下系统位移波形图、相平面轨迹图及庞伽莱截面图。 $T = 22^\circ\text{C}$ 时, 系统处于单周期运动状态; $T = 27.8^\circ\text{C}$ 和 $T = 29^\circ\text{C}$ 时, 系统处于大周期运动状态; $T = 35^\circ\text{C}$ 时, 系统处于混沌运动状态。

3 结 论

(1) 本文用 Melnikov 函数法, 给出非线性动力系统 Smale 马蹄变换意义下出现混沌运动的判据。通过对系统进行数值模拟, 在机械场、电磁场及温度场耦合作用下大挠度简支矩形薄板系统振动方程具有明显的非

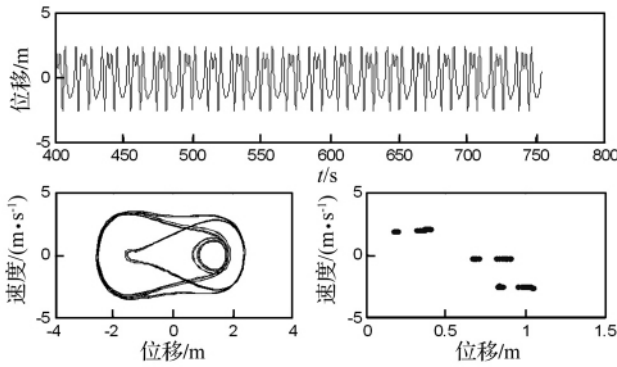


图12 $T_0 = 27.8^\circ\text{C}$ 时的位移波形图, 相平面轨迹图及庞加莱截面图

Fig. 12 Wave diagram of displacement, Phase diagram and Poincaré map ($T_0 = 27.8^\circ\text{C}$)

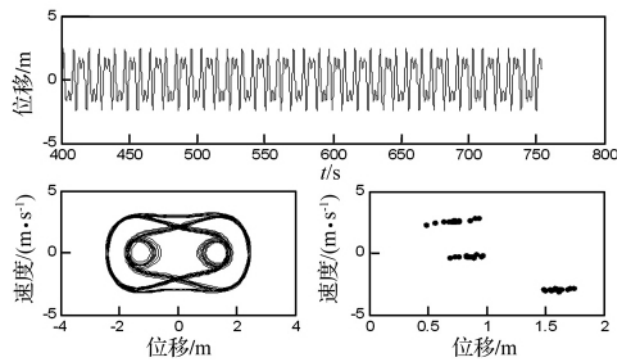


图13 $T_0 = 29^\circ\text{C}$ 时的位移波形图, 相平面轨迹图及庞加莱截面图

Fig. 13 Wave diagram of displacement, Phase diagram and Poincaré map ($T_0 = 29^\circ\text{C}$)

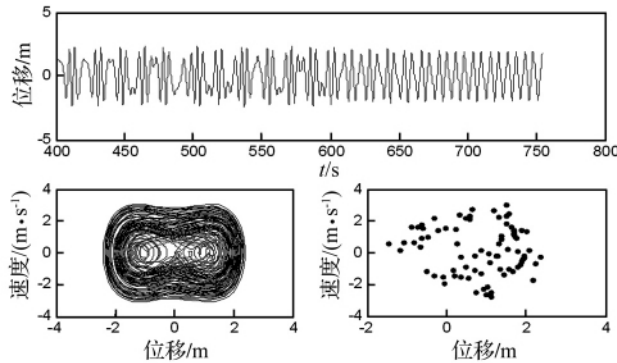


图14 $T_0 = 35^\circ\text{C}$ 时的位移波形图, 相平面轨迹图及庞加莱截面图

Fig. 14 Wave diagram of displacement, Phase diagram and Poincaré map ($T_0 = 35^\circ\text{C}$)

线性运动特性较复杂, 周期区与混沌区交替出现, 混沌区中既有单周期窗口, 也有倍周期窗口、大周期窗口, 混沌与分岔现象较丰富。

(2) 单周期或 k 周期运动, 对应相平面轨迹图为 1 条或 k 条连续的封闭曲线, 在庞加莱截面图上显示 1 个或 k 个点, 位移波形具有明显周期重复性; 混沌运动, 对应的相平面轨迹图轨线较紊乱, 庞加莱截面图呈云片状、杂乱无章, 位移波形呈类随机紊乱状态。

(3) 通过变化机械载荷、电磁参数及温度可使系

统进入混沌运动状态, 或避免混沌运动, 以实现系统振动特性控制。本文研究结果可作为工程在温度环境下电磁结构安全性、可靠性设计参考。

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