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Mathematics Model of the Question of Micro-pits Image with Direction-uniformity Distribution and Area Scrambling Micro-pits Image Design Methods Based on Regular Graphics Arrangement

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Abstract. Mathematical model including design variables, constraint conditions and target functions is raised on the basis of problem description. Using the mathematical model, area scrambling micro-pits image design methods are presented. The methods can generate square area scrambling micro-pits image based on square-aligned-array arrangement, square area scrambling micro-pits image based on square-staggered-array arrangement, and area scrambling micro-pits image based on equilateral hexagon-array arrangement. The uniformity of the last one is the best.

Introduction

Problem description of isotropic distribution of micro-pits is how to design the positions of micro-pits whose number is appointed and whose positions should be no overlap in an appointed plane to make SRa_{dcv} the approximate minimum. SRa_{dcv} referred to here is the coefficient of variation of directional surface arithmetic mean deviation in assessed area[1]. This is a quite complex global optimization problem. The number of possible micro-pits images is very large, but is limited afer all. So there must be an optimal image. In order to simplify the complex problem and acquire a micro-pits image with approximate isotropic distribution, this paper puts forward micro-pits image design methods based on regular graphics arrangement and area scrambling.

Mathematical model

Design variables This paper adopts image evaluation method and uses gray image to represent 3D surface. Each pixel represents the surface of 2×2μm^2 square area. Pixel gray value represents the mean value of the height of the small area. Each gray level represents the height of 0.1μm. The position of the pixel in the image represents the 2D coordinates of the small area.

The sketch of a micro-pit is shown in Fig.1. A micro-pit consists of a ring-shaped lug and the pit. In the sectional view, the lug is simulated by the first half cycle of a sine wave, and the pit is simulated by the next half cycle of a sine wave. The external diameter of the micro-pit is 150μm and internal diameter (the external diameter of the pit) is 100μm. The lug height of the micro-pit is 5μm. The depth of the micro-pit is 20μm. In the platform of the micro-pit, the pixel gray value of the plane part is 200 (This represents the value of the height is 20μm). The pixel gray value of the top of the boss is 250 (This represents the value of the height is 25μm). The pixel gray value of the bottom of the pit is 0 (This represents the value of the height is 0μm). The lighter the colour, the higher the height.

The design coordinate system of micro-pits image is shown in Fig.2. The coordinate unit in the figure is the number of pixels (All the length units without indicating thereinafter are the number of pixels). 1501×1501 area repesents the size of the micro-pits to generate. The disk in the middle whose diameter is 1175 represents assessed area. The disk whose diameter is 75 represents the size of a micro-pit.

In order to compare easily, all the sizes of the micro-pits images generated are 1501×1501. The number of the aligned micro-pits is about 196. The percentage of the cover (the ratio of the total area of the micro-pits to the image area) is about 38.5%. Suppose the 2D coordinates are (x_i, y_i), design variables are represented by 1D vector [x_1,y_1,x_2,$y_2$$\cdots$$x_{391}$,$y_{392}$] including 392 elements.

Constraint conditions According to the requirements that the micro-pits should be all in the area of 1501×1501 and their positions should be no overlap, constraint conditions are as follows:

$$38 \le x_i \le 1464 \qquad i = 1,2\cdots196 \tag{1}$$

$$38 \le y_i \le 1464 \qquad i = 1,2\cdots196 \tag{2}$$

$$\sqrt{(x_j - x_k)^2 + (y_j - y_k)^2} \ge 75 \qquad j,\ k = 1,2\cdots196,\ j \ne k \tag{3}$$

When design a specific micro-pits image, the positions of the micro-pits are often determined by additive constraint conditions. The more the constraint conditions, the less the number of micro-pits images, and the worse the effect.

Target functions After generating the micro-pits image, SRa , a height amplitudes based on area-direction character , can be calculated by rotating and intercepting accessed area (the disk whose diameter is 1175). $SRa(j)$ can be acquired by dividing 180 degrees into 200 equal segments(j = 1,2…200).

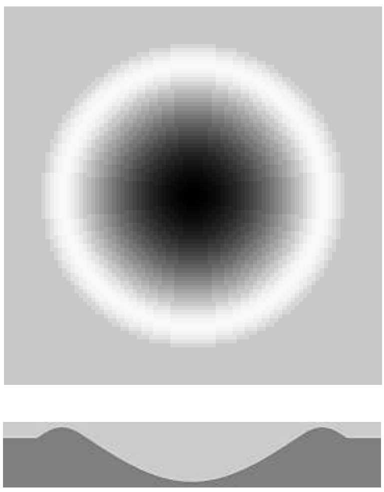

Fig.1 Sketch of a micro-pit

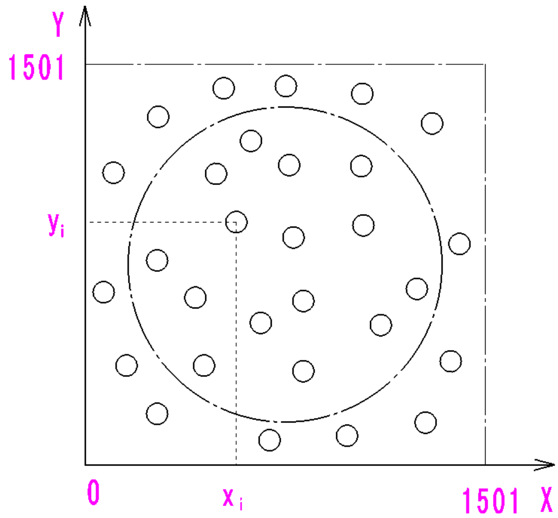

Fig.2 Design coordinate system of micro-pits images

Accessed area consists of 1175 horizontal close aligned rectangles, and each width of the rectangle is a single pixel.

$$SRa(j) = \frac{1}{1175}\sum_{i=1}^{1175} Ra_i(j) \qquad j = 1,2\cdots200 \tag{4}$$

The directional accessed indexes are calculated as follows:

$$SRa_{d\min} = \min[SRa(j)] \qquad j = 1,2\cdots200 \tag{5}$$

$$SRa_{d\max} = \max[SRa(j)] \qquad j = 1,2\cdots200 \tag{6}$$

$$SRa_{drange} = SRa_{d\max} - SRa_{d\min} \tag{7}$$

$$SRa_{dmean} = \frac{1}{200}\sum_{j=1}^{200} SRa(j) \qquad j = 1,2\cdots200 \tag{8}$$

$$SRa_{dstd} = \sqrt{\frac{1}{200}\sum_{j=1}^{200} SRa(j)^2 - SRa_{dmean}{}^2} \qquad j = 1,2\cdots200 \tag{9}$$

$$SRa_{dcv} = \frac{SRa_{dstd}}{SRa_{dmean}} \tag{10}$$

In these formula, SRa_{dmin} is the minimum directional surface arithmetic mean deviation.
SRa_{dmax} is the maximum directional surface arithmetic mean deviation.
SRa_{drange} is the range of directional surface arithmetic mean deviation.
SRa_{dmean} is the mean deviation of directional surface arithmetic mean deviation.
SRa_{dstd} is the standard deviation of directional surface arithmetic mean deviation.
SRa_{dcv} is the coefficient of variation of directional surface arithmetic mean deviation.
SRa_{dcv} is the target funtion to optimize. The requirement is minimized.

Area scrambling micro-pits image design methods based on regular graphics arrangement

First step, the plane is divided into seamless equal segments according to a certain regular graphics, and the micro-pit is aligned in the middle of the regular graphics to generate uniform and regular micro-pits image. This image ensures the area uniformity of the micro-pits distribution.

Second step, the optimization target is to make SRa_{dcv} of the image minimum. Moving the micro-pit ramdomly in the limited area of the regular graphics to acquire micro-pits image of approximate isotropic distribution.

There are three possible regular graphics arrays including square-aligned-array arrangement, square-staggered-array arrangement, and equilateral-hexagon-array arrangement. The optimization method LDSE(Low Dimensional Simplex Evolution Algorithms) is thereinafter adopted[2].

Square area scrambling micro-pits image based on square-aligned-array arrangement. 14×14 micro-pits are aligned by square array on the image whose size is 3×3 mm^2. The distance of the micro-pits is 200μm. These micro-pits are placed in the middle. The center of each micro-pit is changed in the square of 50×50μm^2. The optimization target is to make SRa_{dcv} minimum. After repeated searches, the approximate isotropic distribution of micro-pits image is acquireed. The result is shown in Fig.3. Its graph of *SRa* is shown in Fig.4. Its directional accessed indexes of *SRa* are shown in the data at row 1 in Table 1.

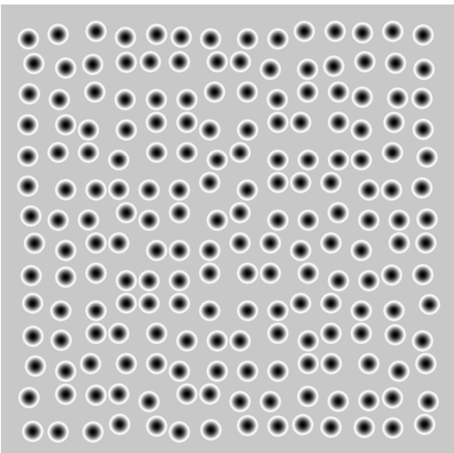

Fig.3 Image based on square- aligned-array and area scrambling of micro-pits

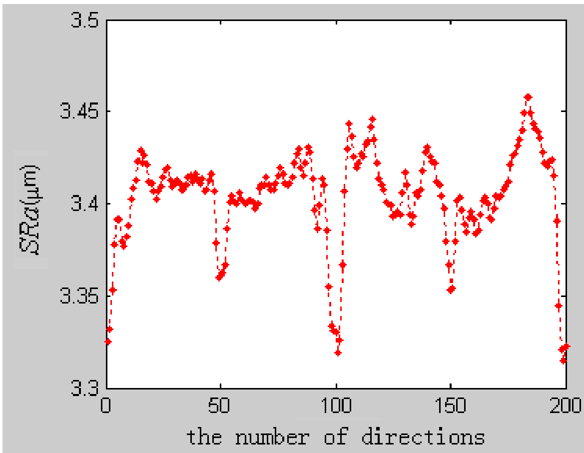

Fig.4 Graph of *SRa* of the image based on square-aligned-array and area scrambling of micro-pits

Square area scrambling micro-pits image based on square-staggered-array arrangement. *SRa* of the micro-pits image based on square-aligned-array have two minimal values in the directions of 90 degree and 180 degree, which cause the value of SRa_{dcv} too big. If initial micro-pits are aligned in a square-staggered-array, the minimal value of *SRa* in the direction of 90 degree can be eliminated, and SRa_{dcv} of *SRa* can be decreased further.

14×14 micro-pits are aligned by square-staggered-array on the image whose size is 3×3 mm^2. The distance of the micro-pits is 200μm. These micro-pits are placed in the middle. The center of each micro-pit is changed in the square of 50×50μm^2. The optimization target is to make SRa_{dcv} minimum. After repeated searches, the approximate isotropic distribution of micro-pits image is acquireed. The result is shown in Fig.5. Its graph of *SRa* is shown in Fig.6. Its directional accessed indexes of *SRa* are shown in the data at row 2 in Table 1.

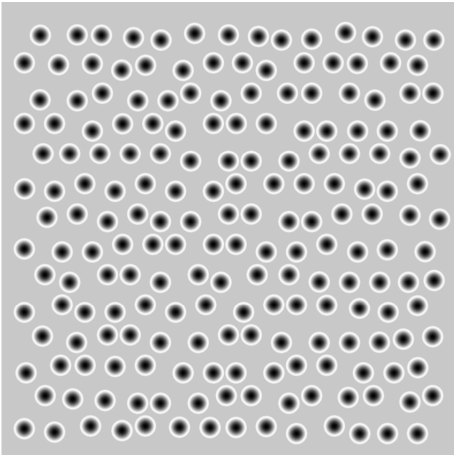

Fig.5 Image based on square-staggered-array and area scrambling of micro-pits

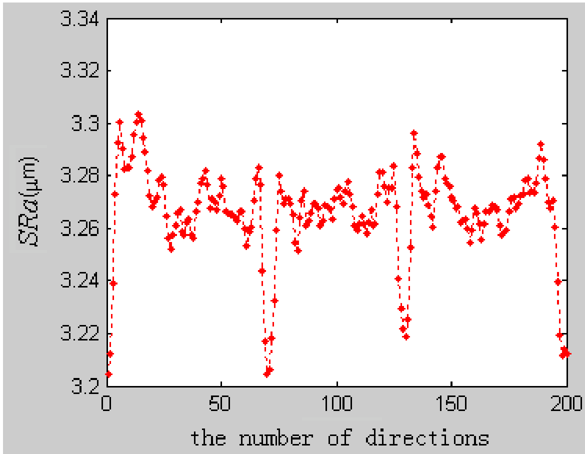

Fig.6 Graph of *SRa* of the image based on square-staggered-array and area scrambling of micro-pits

Area scrambling micro-pits image based on equilateral hexagon-array arrangement. The regular micro-pits image based on equilateral hexagon-array arrangement can ensure uniform distribution in three directions. However, the regular micro-pits image based on square-staggered-array arrangement can only ensure uniform distribution in two directions. SRa_{dcv} of the area scrambling micro-pits image based on equilateral hexagon-array arrangement may smaller.

13×15 micro-pits are aligned by equilateral-hexagon-array on the image whose size is 3×3 mm^2. Horizontal distance of the micro-pits is 216μm and vertical distance of the micro-pits is 186μm. These micro-pits are placed in the middle. The center of each micro-pit is changed in the equilateral hexagon whose side length is 38μm. The optimization target is to make SRa_{dcv} minimum. After repeated searches, the approximate isotropic distribution of micro-pits image is acquired. The result is shown in Fig.7. Its graph of *SRa* is shown in Fig.8. Its directional accessed indexes of *SRa* are shown in the data at row 3 in Table 1.

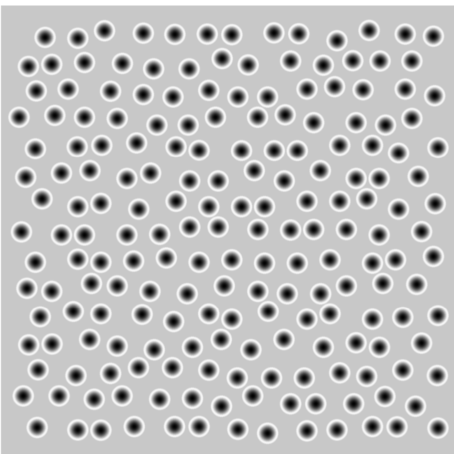

Fig.7 Image based on equilateral hexagon-array and area scrambling of micro-pits

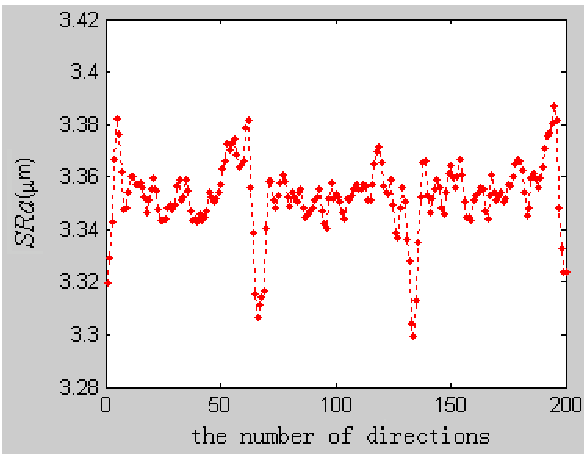

Fig.8 Graph of *SRa* of the image based on equilateral hexagon-array and area scrambling of micro-pits

Table1 Assessed direction indexes of *SRa* of the micro-pits images

SRa_{dmin} (μm)	SRa_{dmax} (μm)	SRa_{drange} (μm)	SRa_{dmean} (μm)	SRa_{dstd} (μm)	SRa_{dcv} (%)
3.3148	3.4579	0.1431	3.4038	0.0264	0.7761
3.2043	3.3032	0.0989	3.2662	0.0178	0.5463
3.2996	3.3871	0.0875	3.3525	0.0135	0.4036

Conclusions

The biggest advantage of the area scrambling micro-pits image design methods based on regular graphics arrangement is convenient for optimization. Distributing micro-pits regularly by area can ensure the area uniformity of micro-pits distribution. Area scrambling micro-pits and the optimization of the distribution can ensure approximate isotropic of micro-pits distribution.

As shown in Table 1, the uniform effect of three micro-pits images are all good. In comparison, the uniform effect of the area scrambling micro-pits image based on equilateral hexagon-array arrangement is the best.

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