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One-Step Direct Aeroacoustic Simulation Using Space-Time Conservation Element and Solution Element Method

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Abstract One-step direct aeroacoustic simulation (DAS) has received attention from aerospace and mechanical high-pressure fluid-moving system manufacturers for quite some time. They aim to simulate the unsteady flow and acoustic field in the duct simultaneously in order to investigate the aeroacoustic generation mechanisms. Because of the large length and energy scale disparities between the acoustic far field and the aerodynamic near field, highly accurate and high-resolution simulation scheme is required. This involves the use of high order compact finite difference and time advancement schemes in simulation. However, in this situation, large buffer zones are always needed to suppress the spurious numerical waves emanating from computational boundaries. This further increases the computational resources to yield accurate results. On the other hand, for such problem as supersonic jet noise, the numerical scheme should be able to resolve both strong shock waves and weak acoustic waves simultaneously. Usually numerical aeroacoustic scheme that is good for low Mach number flow is not able to give satisfactory simulation results for shock wave. Therefore, the aeroacoustic research community has been looking for a more efficient one-step DAS scheme that has the comparable accuracy to the finite-difference approach with smaller buffer regions, yet is able to give accurate solutions from subsonic to supersonic flows. The conservation element and solution element (CE/SE) scheme is one of the possible schemes satisfying the above requirements. This paper aims to report the development of a CE/SE scheme for one-step DAS and illustrate its robustness and effectiveness with two selected benchmark problems.

Key words: aeroacoustics, CAA, CE/SE method, acoustic wave, supersonic jet

INTRODUCTION

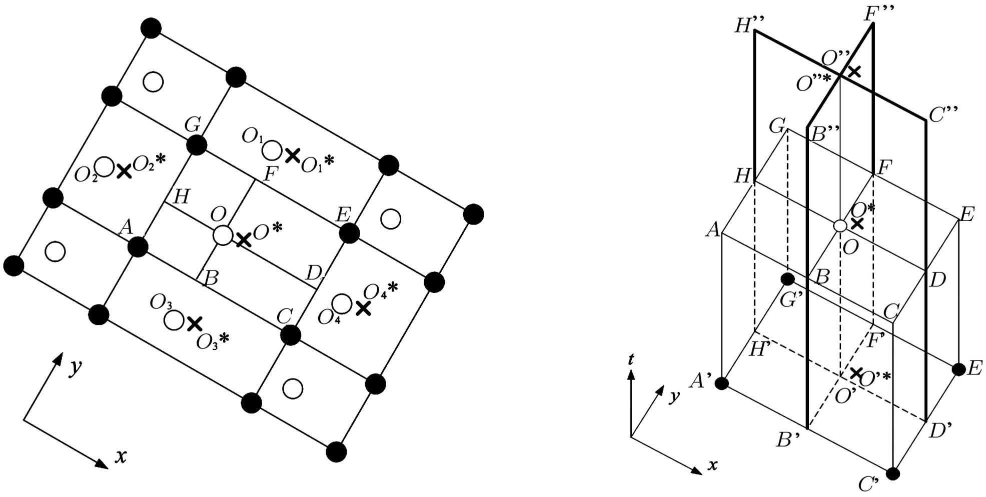
Noise reduction is an important part of engineering design in the transportation industries. Airplane, automobiles, and trains all produce noise that disturbs passengers, operators, and the surrounding communities. Examples of current interest include air-frame noise, cavity acoustics, jet screech, sonic boom, cabin noise, and noise generated by blade/vortex interactions. In particular, the need to meet more stringent community noise-level standards has resulted in recent attention given to the relatively new field of time-domain computational aeroacoustics (CAA), which focuses on the accurate prediction of aerodynamic sound generated by airframe components and propulsion systems, as well as on its propagation and far-field characteristics. Both aspects of the problem, that is, noise generation and propagation, are extremely demanding from a time-domain computation standpoint due to the large number of grid points and small time steps that are typically required. Therefore, if realistic aeroacoustic simulations are to become more feasible, higher-order accurate and optimized numerical schemes have to be sought to reduce the number of grid points required per wavelength while still ensuring tolerable levels of numerically induced dissipation and dispersion.

FORMULATION OF PROBLEM

The development of computational aeroacoustics essentially centers around two approaches [1]. One is based on the application of the acoustic analogy or hybrid approach to the time-dependent computational fluid dynamics data.

Another, known as direct aeroacoustics simulation (DAS), is based on the simultaneous calculation of the aerodynamic and acoustic fields obtained by solving the unsteady compressible Navier-Stokes equations plus the perfect gas equation of state. The solutions of these two approaches are different. The first approach yields the noise radiation in the far field, but the second approach could, in addition, provide a better understanding of the noise source mechanisms. Furthermore, the DAS would also provide the link between turbulence dynamics and the acoustic waves. However, this understanding is obtained at the expense of serious numerical issues that may be difficult to overcome [1,2]. Thus, high-order compact finite difference based numerical schemes are commonly adopted to satisfy these stringent requirements but this would require huge computational resources for accurate results over a reasonably large domain. In addition the need for a large buffer zone to suppress the numerical spurious wave generated at the boundary further increases the computational burden. This paper focuses on the development of a one-step DAS scheme for two-dimensional flows using conservation element and solution element (CE/SE) method. CE/SE, which was first developed by Chang [3], is a numerical scheme that might satisfy the above requirement. It is a highly accurate numerical scheme for solving full compressible Navier-Stokes equations. The present CE/SE scheme only applies first order Taylor expansion in its derivation and this significantly reduces the required computational resources. The method is different from other traditional numerical methods such as finite different, finite volume and finite element. The unique features of the CE/SE scheme include: (1) space and time are treated as same manner; (2) the scheme can ensure local and global flux conservation for both in time and space; (3) Riemann solver is not required for solving shock problem; (4) the scheme can be extended easily to multi-dimensional problems; (5) inlet/outlet boundary conditions can be easily derived and applied for an accurate solution [3].

The CESE scheme requires the decomposition of the computational domain into many small conservation elements (CE) and solution elements (SE). The definition of the elements applied in this study is modified from the original version [3]. Figure 1 shows this modified definition. The advantages of using this definition are as follows. Firstly, the boundaries of the modified CE coincide with grid mesh lines which are not so in the original CE formulation. The present formulation will help reduce computation load and increase accuracy because Riemann solver need not be invoked. Secondly, the volume of the modified CE is only half of the original one under the same grid system, so the modified CE can capture weak and high frequency fluctuations better. Finally, since the modified CE and SE are in rectangular shapes, this leads to easy formulation and programming.



(a) Grid points on the x-y planes

(b) CE & SE definitions

Figure 1: Schematic diagram of CE/SE mesh.

Applying Gauss divergence theorem, the integral form of Navier-Stokes equations can be written as, with all symbols taking their usual meanings,

$$\oint_{S(V)} \boldsymbol{H} \cdot \mathrm{d}\boldsymbol{s} = 0 \tag{1}$$

where $\boldsymbol{H} = (f_{\mathrm{m}} - F_{\mathrm{vm}}, g_{\mathrm{m}} - G_{\mathrm{vm}}, u_{\mathrm{m}})$, $\boldsymbol{u}_m = [\rho, \rho u, \rho v, e]^{\mathrm{T}}$, $\boldsymbol{f}_{\mathrm{m}} = \left[\rho u, \rho u^2 + p, \rho uv, (e+p)u\right]^{\mathrm{T}}$, $\boldsymbol{g}_{\mathrm{m}} = [\rho v, \rho uv, \rho v^2 + p, (e+p)v]^{\mathrm{T}}$, $\boldsymbol{F}_{\mathrm{vm}} = [0, \tau_{xx}, \tau_{xy}, u\tau_{xx} + v\tau_{xy} + q_x]$, $\boldsymbol{G}_{\mathrm{vm}} = [0, \tau_{xy}, \tau_{yy}, u\tau_{xy} + v\tau_{yy} + q_x]$, $S(V)$ denotes the surface around an arbitrary space-time region V and $\mathrm{d}\boldsymbol{s} = \boldsymbol{n}\,\mathrm{d}\sigma$, with n and $\mathrm{d}\sigma$ being the unit normal vector pointing outward and the area of a surface element $S(V)$ respectively. The space-time flux of $\boldsymbol{H}$ leaving region V through surface $S(V)$ is defined as $\boldsymbol{H} \cdot \mathrm{d}\boldsymbol{s}$. For details of the time-matching of solution and information exchange between CEs and SEs, one can refer to Lam et al. [4].

RESULTS AND DISCUSSTIONS

The development of a CE/SE scheme DAS is briefly discussed and its robustness and effectiveness are illustrated with two selected benchmark problems. The first benchmark problem is the calculation of sound radiation from a baffled piston. This problem is able to show the capability of CE/SE DAS in capturing the weak-amplitude vibration of a solid surface and its generated sound field. The numerical results are compared with classical linear acoustic theory [5]. A schematic sketch of the computational domain and some sampled results are shown in Figures 2 and 3 respectively. The second benchmark problem the calculation of shock dynamics of free supersonic jet emerged from a convergent-divergent nozzle and its associated sound generation. In this situation both strong shock waves and weak acoustic waves co-exist in the same computational domain which provide a good case for assessing the capability of CE/SE DAS of solving a problem with disparity in fluctuating strength and Mach number. Essentially the setup given in Hunter [6] is used (Figure 4) whereas sample results are shown in Figure 5. All calculated solutions are in excellent agreement with the results of analytical and previous studies.

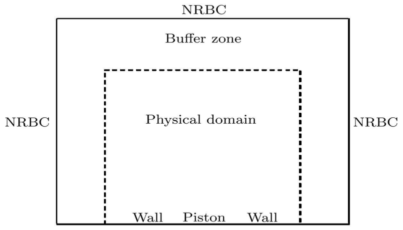
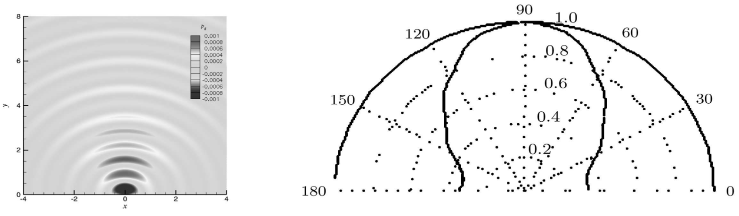


Figure 2: Schematic of sound radiation by a baffled piston.



(a) Acoustic pressure distribution

(b) Directivity pattern

Figure 3: Acoustic radiation driven with $\omega = 5$. (a) Acoustic pressure distribution. (b) Directivity pattern.

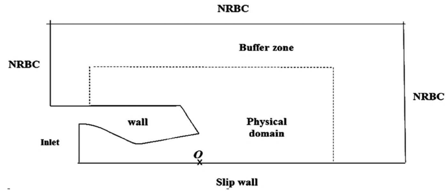
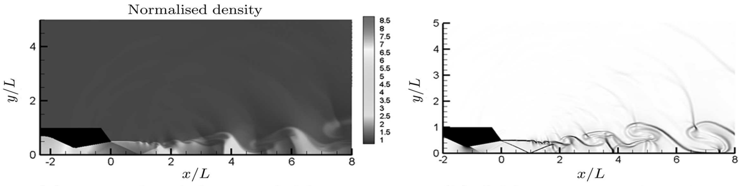


Figure 4: Schematic of supersonic nozzle flow.



(a) Normalized density field ρ

(b) Schlieren image based on density gradient $|\nabla\rho|$

Figure 5: (a) Normalized density field ρ. (b) Schlieren image based on density gradient $|\nabla\rho|$.

CONCLUSIONS

A modified CE/SE numerical scheme has been proposed for one-step DAS. The scheme is applied to two benchmark cases for evaluating its robustness and capability of simulating aeroacoustics. The first case is the calculation of sound radiation from a baffled piston whereas the second case is the calculation of shock dynamics of free supersonic jet emerged from a convergent-divergent nozzle and its associated sound generation. The numerical results agree well with analytical results and previous simulations of the benchmark problems. It illustrates the good accuracy of the CE/SE scheme in capturing the both strong shock waves and weak acoustic waves, indicating it is a good numerical scheme for calculating aeroacoustics ranging from low subsonic to low supersonic Mach numbers

Acknowledgements

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