On the cracks in saturated sand

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Abstract: The mechanism of water film (or crack) in saturated sand is analyzed first by numerical simulations under four given conditions. It is shown that there are only stable water films in the special conditions that there is a fixed thin layer. A simplified method for evaluating the thickness of water film is presented. The numerical results and that computed by the simplified method are compared with that of Kokusho.
Key words: Crack, saturated sand, liquefaction.

It is often occurred on the ground slope that sand deposit translates to lateral spreading or even landslide or debris flow not only, but also after earthquakes. If the sand deposit on a slope are composed of many sublayers, there will be a water film forms once it liquefied [1] which may serves as a sliding surface for postliquefaction failure. As a result, landslide or debris flow may happen on a slope with very gentle slope-angle. Seed [2] was the first to suggest that the existence of "water film" in sand bed is the reason of slope failures in earthquakes. Later, some researchers [3-5] performed some experiments to investigate the formation of "water film" in layered sand or in a sand containing a seam of non-plastic silt. Nevertheless, the mechanism of the formation of cracks or "water film" in a sand with the porosity distributed continuously is not very clear.

In the viewpoints above, a theoretical and numerical analysis is presented in this paper. Firstly, we will present a psudo-three-phase model describing the moving of liquefied sand and give some theoretical analysis and numerical simulations under four initial and boundary conditions. Secondly, we present a simplified method to analyze the evolution of the water film.

1 Formulation of the Problem

It is considered here a horizontal sand stratum, which is water saturated and the porosity changes only vertically. The fine grains may be eroded from the skeleton and the eroding relation is assumed as follows [6]. The x axis is upward.

$$\frac{1}{\rho_s}\left(\frac{\partial Q}{\partial t}+u_s\frac{\partial Q}{\partial x}\right)=\frac{\lambda}{T}\left(\frac{u-u_s}{u^*}-q\right) \qquad \text{if} \qquad -\varepsilon(x,0)\le\frac{Q}{\rho_s}\le\frac{Q_c(x)}{\rho_s} \tag{1}$$

$$\frac{1}{\rho_s}\left(\frac{\partial Q}{\partial t}+u_s\frac{\partial Q}{\partial x}\right)\le 0 \quad \text{otherwise, including } u=0 \tag{2}$$

in which Q is the mass of sand eroded per unit volume of the sand/water mixture, ρ_s is the density of the grains, u and u_s are the velocities of the percolating fluid containing fine sand particles and the sand grains, q is the volume fraction of sand carried in the percolating fluid, T and u^* are physical parameters, λ is a small dimensionless parameter, $\varepsilon(x,\ t)$ is the porosity, $Q_c(x)$ is the maximum Q that can be eroded at *x*.

Considering the eroding of the fine grains, a psudo-three-phase model is presented as follows. The mass conservation Eq.ations are as follows:

$$\frac{\partial(\varepsilon-q)\rho}{\partial t}+\frac{\partial(\varepsilon-q)\rho u}{\partial x}=0 \tag{3}$$

$$\frac{\partial q\rho_s}{\partial t}+\frac{\partial q\rho_s u}{\partial x}=\frac{\partial Q}{\partial t}+u_s\frac{\partial Q}{\partial x} \tag{4}$$

$$\frac{\partial(1-\varepsilon)\rho_s}{\partial t}+\frac{\partial(1-\varepsilon)\rho_s u_s}{\partial x}=-\frac{\partial Q}{\partial t}-u_s\frac{\partial Q}{\partial x} \tag{5}$$

in which ρ is the density of water. A general Eq.ation may be obtained from these three Eq.ations, which is

$$\varepsilon u + (1-\varepsilon) u_s = U(t) \tag{6}$$

in which $U(t)$ is the total mass of fluid and grains at a transect. The momentum Eq.ations may be written as follows

$$[(\varepsilon - q)\rho + q\rho_s]\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right) = -\varepsilon\frac{\partial p}{\partial x} - \frac{\varepsilon^2(u-u_s)}{k(\varepsilon,q)} - [(\varepsilon - q)\rho + q\rho_s]g \tag{7}$$

$$[(\varepsilon - q)\rho + q\rho_s]\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right) + (1-\varepsilon)\rho_s\left(\frac{\partial u_s}{\partial t} + u_s\frac{\partial u_s}{\partial x}\right) = -\frac{\partial p}{\partial x} - \frac{\partial \sigma_e}{\partial x} - [(\varepsilon - q)\rho + q\rho_s]g - (1-\varepsilon)\rho_s g - \left(\frac{\partial Q}{\partial t} + u_s\frac{\partial Q}{\partial x}\right)(u-u_s) \tag{8}$$

in which Eq.(7) denotes the momentum conservation of grains, Eq.(8) denotes the total momentum conservation, the last term on the right hand side of Eq.(8) denotes the momentum caused by the eroded fine grains, p is the pore pressure, k is the permeability, σ_e is the effective stress, θ is the slope, τ is the shear stress on the bed. Here k is assumed to be a function of ε and q in the following form:

$$k(\varepsilon,q) = k_0 f(q,\varepsilon) = k_0(-\alpha q + \beta\varepsilon) \tag{9}$$

in which, α, β are parameters and $1 < \beta << \alpha$, we choose to let α much greater than β, so that changes in q overweighs that of ε.

2 Numerical simulations

Based on the model presented above, we will analyze the occurrence of the crack in saturated sand, This case is about the cracks in a liquefied sand ($\sigma_e = 0, \tau \approx 0$) where the grains sink while the water is pressed to move upward just like the consolidation. Here, the sand column is assumed to be long enough to neglect the boundary effects for the convenience to obtain the solutions. At the same time, we neglect some factors, which may be important in other cases. we will simulate this problem by difference method.

Being an appropriate constant, the mass conservation Eq.ation(6) yield

$$\varepsilon u + (1-\varepsilon) u_s = U(t) = 0 \tag{10}$$

assuming both u and u_s are zero at x=0.

Taking T in (11) as the appropriate characteristic time. Let u_t denote the characteristic velocity and L the characteristic length of the problem. We use these characteristic parameters to make Eq. (11) non-dimensional. Letting

$$\bar{u} = \frac{u}{u_t}, \tau = \frac{t}{T}, \xi = \frac{x}{Tu_t} \tag{11}$$

Instituting Esq. (1), (2), (9), (10) and (12) into Eq..(3) and (4), we may obtain

$$\frac{\partial \varepsilon}{\partial \tau} + \frac{\partial \varepsilon \bar{u}}{\partial \xi} = \bar{u}\frac{u_t}{u*(1-\varepsilon)} - q$$
$$\frac{\partial q}{\partial \tau} + \frac{\partial q \bar{u}}{\partial \xi} = \bar{u}\frac{u_t}{u*(1-\varepsilon)} - q \tag{12}$$

For Tg/u_t >>1, the inertia terms are negligible and the last Eq.ation of Eq. (8) becomes

$$\bar{u} = \left(\frac{1-\varepsilon}{\varepsilon}\right)^2 (\varepsilon - q) f(q,\varepsilon)\frac{k_0\rho_s g(1-\rho/\rho_s)}{u_t} = \left(\frac{1-\varepsilon}{\varepsilon}\right)^2 (\varepsilon - q) f(q,\varepsilon) \tag{13}$$

when u_t is taken to be

$$u_t = k_0\rho_s g(1-\rho/\rho_s) \tag{14}$$

The problem now reduces to finding $\varepsilon(\xi,\tau)$ and $q(\xi,\tau)$ as solutions to Eq. (12). The initial conditions are clearly

$$\varepsilon(\xi,0) = \varepsilon_0(\xi), q(\xi,0) = 0 \tag{15}$$

In order to de-couple the problem from the complication arising from the effect of the consolidation wave initiated from the bottom of the sand column, we assume that the sand column is very tall so that cracks would develop before the consolidation wave arrives.

3 Numerical Results

Here, based on Eq. (12), four initial and boundary conditions are adopted to investigate the conditions under which stable water films may form.

The parameters are as follows:

$\Delta t = 9\times10^{-4}$, $\Delta\zeta = 5\times10^{-3}$, $\beta = 46 \sim 56$, κ=50.0, $a = 0.08$, $\rho_s = 2400\,kg/m^3$, $\rho_w = 1000\,\mathrm{kg/m^3}$, $u^* = 0.04$, $k_0 = 4\times10^{-6}\,\mathrm{m/s}$, $\alpha = 1$

The boundary conditions are as follows:

1) The initial porosity distribution is $\varepsilon_0(x) = \bar{\varepsilon}_0\left(1 - a\tanh\left((x-0.5L)/2\right)\cdot\kappa\right)$, in which $\bar{\varepsilon}_0 = 0.4$, $L = 1$, $0 \le x \le 1$, L is the length of sand column. There is an assumption that u keeps zero once it drops to zero. if $\varepsilon = 1$ then $u = 0, q = 0, \beta = 56$, if $\varepsilon = 1$, then $q = 0$.

2) The distribution of initial porosity is the same as that in condition 1, there is no assumption.

3) There are three layers of sand: the upper is dense sand with $\varepsilon_0 = 0.3$, the middle is dense sand with $\varepsilon_0 = 0.2$ and the length is $L/50$, the lower is loose sand with $\varepsilon_0 = 0.4$. There are any assumptions.

4) The distribution of initial porosity is the same as that in condition 3, There are not any assumptions.

Fig.1 shows that if we assume that once the sand column at some point is jammed, they keep this state forever, then the sand above the jammed position will be prevented to drop cross the jammed point and so the porosity becomes smaller and smaller, while the sand below the point will settle gradually and makes the crack extends gradually. But if we do not adopt the assumption as in Fig.1, the crack (water film) will form first and then disappear gradually (Fig.2). Fig.3 and Fig.4 show that when the porosity of the upper sand column is 0.3, that of the middle is 0.2, and that of the lower is 0.4, there is a stable water film formed when there is the same assumption (Fig.3) as that in Fig.1. Nevertheless, if there are no any assumptions, there will be no stable water film forming (Fig.4). The results above show that the conditions for the forming of stable water film are: (1) the porosity of the upper part of the sand column must be smaller than that of the lower. (2) The keeping of the jamming state or the effective stress to prevent the free dropping of the grain or the skin friction in Kokusho's experiments is needed. At last, we compare the results with the experimental data of Kokusho[7] (Fig. 5, the parameters used in computing is shown in Table 1). It is shown that they are agreement with qualitatively although their experiments were carried out in a finite tube with free surface while our analysis is based on a infinite sand column. There is obvious changes of pore pressure near the water film, and the water film thickness develops fast to a biggest value and then gradually decreases.

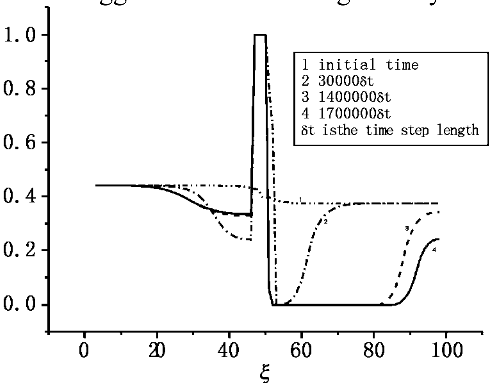

Fig. 1. The evolution of cracks in the initial and boundary condition 1

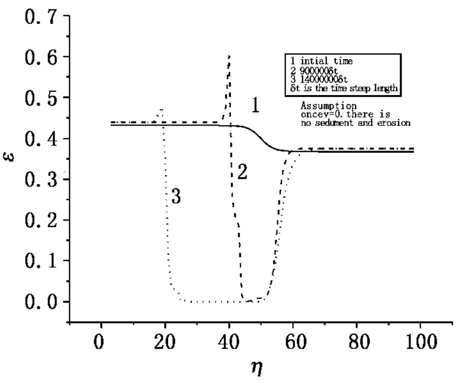

Fig. 2 The evolution of cracks in the initial and boundary condition 2

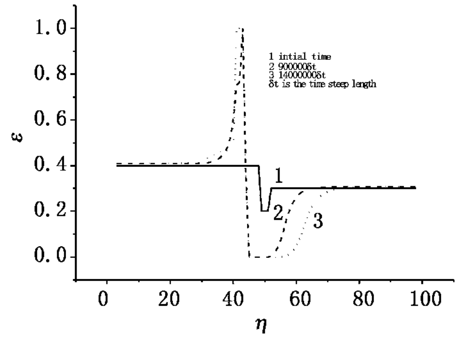

Fig. 3. The evolution of cracks in the initial and boundary condition 3

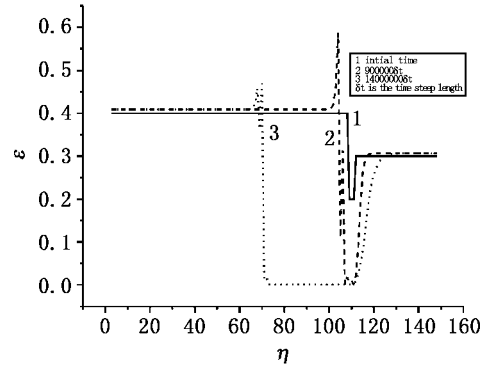

Fig. 4. The evolution of cracks in the initial and boundary condition 4

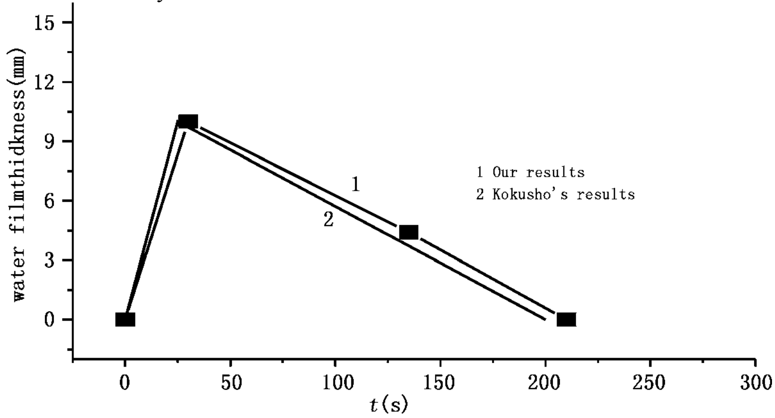

Fig. 5 .The comparison of our results with the experimental of Kokusho

4 A Simplified evaluation method

Although the numerical simulation may give more information about the evolution of the water film, a simplified method is needed to analyze the evolution of the water film fast. Florin and Ivanov[8] pointed out that when the settling particles reach solid material, which is usually the unliquefied underlying soil, or the container base in a experiment, they accumulate to form a solidified zone which increases in thickness with time. A solidification front therefore moves upward until it reaches the surface, or overlying unliquefied material. Scott et al [9]had analyzed the development of the solidification. Here, we present a simplified analytical method of the water film in saturated sand with initial non-uniform grade series.

Assuming that the whole mass reaches its terminal velocity, k, which is the permeability, instantaneously at the end of liquefaction, Florin have given an expression for the constant velocity $\dot{z}$, of the solidification front：

$$\dot{z} = \frac{\rho}{\rho_w}\frac{1-n_1}{n_2-n_1}k \tag{16}$$

in which $\rho = \rho_s - \rho_w$ is the buoyant unit weight of the liquefied soil，n_1 is the porosity of the liquefied soil，n_2 is the porosity of the solidified soil.

From Eq.(16), we can obtained the duration of liquefaction and subsEq.ent excess pore pressure decline for any point in the soil column.

$$t = \frac{\rho_w}{\rho}\frac{n_2-n_1}{1-n_1}\frac{h}{k} \tag{17}$$

in which h is the height of any point in the soil column.

The final settlement of the top surface of the sand layer is

$$\Delta L = \frac{n_0-n_1}{1+n_0}H \tag{18}$$

This occurs in time given by Eq.(17), so the rate of settlement is

$$\dot{s}_a = \frac{\gamma' k}{\gamma_w} \tag{19}$$

The settlement at any time is

$$s_a = \frac{\gamma^{'} k}{\gamma_w} t \tag{20}$$

The settlement velocity v_e of the elements above the water film is determined by the combined permeability k_{es} of the middle layer and the upper layer as:(Kokusho, 2002)

$$k_{es} = \sum_1^m L_i \bigg/ \sum_1^m L_i / k_i \tag{21}$$

The upward seepage flow with the same velocity：

$$v = k_{es} i_e \tag{22}$$

where，i_e is the average hydraulic gradient.

In fact, the particles and solidified soil are compressible; the skeleton might consolidate by the geostatic stress after the solidification. If the permeability is small, the super pore pressure does not disperse immediately with the increase of solidification zone. The deform of the skeleton of the sand by the geostatic stress in the solidification zone may be expressed as [5]

$$s_2 = \frac{1}{2} \frac{\rho^{'} g}{m_s} x^2(t) \tag{23}$$

in which the compressible modulus m_s is assume as a constant. The percolation is assumed to obey the Darcy lay. The total deformation is

$$\Delta s = \Delta s_1 + \Delta s_2 \tag{24}$$

Institute Eqs.(16) and (23) into eEq.(24)：

$$\Delta s = \frac{n_{20} - n_{21}}{1 - n_{20}} (\Delta z + \Delta s) + \frac{\rho g}{m_s} z \Delta z \tag{25}$$

The settling velocity of the surface is

$$\frac{\Delta s}{\Delta t} = k \frac{\rho^{'}}{\rho_w} \tag{26}$$

The increase velocity of the thickness of solidification：

$$\frac{\Delta z}{\Delta t} = \frac{k\rho}{\rho_w} \bigg/ \left(\frac{n_0 - n_1}{1 - n_1} + \frac{1 - n_1}{1 - n_0} \frac{\rho\ g}{m_s} z \right) \tag{27}$$

The duration of any location that the solidification front arrives：

$$t = \frac{\rho_w}{k\rho} \left(\frac{n_0 - n_1}{1 - n_1} z + \frac{1}{2} \frac{1 - n_1}{1 - n_0} \frac{\rho g}{m_s} z^2 \right) \tag{28}$$

If it should be considered，the side friction may be expressed as

$$\sigma_s = \mu K_0 \sigma_z \tag{29}$$

This effect should be considered in the pore-pressure-gradient.

The effect of the changes of porosity on the permeability is considered as a linear relation:

$$k = k_0 [1 - \alpha (n_0 - n)] \tag{30}$$

in which k_0 is the initial porosity, α is a parameter, n_0 is the initial porosity, k is the permeability when the porosity is n.

By considering Eqs.(29) and (30) in the pore pressure gradient and considering the consolidation of the solidification zone, we can compute the development of the water film.

5 Comparison with the experimental results

It is shown that the results computed by numerical method and the simplified method are close to the experimental results. The simplified method presented in this paper may be used to computing the evolution of the water film. The data of the parameters are shown in Table 1.

Tab.1 The data of the parameters

sand	Thickness /cm	Relative density /%	Permeability coefficient /(cm • s^{-1})	Initial pore ratio	The max. strain /%
The upper layer of sand	103.6	14	0.04	0.924(0.48)	2.4
seam	0.4		1.8E-4	1.5(0.6)	2.4
The lower layer of sand	96	39	0.04	0.831(0.454)	0.95

$\rho_s = 2600\,\text{kg/m}^3$, $\rho_w = 1000\,\text{kg/m}^3$, $\mu K_0 = 0.125$, $\alpha = 20$

6 Conclusions

By numerical simulations under four given conditions, we can see that there are stable water films only in the conditions that: (1) the porosity of the upper part of the sand column must be smaller than that of the lower. (2) The keeping of the jamming state or the effective stress to prevent the free dropping of the grain or the skin friction in Kokusho's experiments is needed. A simplified method for evaluating the thickness of water film is presented. It is shown that the results computed by the simplified method are close to the experimental results.

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