A FINITE VOLUME SIMULATOR FOR SINGLE-PHASE FLOW IN FRACTURED POROUS MEDIA

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We present a cell-centered finite volume method for the fully coupled discretization of single-phase flow in fractured porous media. Fractures are discretely modeled as lower dimensional elements. The method works on structured/unstructured and hybrid grids. An explicit time discretization is employed to solve steady/unsteady problems. Results from two-dimensional and three-dimensional simulations are presented.

INTRODUCTION

Modeling of flow in fractured porous media plays an integral role in many areas of the geosciences, ranging from groundwater hydrology to oil [1, 2]. It is also of high importance in radioactive waste management and hydraulic fracturing.

Flow simulations in fractured media are challenging for several reasons such as uncertainty in fracture location, complexity in fracture geometry, dynamic nature of fractures, etc. A discretization of a fractured medium domain with volumetric elements in the fractures requires a mesh which resolves the geometry of the problem. Besides, the large contrast in the rock matrix and fracture permeability coupled with small fracture openings makes the numerical simulation challenging. Therefore, a mixed-dimensional finite volume discretization method is applied [3], which realizes fractures as lower-dimensional elements. One-dimensional elements are used for fractures in two-dimensional domains and two-dimensional fracture elements in three-dimensional domains. This method is suited for structured/unstructured grids and locally refined grids.

There are two types of finite volume methods and the cell-centered one is employed in order to model hydraulic fracturing in future. It's considered that the cell-centered type has a better numerical stability than the node-centered one in many aspects though a bit less accuracy than the latter one.

In this study, we first present the governing equations of single-phase flow, and then introduce the numerical models. After model introduction, several numerical simulations are tested. Results from two-dimensional and three-dimensional simulations are both presented. Some conclusions are drawn afterwards.

GOVERNING EQUATIONS

Darcy's law, in its simplified form and ignoring gravitational forces, can be used to obtain velocity field,

$$\mathbf{v} = -\frac{k}{\mu}\nabla p \tag{1}$$

where $\mathbf{v}$ denotes the velocity field and p represents the pressure values. The pressure equation for single-phase flow in porous media in the absence of gravity and capillary forces can be written as

$$\phi c \frac{\partial p}{\partial t} + \nabla \cdot \mathbf{v} = q \tag{2}$$

where q stands for external sinks and sources. In the above equations, k, μ, c and ϕ are physical properties of porous media and they stand for permeability, viscosity, compressibility and porosity respectively.

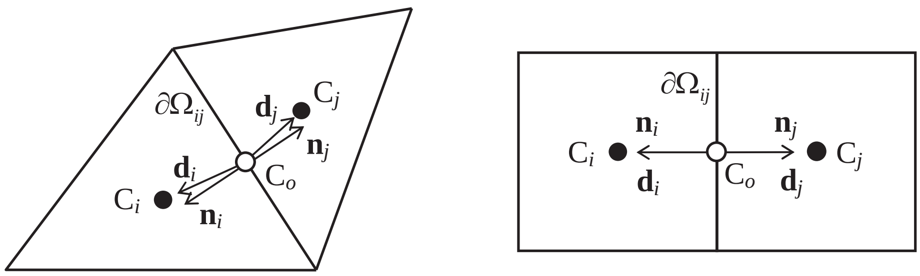
Equations (1) and (2) work on flow in fractures as well despite the different physical properties from matrix. The porosities of fractures equal unity and the permeabilities can be written in the following form [4]

$$k_f = \frac{a^2}{12} \tag{3}$$

where a denote the apertures of fractures.

NUMERICAL MODELING

To derive the set of finite volume mass balance equation for pressure equation, consider a grid cell Ω_i in the domain denoted by Ω. Ω_j is one of the neighboring cells of Ω_i and they own the same interface $\partial\Omega_{ij} = \Omega_i \cap \Omega_j$. They are shown in the Fig 1 below. The centers of the two cells are C_i and C_j respectively. C_o is the middle point of $\partial\Omega_{ij}$.



(a) Unstructured mesh (b) Structured mesh

Fig 1. Geometry representation of two adjacent cells in two dimensions – the solid dot indicates the control volume and the hollow dot indicates a virtual and auxiliary point

The finite volume method is derived by obtaining the relationship between flux Q_{ij} across the interface $\partial\Omega_{ij}$ and pressures p of the two adjacent cells. Let $K = k / \mu$ for simplicity. The flow velocities along line segments C_iC_o and C_oC_j can be obtained from Darcy' law,

$$\mathbf{v}_{io} = -K_i \nabla p_{io} = -K_i \frac{p_o - p_i}{D_i}(-\mathbf{d}_i) \tag{4}$$

$$\mathbf{v}_{oj} = -K_j \nabla p_{oj} = -K_j \frac{p_j - p_o}{D_j}\mathbf{d}_j \tag{5}$$

Flux across interface $\partial\Omega_{ij}$ can be calculated by taking the following integral,

$$Q_{ij} = Q_{io} = \int_{\partial\Omega_{ij}} \mathbf{v}_{io} \cdot (-\mathbf{n}_i) dS = AK_i \frac{p_i - p_o}{D_i} (\mathbf{d}_i \cdot \mathbf{n}_i) \tag{6}$$

$$Q_{ij} = Q_{oj} = \int_{\partial\Omega_{ij}} \mathbf{v}_{oj} \cdot \mathbf{n}_j dS = AK_j \frac{p_o - p_j}{D_j} (\mathbf{d}_j \cdot \mathbf{n}_j) \tag{7}$$

where A is the area of the interface between the adjacent cells, K_i is the intrinsic permeability of cells i, D_i is the distance between the cell center and the middle point of the interface, $\mathbf{n}_i$ is the unit normal vector to the interface and $\mathbf{d}_i$ is the unit direction vector along C_oC_i.

We can obtain $(p_i - p_o)$ from equation (6) and $(p_o - p_j)$ from equation (7). By adding the two terms, we get

$$p_i - p_j = \left[\frac{D_i}{AK_i(\mathbf{d}_i \cdot \mathbf{n}_i)} + \frac{D_j}{AK_j(\mathbf{d}_j \cdot \mathbf{n}_j)} \right] Q_{ij} \tag{8}$$

By denoting $\alpha_i = \frac{AK_i}{D_i} \mathbf{d}_i \cdot \mathbf{n}_i$, $\alpha_j = \frac{AK_j}{D_j} \mathbf{d}_j \cdot \mathbf{n}_j$ and $T_{ij} = \frac{\alpha_i \alpha_j}{\alpha_i + \alpha_j}$, the above equation can be simplified below

$$Q_{ij} = T_{ij}(p_i - p_j) \tag{9}$$

where T_{ij} represents the geometric transmissibility between cell i and cell j.

For steady flow, mass balance equation can be obtained by taking the following integral

$$\int_{\Omega_i} (\nabla \cdot \mathbf{v} - q) dV = 0 \tag{10}$$

Using divergence theorem, the equation (10) transforms into the following

$$\oint_{\partial\Omega_i} \mathbf{v} \cdot \mathbf{n} dV = \int_{\Omega_i} q dV \tag{11}$$

The left term of equation (11) can be calculated by equation (9),

$$\oint_{\partial\Omega_i} \mathbf{v} \cdot \mathbf{n} dV = \sum_j T_{ij}(p_i - p_j) \tag{12}$$

By denoting $\int_{\Omega_i} q dV = Q_i$, we get the numerical scheme for steady flow

$$\sum_j T_{ij}(p_i - p_j) = Q_i \tag{13}$$

where Q_i is the volumetric flux of cell i. The j denotes the number of interfaces of the cell.

For unsteady flow, an explicit time discretization is used. By taking the integral of equation (2) over cell *i* with the difference form of pressure derivative, the numerical scheme for unsteady flow can be given below

$$\int_{\Omega_i} \phi c \frac{p^{n+1} - p^n}{\Delta t} dV = Q_i - \sum_j T_{ij} (p_i - p_j) \tag{14}$$

Thus, we get an explicit scheme by transforming the above equation into the following

$$p^{n+1} = p^n + \frac{\left[Q_i - \sum_j T_{ij} (p_i - p_j) \right]}{\phi_i c_i V_i} \Delta t \tag{15}$$

Consider a system with N degrees of freedom where N is the sum of the number of the matrix and fracture cells. The scheme can be rewritten as the form of matrix

$$\mathbf{P}^{n+1} = \mathbf{P}^n + \mathbf{C}^{-1} (\mathbf{Q}^n - \mathbf{T}\mathbf{P}^n) \Delta t \tag{16}$$

where $\mathbf{P}^n$ is the pressure vector with dimension $N \times 1$, $\mathbf{C}$ is the compressibility matrix which is diagonal and Nth-ordered, $\mathbf{Q}^n$ is the flux vector with dimension $N \times 1$ as well and $\mathbf{T}$ is the transmissibility matrix of dimension $N \times N$.

The above equations work on flow in three dimensions and coupling flow of two and three dimensions as well, but some treatment should be done as shown in Fig 2 and Fig 3.

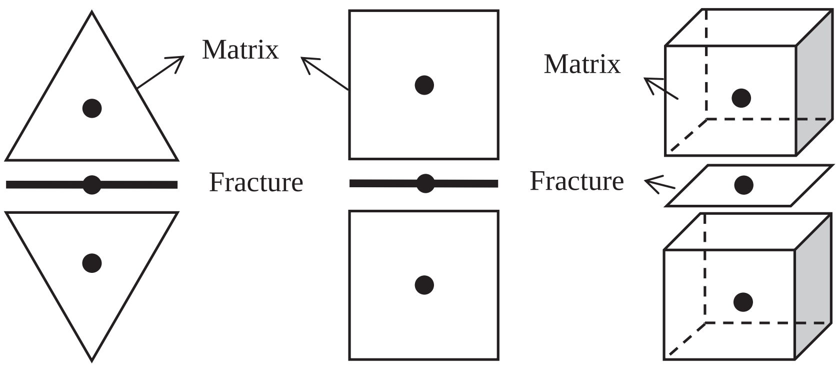


Fig 2. Treatment of coupling between matrix and fractures

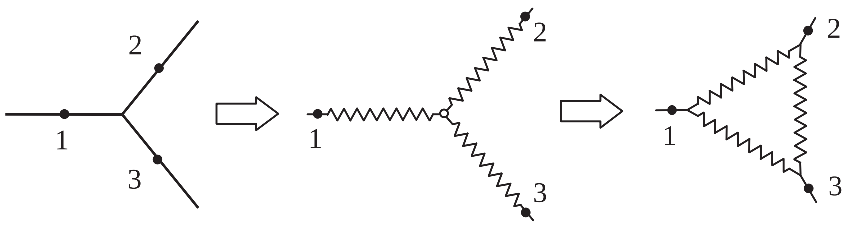
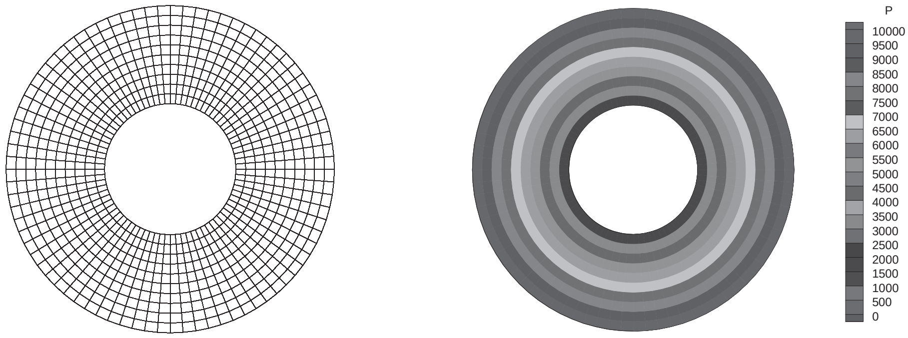


Fig 3. Fracture intersection – using the star-delta transformation [5]

NUMERICAL SIMULATIONS

Several numerical simulations are given to test the scheme.

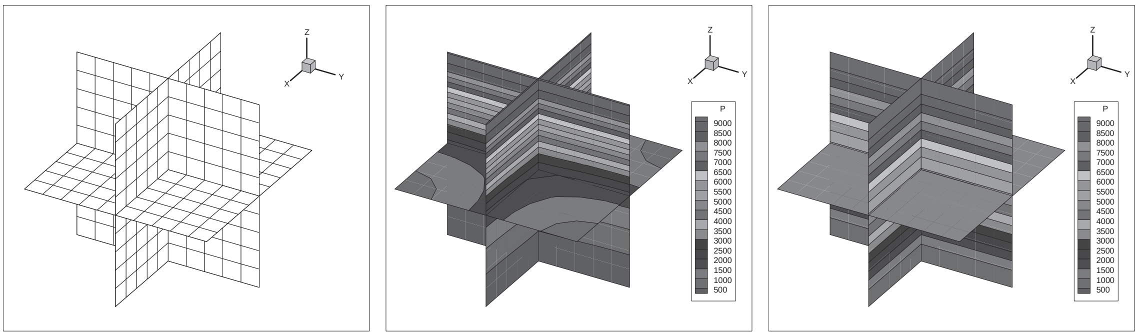
Case 1. At first an example of flow through matrix in three dimensions is shown below. It’s a ring flow and it has been tested right.



(a) Grids for calculation (b) Results from calculation by the scheme

Fig 4. A 3D case: grids and results of flow through matrix

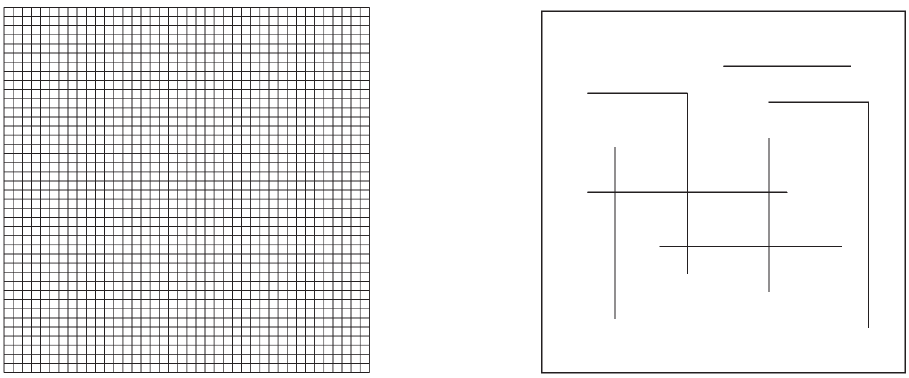
Case2. Secondly, we present an example of flow through fractures in three dimensions.



(a) Grids of fracture domain (b) Unsteady-state result (c) Steady-state result

Fig 5. A 3D case: grids and results of flow through fracture

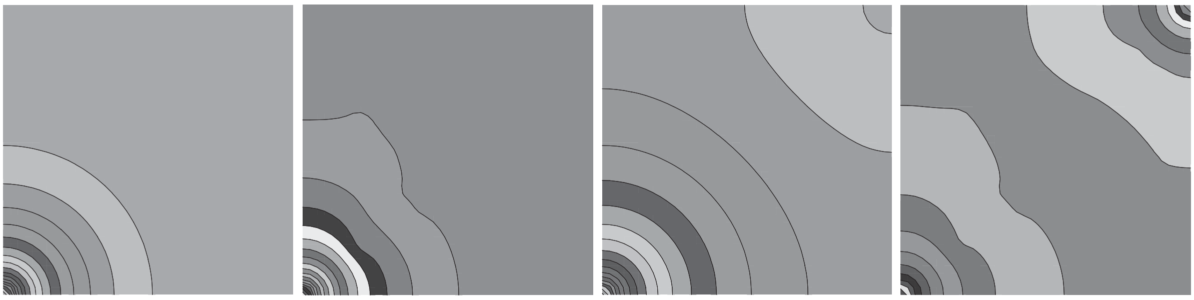
Case 3. At last, an example of structured grids in two dimensions is given below.



(a) Structured grids for calculation (b) Distribution of fractures in matrix

Fig 6. Grids for calculation in case 1

We give two different simulation results.



(a) t=5s Matrix (b) t=5s Matix-fracture (c) t=200s Matrix (d) t=200s Matrix-fracture

Fig 7. Results from Case 1: Matrix v.s. Matrix/Fracture

The results show that the fractures have a significant influence on flow. When fluid flows from matrix to fractures, the front becomes sharp as show in Fig 7.

CONCLUSION

The three cases we give show that this cell-centered finite volume scheme is suitable for calculation of single-phase flow in fractured porous media. The features of the presented method can be summarized as follows:

- Discrete fracture model in 2D and 3D domains.
- Structured/unstructured and hybrid grids.
- Fully coupled mass-conserving cell-centered finite volume discretization.
- Explicit time discretization.
- Steady/unsteady flow in matrix-matrix, matrix-fracture, and fracture-fracture.

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