A COMPUTATIONAL MODEL WITH FRACTURE ENERGY IN CONTINUUM-DISCONTINUUM ELEMENT METHOD

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Energy function with fracture energy, frictional energy, kinetic energy and elastic energy is proposed. And dynamic equations for continuum-discontinuum model in solid are obtained with the energy function and progressive failure constitutive model based on strain strength distribution. Combined with spring element model, computational scheme for spring force and nodal force in continuum-discontinuum element method is established with consideration of energy dissipation. Relationship between continuum mechanics model and discrete particles are discussed and the critical parameters are determined. Characteristic fracture length in representative volume element (RVE) is introduced, and theoretical expressions for progressive failure process of the element are given. Numerical examples such as uniaxial compression and uniaxial tension test are calculated with this model for verification.

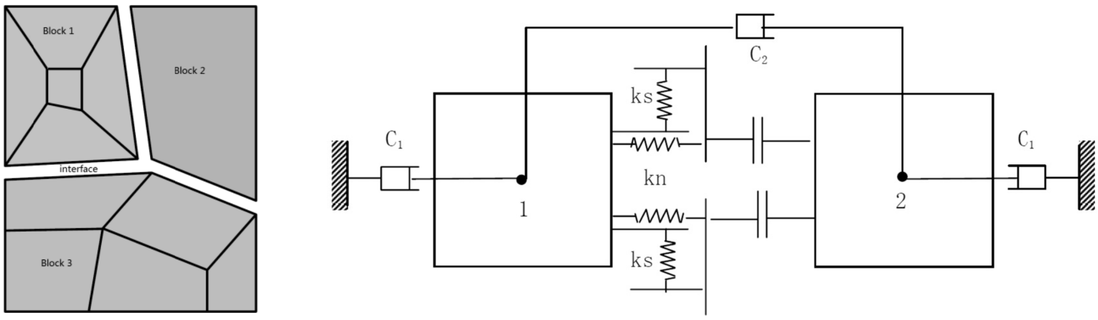
INTRODUCTION

In traditional discontinuum computational model or continuum-discontinuum coupled model, fracture in material is usually described with the breakage of springs. In these models, energy dissipation caused by fracture is not considered, and fracture process is transformed into energy transmission between kinetic energy and potential energy, which is apparently unreasonable. Especially when continuum system is modeled by particles, energy released from local fracture may cause chain-reaction, which could accelerate the failure process. So it is very important to introduce fracture energy into numerical method.

The objective of this paper is to introduce a new computational model in continuum-discontinuum element method. Different from the traditional discrete element method and original continuum-based discrete element method proposed by Li et al.[1,2], energy dissipation caused by facture is considered. A new energy function with fracture energy, kinetic energy and elastic energy is proposed, that can be adopted to spring element model[3], and dynamic equations for continuum-discontinuum model in solid are developed with the energy function and progressive failure constitutive model based on strain strength distribution proposed by Li and Zhou[4,5,6]. With spring element model, linear relationship between continuum mechanics system and discrete particles are established and critical parameters are determined. Algorithm for nodal force in element considering local fracture is given in uniform distribution assumption for strain strength. Characteristic fracture length in representative volume element is introduced and maximum fracture area in the element is defined. Based on these concepts, relationship between fracture area and volume is established, and expressions for progressive process of an element are developed. Program realization of these new models relies on VC++ and original CDEM code. Numerical examples are calculated in the last section for verification.

SOME BASIC MODELS AND THEORIES

Continuum-based discrete element method(CDEM). Continuum-based discrete element method (CDEM)[2] is a kind of numerical approach which is a combination of continuum element model and discrete element method. The algorithm is based on time-dependent explicit iteration using dynamic relaxation. Deformation of elements is calculated with continuum model such as finite element method, finite volume method or spring element method. While interface is calculated with bond spring model or contact spring model. Progressive failure process of materials or structures from continuous deformation, crack growth to totally breakdown can be modeled with this method. Some basic concepts and model are shown in Fig 1..



(a) Geometrical concepts (b)Interface model

Fig 1. concepts and models in CDEM

Spring element method(SEM). Spring element method[3] is a new approach to calculate the inner force and deformation of an element. A tetrahedron element consists of three basic orthogonal springs. Poisson and pure shear effect between springs are considered and only 15 independent stiffness parameters are enough to describe the linear mechanical behavior of the element. Result of spring element method is exactly consistent with finite element method, but the efficiency is greatly improved. Geometry of the spring element is shown in Fig 2..

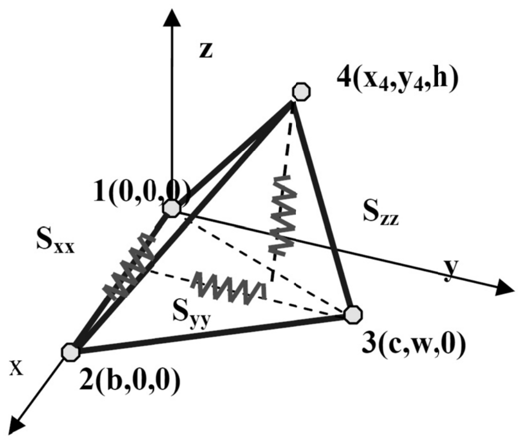


Fig 2. Geometry of spring element

Strain strength distribution criterion. Progressive failure constitutive model or criterion based on strain strength distribution was recently proposed by Li and Zhou[4,5,6]. This theory assumes that strain strength is distributed in material in mesoscopic view. Any section of representative volume element can be composed of elastic microplanes and fractured microplanes, as shown in Fig 3. Complicated macroscopic mechanical behavior of material such as nonlinear and strain softening is naturally obtained through mesoscopic fracture and interaction mechanism.

Fracture and friction are quantitatively determined in statistic way and with the definition of fractured microplanes.

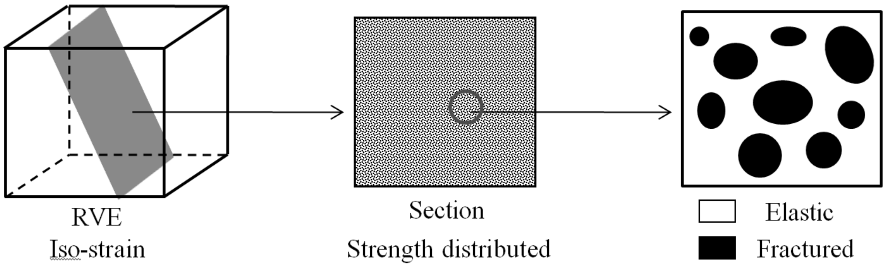


Fig 3. Concept of strain strength distribution criterion

EXPRESSION FOR FRACTURE ENERGY IN RVE

Characteristic fracture length and maximum fracture area. In order to describe fracture energy in an element quantitatively, characteristic fracture length should be defined according to the property of a material. Given distributive law for strain strength, fracture area can be calculated with characteristic fracture length.

Assume that the RVE is equivalent to a certain number of micro spheres with the same size, as shown in Fig 4. The number of micro spheres can be expressed as

$$n = \frac{V}{V_m} = \frac{6V}{\pi D^3} \tag{1}$$

Where V represents volume of the RVE, V_m is the volume of a micro sphere and D is the diameter of a micro sphere.

Strain strength of these micro spheres are distributed and each sphere can break only once through the center. Diameter of the microsphere is defined as the characteristic fracture length. Then maximum fracture area can be written as

$$S_{\max} = n\frac{\pi D^2}{4} = \frac{3V}{2D} \tag{2}$$

Fracture degree and fracture energy. Accoding to strain strength distribution criterion, fracture degree of RVE can be expressed as

$$D_b = \begin{cases} 0 & \varepsilon \le \varepsilon_{\min} \\ \int_{\varepsilon_{\min}}^{\varepsilon} f(\varepsilon)d\varepsilon & \varepsilon_{\min} < \varepsilon \le \varepsilon_{\max} \\ 1 & \varepsilon \ge \varepsilon_{\max} \end{cases} \tag{3}$$

Where $f(\varepsilon)$ is the distributive density function of strain strength, $\varepsilon_{\min}$ and $\varepsilon_{\max}$ represent the minmum and maximum strain strength respectively.

Current fracture area under a certain strain can be written as

$$S_f = D_b S_{\max} \tag{4}$$

And fracture energy can be written as

$$\Pi_G = G_f S_f = D_b G_f S_{\max} \tag{5}$$

Where G_f represents fracture energy per unit area.

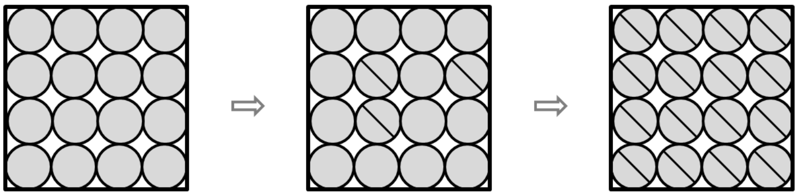


Fig 4. Determination of fracture area with micro sphere model

ENERGY FUNCTION IN CONTINUUM-DISCONTINUUM ELEMENT METHOD

Elastic system. The energy function for elastic system in CDEM can be written as

$$\begin{aligned}\Pi &= \Pi_e + \Pi_m + \Pi_\mu + \Pi_f + \Pi_{\bar{T}} \\ &= \int_V \sigma_{ij}\varepsilon_{ij} dV + \frac{1}{2}\int_V \rho \dot{u}_i^2 dV + \int_V \mu \dot{u}_i u_i dV - \frac{1}{2}\int_V f u_i dV - \int_S \bar{T} u_i ds\end{aligned} \tag{6}$$

According to Lagrange equation,

$$\frac{\partial}{\partial t}\left(\frac{\partial \Pi}{\partial \dot{u}_i}\right) - \frac{\partial \Pi}{\partial u_i} = 0 \tag{7}$$

Dynamic equation and boundary condition can be derived and written as

$$\sigma_{ij,j} + f_i - \rho \ddot{u}_i - \mu \dot{u}_i = 0 \tag{8}$$

$$\sigma_{ij} n_j - \bar{T} = 0 \tag{9}$$

With energy function above, nodal forces of the element can be expressed as

$$F_i = \frac{\partial \Pi}{\partial u_i} \tag{10}$$

Nonlinear system with damage and fracture energy dissipation. Energy function with damage and fracture energy is shown in equation(11).

$$\begin{aligned}\Pi &= \Pi_e^r + \Pi_m + \Pi_\mu + \Pi_f + \Pi_{\bar{T}} + \Pi_G \\ &= \int_V \bar{\sigma}_{ij}\varepsilon_{ij} dV + \frac{1}{2}\int_V \rho \dot{u}_i^2 dV + \int_V \mu \dot{u}_i u_i dV - \frac{1}{2}\int_V f u_i dV - \int_S \bar{T} u_i ds + D_b G_f S_{\max}\end{aligned} \tag{11}$$

Damaged stress can be determined $\bar{\sigma}_{ij}$ with progressive failure constitutive model proposed by Li and Zhou[6], and then the residual elastic energy is obtained. Assume that strain strength complies with uniform distributive law, in the progressive failure stage, frac-fracture energy is written as

$$\Pi_G = G_f S_{max} \frac{\varepsilon - \varepsilon_{min}}{\varepsilon_{max} - \varepsilon_{min}} \tag{12}$$

In spring element model, force of the basic springs caused by fracture can be expressed as partial derivation of relative displacement to fracture energy.

$$F_i^j = \frac{\partial \Pi_G}{\partial \Delta u_i^j} \tag{13}$$

Consider octahedral shear failure, the basic spring forces can be written as

$$F_i^j = \sqrt{\frac{2}{3}} \frac{G_f \overline{S}}{L_i \Delta\gamma} \cdot \frac{3\varepsilon_{ij} - \delta_{ij} I_1'}{\sqrt{J_2'}} \tag{14}$$

Where L_i represents the length of i_{th} basic spring, I_1' represents the first invariant of strain, J_2' represents the second invariant of strain deviator. $\Delta\gamma$ is the interval of shear strain strength distribution.

While in maximum tensile failure, the expression is

$$F_i^j = \frac{G_f \overline{S}}{L_i \Delta\varepsilon} \cdot N_{ij} \tag{15}$$

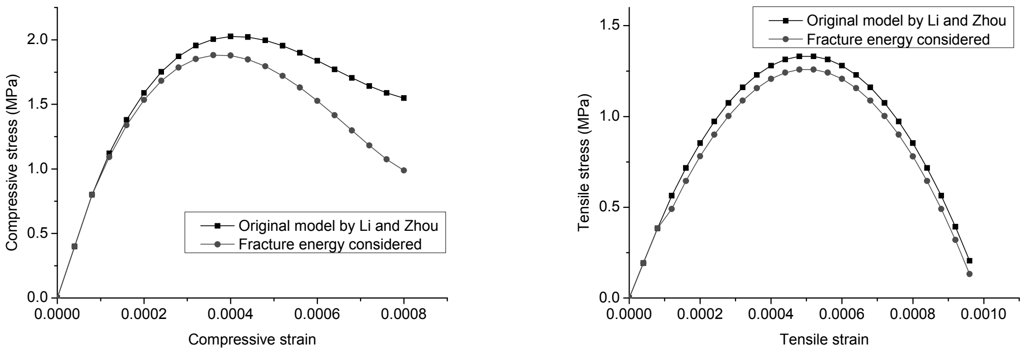
Where $N_{ij} = n_i n_j$ is the projective tensor of maximum tensile strain, $\Delta\varepsilon$ is the interval of tensile strain strength distribution.

PROGRAM REALAIZITION OF THESE MODELS AND ITS APPLICATIONS

The computational model is realized with spring element model and original CDEM code on VC++ platform. Uniaxial compression test and uniaxial tension test are simulated with the improve continuum-discontinuum element method. Numerical sample and computational result are shown in Fig 5. And stress-strain curves are shown in Fig 6., which indicate that the stress and strength are reduced when fracture energy is considered.

(a)Model and mesh (b) Compressive deformation (c)Tensile failure

Fig 5. Numerical model and cloud map of displacement



(a)Compression (b) Tension

Fig 6. Stress-strain curve of compression and tension simulated by continuum-discontinuum element method with different model: (a) Compression, (b) Tension.

CONCLUSION

A new computational model in continuum-discontinuum element method is introduced. A new energy function considering fracture is applied in this model. Characteristic fracture length is defined and relationship between fracture area and volume of the element is established. With consideration of fracture energy dissipation, questions for energy transmission on traditional spring model in DEM could be answered. Combined with strain strength distribution model and spring element model, continuum-discontinuum element method can solve wide range of nonlinear problems with damage and fracture effectively.

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