A Theoretical and Experimental Study of the Thermal Buckling Behaviour of the Fully-clamped Sandwich Panel with Metal-truss Core[1]

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This article presents a theoretical analysis and experimental study of thermal buckling behaviour of sandwich panels with metal truss cores under fully-clamped boundary conditions subject to uniform temperature rise. The theoretical analysis adopts Ressiner model and assumes the truss core is a continuous material. We ignore the flexural rigidity and bending stiffness of the core, consider the shear stiffness of the sandwich panel is only contributed by the core. By using the double Fourier expansion of the virtual deformation mode, we obtained the critical temperature of sandwich panels under clamped boundary conditions numerically. We carried out thermal buckling experiments on sandwich panels with a specifically designed fixture which can introduce in-plane load in the sandwich panel using thermal mismatch between the panel and the frame. High temperature strain gauges were used to measure the local strains of the panels from which the critical buckling temperature can be determined with Southwell method.

Keywords: *lattice material, sandwich panel, Ressiner model, thermal buckling*

1 Introduction

During the past few years there has been increasing interests for the design of light-weight structures. Sandwich panels with metal truss cores have significant multifunctional advantages such as light weight, high specific strength, high specific stiffness, thermal insulation and shock resistance. They have been considered as one of the most promising candidates for light-weight design and thermal protection structures utilized in the hypersonic aircraft. When being used as load bearing components in a thermal protection system, the sandwich panels often experience a large temperature change, and may buckle due to the in-plane load caused by the constrained thermal expansion. Therefore, it is necessary to carry out theoretical and experimental analysis on the thermal buckling of sandwich panels with metal truss cores.

Previous researches mainly focus on the analysis of buckling and post-buckling of thin plates and laminates. Mossavarali and Eslami[1] studied the post buckling of thin plate which has the initial flaws. Fu[2] solved the buckling problem of plates at various complex edge conditions by using reciprocal theorem. Su[3] analyzed the buckling of sandwich panels with metal truss cores under fully simply-supported conditions.

At present the most common truss cores are in pyramid, tetrahedral and Kagome configurations. There are plenty of studies on the analysis of fundamental mechanical properties of the sandwich panels with metal truss cores. Deshpande and Fleck [4] derived the three dimensional elastic constitutive relation of the lattice truss cores. But to our knowledge there are little theoretical analysis and experimental studies reported on thermal buckling of this kind of structure especially under fully-clamped boundary conditions subjected to uniform temperature rise.

So far, there have been a number of theoretical models on deformation of sandwich panels [5]. This article adopts fist-order deformation theory and assumes the truss core is continuous. By using the Ressiner model, the critical temperature is determined through the method of double Fourier expansion of the virtual deformation mode. Thermal buckling experiments are also carried out on sandwich panels by way of high temperature strain gauge technique.

2 Theoretical analysis

In this section a Ressiner model is developed to study the critical temperature at which the sandwich panels with lattice truss cores lose stability.

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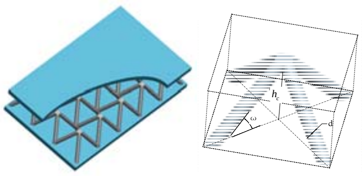
2.1 Model assumption

The object we studied is a sandwich panel with metal truss core as shown in Fig. 1. The face sheets and cores of the sandwich panels are made of isotropic materials, and the face sheets are very thin. Figure 2 shows the equivalent analytical model with a length a width b, the thickness of the cores h_c and the face sheets thickness h_p. Because the shear stiffness of the core is relatively small, and the flexural rigidity of the face sheets is large, the following assumptions are made:



(a) Sandwich panels (b) Pyramid unit cell

Figure 1: Schematic illustration of the sandwich panel with pyramid truss core (a) sandwich panel (b) pyramid unit cell

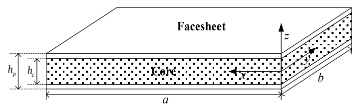


Figure 2: Equivalent analytical model

1).The size of the unit cell is very small in comparison with the size of the sandwich panels, so we assumes the truss core is a continuous material and is homogeneous in properties.

2).The truss cores are pin-jointed and make no contribution to the overall flexural rigidity.

3).The transverse shear stiffness of the face sheet is neglected.

4).The deformation of the sandwich panels is very small.

2.2 Fundamental equations

Under the application of a temperature load arising out of a uniform rise in temperature, the compressive in-plane force can expressed as:

$$N_x = N_y = -\frac{E\alpha}{1-\mu}(h_p - h_c)\Delta T = N\,,\quad N_{xy} = 0 \qquad (1)$$

Where E and α are the modulus of elasticity and coefficient of thermal expansion of the sandwich panels.

Following the assumptions above, the equations of the sandwich panels with metal truss cores can be obtained:

$$D(\frac{\partial^2\phi_x}{\partial x^2} + \frac{1-v}{2}\frac{\partial^2\phi_x}{\partial y^2} + \frac{1+v}{2}\frac{\partial^2\phi_y}{\partial x\partial y}) + C(\frac{\partial w}{\partial x} - \phi_x) = 0$$

$$D(\frac{\partial^2\phi_y}{\partial y^2} + \frac{1-v}{2}\frac{\partial^2\phi_y}{\partial x^2} + \frac{1+v}{2}\frac{\partial^2\phi_x}{\partial x\partial y}) + C(\frac{\partial w}{\partial y} - \phi_y) = 0$$

$$D(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} - \frac{\partial\phi_x}{\partial x} - \frac{\partial\phi_y}{\partial y}) + N\nabla^2 w = 0 \qquad (2)$$

where ϕ_x and ϕ_y are the rotations of the normal in the xz and yz planes, respectively, w is the displacement in the z direction, C and D are the shear stiffness and flexural rigidity of the sandwich panels respectively, and G_c is the equivalent shear modulus of the lattice truss core which can be derived through equivalent method.

Consider the sandwich panels to be full-clamped, the boundary conditions are written as:

$$x = 0,\ a: w = \phi_x = \phi_y = 0$$

$$y = 0,\ b: w = \phi_x = \phi_y = 0 \qquad (3)$$

The following solutions are assumed:

$$w = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn}\sin\alpha_m x\sin\beta_n y$$

$$\phi_x = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} B_{mn}\sin\alpha_m x\sin\beta_n y$$

$$\phi_y = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} C_{mn}\sin\alpha_m x\sin\beta_n y \qquad (4)$$

where A_{mn} , B_{mn} and C_{mn} are Fourier constant coefficients, and α_m and β_n are defined as $\frac{m\pi}{a}$ and $\frac{n\pi}{b}$ respectively.

Substituting Eq. (4) into Eq. (2), then we expand $\cos(\alpha_m x)$, $\cos(\beta_n y)$ and $\cos(\alpha_m x)\cos(\beta_n y)$ into the following forms [6]:

$$\cos(\alpha_m x)\cos(\beta_n y) =$$

$$\sum_{r=1}^{\infty}\sum_{s=1}^{\infty} h_{rm}h_{sn}\sin(\gamma_r x)\sin(\psi_s y), 0 < x < a,\quad 0 < y < b$$

$$\cos(\alpha_m x) = \sum_{r=1}^{\infty} h_{rm}\sin(\gamma_r x), 0 < x < a$$

$$\cos(\beta_n y) = \sum_{s=1}^{\infty} h_{sn}\sin(\psi_s y), 0 < y < b$$

where

$$h_{rm} = \frac{4m}{\pi(m^2 - r^2)},\ h_{sn} = \frac{4n}{\pi(n^2 - s^2)}$$

$$\gamma_r = \frac{r\pi}{a},\ \psi_s = \frac{s\pi}{b}. \qquad (5)$$

We can get the following:

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}(\frac{\mu-1}{2}D\beta_n^2 - D\alpha_m^2 - C)B_{mn} + \sum_{r=1}^{\infty} h_{rm}CA_{rn}\gamma_r +$$

$$\sum_{r=1}^{\infty}\sum_{s=1}^{\infty} h_{rm}h_{sn}\frac{1+\mu}{2}DC_{rs}\gamma_r\psi_s = 0$$

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\{(\frac{\mu-1}{2}D\alpha_m^2 - D\beta_n^2 - C)C_{mn} + \sum_{s=1}^{\infty} h_{sn}CA_{ms}\psi_s +$$

$$\sum_{r=1}^{\infty}\sum_{s=1}^{\infty} h_{rm}h_{sn}\frac{1+\mu}{2}DB_{rs}\gamma_r\psi_s\} = 0$$

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\{-[C(A_{mn}\alpha_m^2 + A_{mn}\beta_n^2)] - \sum_{s=1}^{\infty} h_{sn}CC_{ms}\psi_s$$

$$-\sum_{r=1}^{\infty} h_{rm}CB_{rn}\gamma_r\} - N\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}[(A_{mn}\alpha_m^2 + A_{mn}\beta_n^2)] = 0 \qquad (6)$$

A computer program is developed using FORTRAN to solve Eq. (6) in which the subroutine GVCRG of IMSL is used to solve the Eigen value problem.

2.3 Convergence characteristics

In order to study the convergence of this double Fourier expansion, critical temperatures are computed with *m*=*n*. Figure 3 shows the variation of critical temperature with the values of *m* and *n* increasing. A reasonable convergence is observed when $m = n \geq 10$. A good agreement of the analytically determined critical temperature with that from finite element method (FEM) is shown in Fig. 4. Table 1 shows the parameters in the numerical and finite element analysis.

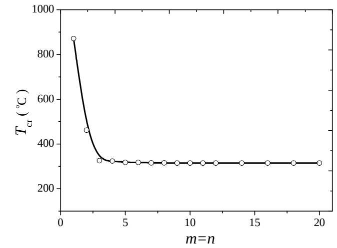


Figure 3: Convergence behavior of critical temperature versus *m* and *n*

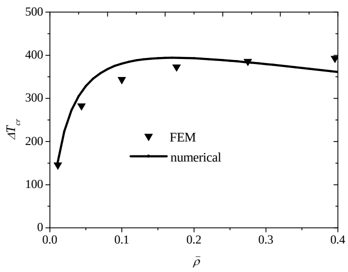


Figure 4: Comparison of theoretical and finite element analysis results

Table 1: Parameters in computing critical temperature

h_c (mm)	h_p (mm)	E (Gpa)	μ	α (10^{-5})	a (mm)	b (mm)
8	10	200	0.3	1.7	300	300

3 Thermal buckling experiments

3.1 Sandwich panel fabrication

A pyramid lattice core with a relative density of 3% was fabricated from 0.7 mm thick commercially available perforated stainless steel sheet. The perforated sheets were press brake formed using a 60º die angle to create a pyramidal truss structure which has some differences from the pyramidal configuration studied in the theoretical analysis above. The core was then bonderd to 0.9 mm thick stainless steel face sheets by braze welding to create the sandwich panel.

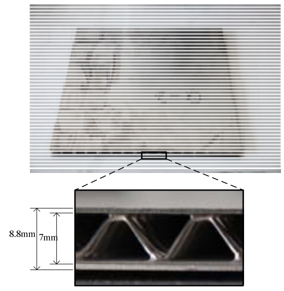


Figure 5: The sandwich panel

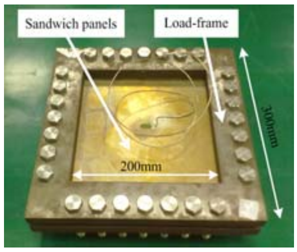


Figure 6: The assembly of sandwich panel and load frame

3.2 Experimental method

Figure 6 shows the assembly of sandwich panel and load frame to induce in-plane thermal loading. The frame has two identical pieces that are bolted together to provide a clamped boundary condition along the border of the sandwich panel. The central concept of the frame is to develop in-plane loads in the specimen by mismatched coefficient of thermal expansion (CTE). The sandwich panels are made of stainless steel and the load-frames are cast iron which has a lower CTE than the former. The panel-frame assembly was subjected to a uniform thermal loading in a furnace. The in-plane load is given by

$$N_T = \int \frac{E(z,t)(\alpha_{\text{frame}} - \alpha_{\text{panel}})\Delta T}{1-\mu} dz \tag{7}$$

where α_{panel} and α_{frame} are CTEs of panel and frame respectively.

Two high temperature strain gauges, orthogonal to each other, were attached in the center of the front and back surface of the specimen. The strain versus temperature history was monitored during the heating and the critical temperature for buckling was determined with a Southwell plot analysis [7] of the bifurcation point of strain gauge pair.

3.3 Method validation

To verify the accuracy of this technique, a square stainless steel of the thickness 2.7mm was placed in the frame and subjected to a quasi-statically increasing uniform temperature field. Table 2 shows the critical temperature measured from experiment and the theoretical prediction from thin plate theory. The data indicates the agreement between the theory and experiments is within 12%.

Table 2: Critical temperature of thin plate

—	ΔT_{cr}	Difference
Theory	68 ºC	—
Plate1	76 ºC	11.7%
Plate2	64 ºC	6.0%

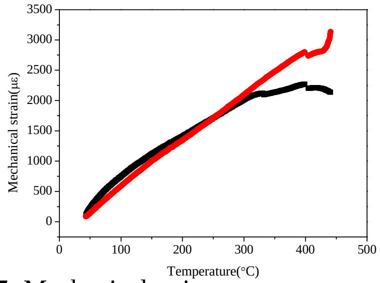


Figure 7: Mechanical trains versus temperature history at the center of the face sheet of sandwich panel

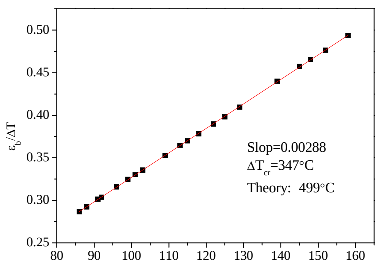


Figure 8: Southwell plot of sandwich panels

3.4 Buckling experiment of sandwich panels

The mechanical strains revised from the aggregate strains versus temperature history are shown in Figure 7. The strains of two stain gauges appear consistent at low temperature, but bifurcate around 300 ºC due to the buckling of the sandwich panels. The Southwell plot in Figure 8 shows the critical temperature is 347 ºC. Due to the boundary conditions of the sandwich panels tested in this experiment are not totally fully-clamped, the critical temperature should be obtained by using equation (7) as compressive in-plane force instead of equation (1). In addition, the cores of the specimen are not completely pyramid configuration. According to these factors, the critical temperature solved by the theoretical analysis should be 499 ℃, rather than 320 ℃ in Figure 3.

4. Conclusions

An analytical method for the thermal buckling of fully-clamped sandwich panels with metal truss cores subjected to a uniform temperature field is developed by using the Ressiner model. The theoretical results are in good agreement with those of finite element analysis. With the aid of a specifically designed load frame, the thermal buckling behavior of sandwich panel with pyramid truss core was investigated experimentally. The critical temperature for buckling was determined from strain measurements at the center of the face sheet by using the Southwell method.

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