Topological structure bifurcation of temperature field of deformable drop in Marangoni migration

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Abstract The unsteady processes of the Marangoni migration of deformable liquid drops are simulated numerically in a wider range of Marangoni number (up to $Ma = 500$) in the present work. A steady terminal state can always be reached, and the scaled terminal velocity is a monotonic function decreasing with increasing Marangoni number, which is generally in agreement with corresponding experimental data. The topological structure of flow field in the steady terminal state does not change as the Marangoni number increases, while bifurcation of the topological structure of temperature field occurs twice at two corresponding critical Marangoni numbers. A third critical value of Marangoni number also exists, beyond which the coldest point jumps from the rear stagnation to inside the drop though the topological structure of the temperature field does not change. It is found that the inner and outer thermal boundary layers may exist along the interface both inside and outside the drop if $Ma > 70$. But the thickness decreases with increasing Marangoni number more slowly than the prediction of potential flow at large Marangoni and Reynolds numbers. [doi:10.1063/2.1103205]

Keywords Marangoni migration, liquid drop, topological structure, thermal boundary layer

A fluid particle (gas bubble or liquid drop) floating in an immiscible bulk fluid with a temperature gradient can be moved by the non-uniform surface tension at the particle interface. This motion is well known as the thermocapillary or Marangoni migration. It is a classical problem in fluid mechanics playing an important role in many natural physical processes as well as a host of industrial activities, particularly in space material processing and many other scientific and engineering applications in microgravity. Thus, it attracts much interests of researchers all over the world along with the progress of human space activities.[1]

Marangoni migration was first analyzed by Young et al.[2] in the case of infinitesimal Reynolds and Marangoni numbers, in which convective transport of momentum and heat can be neglected compared with molecular transport of these quantities and the governing equations can thus be linearized. They derived the named YGB theory to predict the following steady migration velocity

$$V_{\mathrm{YGB}} = \frac{2U}{(2+3\alpha)(2+\beta)}, \tag{1}$$

where $U = -\sigma_T \nabla T_\infty R/\mu_1$ is the named thermocapillary velocity. σ_T, ∇T_∞ and R denote the variation of interfacial tension with the temperature, the temperature gradient imposed in the continuous bulk fluid, and the drop radius, respectively. $\alpha = \mu_2/\mu_1$ and $\beta = k_2/k_1$ are the dynamic viscosity ratio and the thermal conductivity ratio between the two phases. The subscripts 1 and 2 denote the continuous bulk fluid and the liquid drop, respectively. There are also many other dimensionless parameters, such as the Reynolds number $Re = UR/\nu_1$, the Marangoni number $Ma = UR/\lambda_1$, the Prandtl number $Pr = \nu_1/\lambda_1$, the capillary number $Ca = \mu_1 U/\sigma$, the density ratio $\xi = \rho_2/\rho_1$, and the thermal diffusivity ratio $\chi = \kappa_2/\kappa_1$. Not all of them are independent, for example, one can easily obtain $Ma = PrRe$.

The analysis of Young et al.[2] was extended by many others to include the convective factor. For example, using asymptotic expansion technique, the migration velocity of a nondeformable bubble or drop for small but non-zero convective heat transfer in the limit of zero Reynolds number was obtained by Subramanian.[3,4] He found that the migration velocity was reduced by the inclusion of the effect of convective transport of energy when Marangoni number was small. Balasubramaniam and Subramanian[5] found a linear increase of the migration velocity of a liquid drop with increasing Marangoni number in the case where the convective transport of energy is predominant both in the drop phase and in the continuous phase, i.e. when the Marangoni number and the Reynolds number are large enough. They employed the method of matched asymptotic expansions to solve the conjugate heat transfer problem in the two phases. Thin thermal boundary layers were assumed both outside and within the drop, while the velocity fields were given by a potential flow field in the continuous phase and a Hill's vortex inside the drop. However, experimental observations, e.g. Hadland et al.[6] and Xie et al.[7] did not support this prediction. A monotonous de-

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crease of the migration velocity of drop with increasing Marangoni number was observed. Because of the strong non-linearity of the problem with large Reynolds and Marangoni numbers, it can be observed experimentally only in microgravity environment in order to avoid the buoyant convection. Restricted by the scarce opportunity of space experiment, data are not sufficient, though experiments reported in the literature have covered a wide range of Reynolds and Marangoni numbers.

There are also lots of numerical simulations on this subject. The followings are some examples focusing upon the case of liquid drops. Ma et al.[8] analyzed numerically the quasi-steady Marangoni migration of a nondeformable spherical drop, and concluded that the migration velocity initially decreases as the Marangoni number increases, but increases after it reaches a certain value (around 50−200). They stated that the computed drop velocities at large Marangoni numbers were qualitatively in agreement with the prediction of Balasubramaniam and Subramanian.[5] Unfortunately, as pointed out later by Balasubramaniam and Subramanian,[5] a typographical error in their earlier version resulted in a lower velocity by a factor of 4. Using the front-tracking method, Yin et al.[9] investigated numerically the unsteady Marangoni migration of a nondeformable spherical drop. It has been found that fairly large Marangoni numbers may lead to fluctuations in the drop velocity at the beginning of simulations. A monotonous decrease of the terminal migration velocity, however, was observed with increasing Marangoni number. Using the level-set method to catch the interface, Haj-Hariri et al.[10] simulated numerically the unsteady Marangoni migration of a deformable liquid drop at finite Reynolds and Marangoni numbers (up to $Ma = 100$). They also found that the heat convection may retard the Marangoni migration of the drop, and that the terminal migration velocity decreases monotonously with increasing Marangoni number.

In the present paper, the unsteady processes of the Marangoni migration of deformable liquid drops are studied in a wider range of the Marangoni number (up to $Ma = 500$), while the major effort is concentrated on the steady terminal states, particularly on the topological structures of the flow and temperature fields in the drop. The axisymmetric model is adopted in our simulations, and the level-set method is employed to catch the interface between the drop and the continuous phase. The detailed numerical method is the same as that used in Zhao et al.[11] and not included here for brevity. Constant material parameters are assumed, namely $\xi = 1.89$, $\alpha = 0.14$, $\beta = 0.47$, $\chi = 0.69$, and $Pr = 83.3$, which are based on the fixing of the space experimental materials in Hadland et al.[6] and Xie et al..[7] Furthermore, a slightly larger value for the capillary number, i.e. Ca=0.2, is used here to accelerate the convergence of the numerical process and to prevent the virtual flow caused by the strong jump of the normal stress across the interface. Although the capillary number in the space experiments is of the order of 10^{-1} or

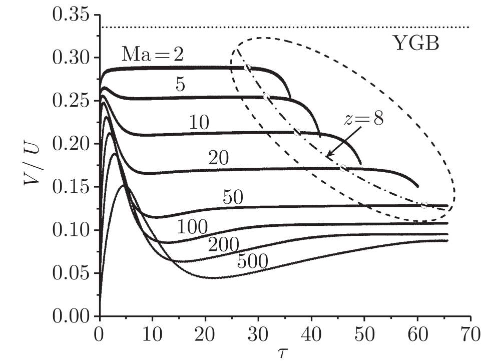


Fig. 1. Evolutions of the scaled migration velocity versus the dimensionless time.

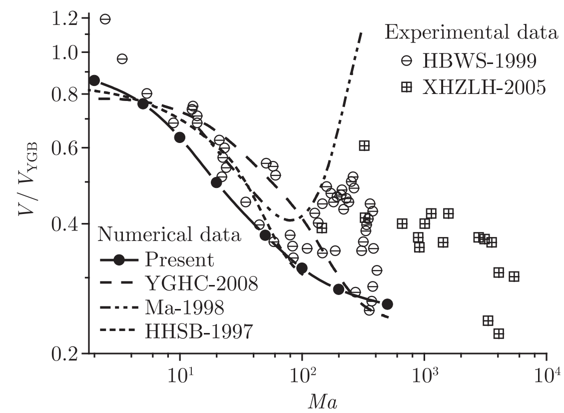
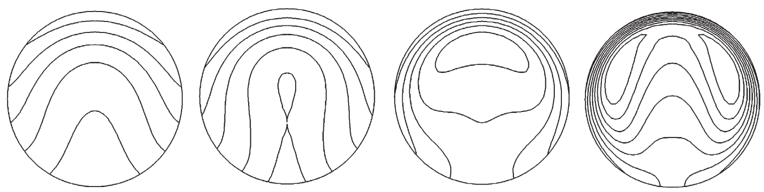


Fig. 2. Comparison of the present predictions of the steady terminal migration velocity with space experimental data and other numerical data reported in the literature.

less, the above value is still much less than 1 to guarantee not distinct deformation of the drop, and then not distinct difference caused by the drop deformation. It is verified by the computed results that the biggest variation of the aspect ratio between the longitudinal and transverse lengths of the drops at the steady terminal state is no more than 0.8 %.

Figure 1 shows the evolutions of the scaled migration velocity of the drops versus the dimensionless time at different Marangoni numbers. It oughts to be pointed out first that the fall of the scaled velocities in the marked range is due to the influence of the upper wall on the flow and temperature fields. Further analysis shows that this influence will not be evident unless the position of the drop center does not get across the line $z = 8$, or the dimensionless distance between the drop center and the upper wall is less than 4. Thus, a reasonable terminating distance oughts to be adopted before the wall influence becomes obvious. Notwithstanding this effect, it is still evident that a steady terminal state can always be reached for the Marangoni migration of the drops, and the scaled terminal velocity is a monotonic function decreasing with increasing Marangoni number.



(a) $Ma=5$ (b) $Ma=10$ (c) $Ma=50$ (d) $Ma=300$

Fig. 3. Evolution of topological structure of temperature field at different Marangoni numbers.

In Fig. 2, the terminal migration velocities, which are scaled using the corresponding values predicted by the YGB theory, are compared with the experimental data of Hadland et al.[6] and Xie et al.[7] These data are labeled respectively as HBWS-1999 and XHZLH-2005 for brevity. Generally, good agreements are evident. Numerical results of Ma et al.,[8] Yin et al.,[9] and Haj-Hariri et al.[10] labeled respectively as Ma-1998, YGHC-2008, and HHSB-1997, are also shown in Fig. 2. Everyone except the prediction of Ma et al.[8] shows a monotonic decrease of the terminal migration velocity with increasing Marangoni number. Considering the different values of physical parameters used in these simulations, the differences among the present results and those of Yin et al.[9] and Haj-Hariri et al.[10] are reasonable.

A Hill's vortex is formed inside the drop, which can be seen from the streamlines in the local reference frame attached to the center of the drop. The center of the vortex locates near the interface. Its transverse position in the local reference frame is not changed with the Marangoni number, while the longitudinal position moves downstream with increasing Marangoni number. The topological structure of flow field does not change as the Marangoni number increases.

On the contrary, bifurcation of the topological structure of temperature field occurs twice at certain critical values of Marangoni number (Fig. 3). According to the YGB theory, a uniform and straight layer-type structure of temperature field exists as $Re \to 0$ and $Ma \to 0$, and thus the coldest point in the drop locates at the rear stagnation. As the Marangoni number increases, the isotherm begins to wrap inside of the drop due to convective transport, and a distorted layer-type structure appears (Fig. 3a). The coldest point in the drop still locates at the rear stagnation. Beyond the first critical value of Marangoni number, a local cooler zone appears around the center of the drop, and thus the first bifurcation of the topological structure of temperature field occurs (Fig. 3b). The coldest point, however, still locates at the rear stagnation. Further increase of the Marangoni number results in reduction of the temperature inside the local cooler zone, and also in expansion of this zone, particularly in the transverse direction. After the Marangoni number exceeds the second critical value, a cap-type structure of the local cooler zone can be observed (Fig. 3c), and the coldest point jumps to inside the drop, even locates at a position above the center of the drop though the topological structure of temperature field does not change. With further increasing Marangoni number, the cap-type structure of the local cooler zone moves upwards with a transverse expanding and a longitudinal shrinking, leading to a shell-type one. The center part of the shell-type cooler zone inside the drop will become thinner and thinner. It will finally rupture from the central point and then form a torus-type one if the Marangoni number exceeds the third critical value (Fig. 3d), which corresponds to the second bifurcation of the topological structure of temperature field. Then the coldest line, not a sole point, inside the drop will depart from the symmetrical axis of the drop. Figure 4 shows the coldest locations inside the drop at different Marangoni numbers.

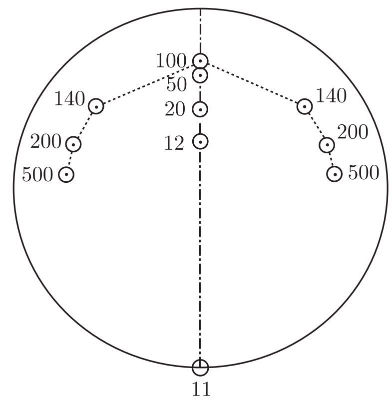


Fig. 4. Movement of the coldest point inside the drops with increasing Marangoni number.

If the concepts of the inner and outer thermal boundary layers attached to the interface are introduced formally, the thickness along the interface can be determined straightforwardly. Figure 5 shows the variation of the thickness of the inner and outer thermal boundary layers versus the attack angle of $\pi/3$, which decrease with increasing Marangoni number. There is no doubt that the inner and outer thermal boundary layers exist in the common meaning if $Ma > 70$. It is obvious that the thickness of the outer thermal boundary layer $\delta_T \sim O\left(Ma^{-2/3}\right)$, and a negative, smaller exponent very close to 0 can be found for the inner one. $\delta_T \sim O\left(Ma^{-1}\right)$, however, was assumed for both inner and outer thermal boundary layers in Balasubramaniam

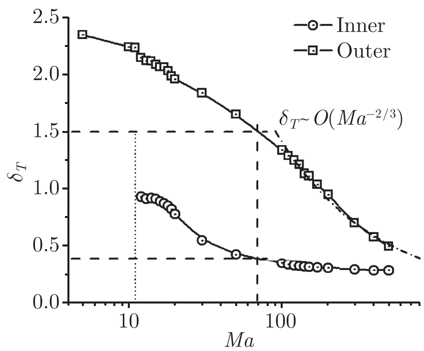


Fig. 5. Thickness of the inner and outer thermal boundary layers at different Marangoni numbers.

and Subramanian.[5] The maximum of Reynolds number in the present work is about 6, which is not large enough for the retention of potential flow in the continuous phase. This may be the reason for the fact that the trend of the migration velocity is inconsistent with the prediction of Balasubramaniam and Subramanian.[5]

The above thermal structures have also been obtained in the previous numerical simulations, but no allegation on detailed process of its evolution has been made in the literature. It is observed that the evolution of the topological structure of temperature field reported here is consistent with that in Haj-Hariri et al.[10] in the same range of Marangoni number. However, a relatively slower evolution was found in Yin et al.,[9] while a much quicker one in Ma et al..[8] In the latter work, the second bifurcation of the topological structure of temperature field occurred at a much smaller Marangoni number, and then the thermal structure at $Ma = 100$ (shown in Fig. 9 of Ma et al.[8]) was close to that at $Ma \approx 500$ found in the present work. Thus, much quicker decrease of the thickness of thermal boundary layers both inside and outside the drop was observed in Ma et al..[8] Although it is not clear for the reason of the difference, the validity of the present results may be guaranteed by the agreements with other numerical simulations as well as with experimental data.

In summary, the unsteady processes of the Marangoni migration of deformable liquid drops are simulated numerically in a wider range of Marangoni number (up to $Ma = 500$) in the present work. A steady terminal state can always be reached, and the scaled terminal velocity is a monotonic function decreasing with increasing Marangoni number, which is generally in agreement with space experimental data and some previous numerical predictions except those of Ma et al..[8] The topological structures of the flow and temperature fields in the drop are analyzed in detail. The topological structure of flow field does not change as the Marangoni number increases, while bifurcation of the topological structure of temperature field occurs twice at two critical Marangoni numbers. A third critical value of the Marangoni number also exists, beyond which the coldest point jumps from the rear stagnation to inside the drop though the topological structure of the temperature field does not change. The existence and the thickness of thermal boundary layers along the interface both inside and outside the drop are also discussed. It is found that the inner and outer thermal boundary layers may exist if $Ma > 70$. But the thickness decreases with increasing Marangoni number more slowly than the prediction of potential flow at large Marangoni and Reynolds numbers.

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