Acta Mechanica Sinica (2014) 30(4):480–484
DOI 10.1007/s10409-014-0049-2

RESEARCH PAPER

A multifractal model for linking Lagrangian and Eulerian velocity structure functions

Yu-Feng Dong · Guo-Dong Jin

Received: 12 December 2013 / Accepted: 3 January 2014

Abstract A multifractal model is developed to connect the Lagrangian multifractal dimensions with their Eulerian counterparts. We propose that the characteristic time scale of a Lagrangian quantity should be the Lagrangian time scale, and it should not be the Eulerian time scale which was widely used in previous studies on Lagrangian statistics. Using the present model, we can obtain the scaling exponents of Lagrangian velocity structure functions from the existing data or models of scaling exponents of Eulerian velocity structure functions. This model is validated by comparing its prediction with the results of experiments, direct numerical simulations, and the previous theoretical models. The comparison shows that the proposed model can better predict the scaling exponents of Lagrangian velocity structure functions, especially for orders larger than 6.

Keywords Lagrangian multifractal · Eulerian multifractal · Intermittency · Velocity structure functions

1 Introduction

Understanding the statistical properties of fully developed turbulent flows from the Lagrangian viewpoint is naturally connected with the problems of turbulent dispersion of contaminants and turbulent mixing of passive scalars [1, 2]. However, investigation of turbulent flows from the Lagrangian viewpoint is a challenging problem in both experiments and direct numerical simulations (DNS) since it needs to follow the trajectories of tracer particles and resolve time scales ranging from Kolmogorov time scale τ_η to integral time scale T [3]. The ratio of the largest time scale to the smallest one is $T/\tau_\eta \sim Re^{1/2}$ which increases with Reynolds number, where Re is the Reynolds number based on the integral length scale L. In addition, the width of Lagrangian inertial range $T/\tau_\eta \sim Re^{1/2}$ is smaller than the Eulerian inertial range $L/\eta_{\rm K} \sim Re^{3/4}$ (here $\eta_{\rm K}$ is the Kolmogorov length scale), which implies that higher Reynolds numbers flows are needed to investigate the Lagrangian statistics of turbulence [2]. This requirement raises challenges to current DNS and particle detection techniques in experiments. However, the Eulerian quantities have been intensively studied [4–8]. Therefore, it is of great significance to build a bridge between the easily measured or known Eulerian statistics and the difficultly measured or unknown Lagrangian statistics. The recent development on space-time correlation models [9–12] is the work aligned in this direction.

In recent years, progresses have been made in experimental measurements [13–17] and numerical simulations [18–23] of Lagrangian statistics (see Ref. [3] for a review and comparison of experimental and numerical data). These experimental and DNS results provide theorists the chance to study the Lagrangian properties of turbulence, and motivate a number of theoretical studies on Lagrangian turbulence [24–31]. Under this background, Borgas [24], Boffetta et al. [25], and Biferale et al. [26] have bridged the Lagrangian and Eulerian velocity increments statistics by using the multifractal formalism [32]. Schmitt [33] reviewed these Lagrangian multifractal formalism: Borgas used a corollary of ergodic hypothesis [24, 34] and assumed the characteristic time scale of fluid particles along trajectories is the Eulerian decorrelation time scale $\tau_{\rm E} = (\nu/\bar{\epsilon})^{1/2}$ (which is identical to the Kolmogorov time scale), where $\bar{\epsilon}$ is the globally mean energy dissipation rate per unit mass, ν is the kinematic viscosity; Boffetta et al. [25] and Biferale et al. [26]

The project was supported by the National Natural Science Foundation of China (11072247, 11021262, and 11232011), National Natural Science Associate Foundation of China (NSAF) (U1230126), and 973 program of China (2013CB834100).

Y.-F. Dong · G.-D. Jin (✉)
LNM, Institute of Mechanics,
Chinese Academy of Sciences, 100190 Beijing, China
e-mail: gdjin@lnm.imech.ac.cn

assumed the time lag τ in Lagrangian velocity structure functions $\langle(\delta_\tau v_i)^p\rangle \equiv \langle[v_i(t+\tau) - v_i(t)]^p\rangle$ has the same order as Eulerian characteristic time scale, i.e., $\tau \sim r/\delta_r u$, and $\delta_r u \sim \delta_\tau v_i$, where $\delta_r u \equiv [\boldsymbol{u}(\boldsymbol{x}+\boldsymbol{r}) - \boldsymbol{u}(\boldsymbol{x})] \cdot \boldsymbol{r}/r$, $r = |\boldsymbol{r}|$, v_i ($i = x, y, z$) is the velocity component along a single particle trajectory, and the average $\langle\cdot\rangle$ is defined over the ensemble of trajectories of fluid particles. Due to isotropy and stationarity, the Lagrangian velocity structure functions $\langle(\delta_\tau v_i)^p\rangle$ are only dependent on the time lag τ.

In the studies of Borgas, Boffetta et al. and Biferale et al. [24–26], a physical assumption that the Lagrangian characteristic time scale is approximately equal to the Eulerian characteristic time scale has been used. However, one can find that the ratio of the Lagrangian time microscale $\tau_{\mathrm{L}} = V(\nu/\overline{\epsilon}^3)^{1/4}$ (here V is the root-mean-square of Eulerian velocity fluctuations which is independent of Reynolds number) [35, 36] to its Eulerian counterpart τ_{E}, $\tau_{\mathrm{L}}/\tau_{\mathrm{E}} \sim \nu^{-1/4} \sim Re^{1/4}$, increases with Reynolds number. Therefore, the assumption that the Lagrangian characteristic time scale approximates to its Eulerian counterpart may cause errors in building the bridge between the Lagrangian and Eulerian statistics, especially at high Reynolds numbers.

In this paper, we propose a group of more reasonable assumptions, instead of the assumptions that were used in Refs. [24–26], to develop a new model for linking the Lagrangian and Eulerian velocity structure functions. Our assumptions are based on that the characteristic time scale of Lagrangian quantities should be the Lagrangian decorrelation time scale, not be the Eulerian decorrelation time scale. Those assumptions are used to develop Lagrangian multifractal models for dissipation rates (Sect. 2) and velocity increments (Sect. 3). The model for dissipation rates can be transformed to the model for velocity increments (Sect. 4). The results obtained are validated by comparing their prediction with the experimental and DNS results.

2 A Lagrangian multifractal model for dissipation rates

In this section, we will develop a Lagrangian multifractal model for dissipation rates. It is assumed in the multifractal model for dissipation rates that the local dissipation rate ϵ_α at viscous length scale η_α, the local dissipation rate ϵ_κ at viscous time scale τ_κ, and their probability density functions (PDFs) are scaled as

$$\begin{aligned} \epsilon_\alpha &\sim \overline{\epsilon}\left(\frac{\eta_\alpha}{L}\right)^{\alpha-1}, \quad P(\epsilon_\alpha) \sim \left(\frac{\eta_\alpha}{L}\right)^{1-f(\alpha)}, \\ \epsilon_\kappa &\sim \overline{\epsilon}\left(\frac{\tau_\kappa}{T}\right)^{\kappa-1}, \quad P(\epsilon_\kappa) \sim \left(\frac{\tau_\kappa}{T}\right)^{1-\tilde{f}(\kappa)}, \end{aligned} \tag{1}$$

where $f(\alpha)$ is the Eulerian fractal dimension spectrum, and $\tilde{f}(\kappa)$ is the Lagrangian fractal dimension spectrum [24, 32].

To relate Lagrangian multifractal with Eulerian multifractal, Borgas assumed that

$$\langle(\epsilon_\alpha)^q\rangle = \langle(\epsilon_\kappa)^q\rangle, \tag{2}$$

$$\eta_\alpha = (\nu^3/\epsilon_\alpha)^{1/4}, \tag{3}$$

$$\tau_\kappa = (\nu/\epsilon_\kappa)^{1/2}. \tag{4}$$

Equation (2) is the consequence of the ergodic hypothesis (see Ref. [24] or [34]); Equation (3) is based on the Kolmogorov length scale $\eta_{\mathrm{K}} = (\nu^3/\overline{\epsilon})^{1/4}$; Equation (4) is based on the Eulerian decorrelation time scale $\tau_{\mathrm{E}} = (\nu/\overline{\epsilon})^{1/2}$, which is also identical to the Kolmogorov time scale [35, 36]. Equations (3) and (4) lead to

$$\eta_\alpha = LRe^{-3/(3+\alpha)}, \tag{5}$$

$$\tau_\kappa = TRe^{-1/(\kappa+1)}. \tag{6}$$

Borgas uses Eqs. (5) and (6) and the saddle-point approximation to evaluate $\langle(\epsilon_\alpha)^q\rangle$ and $\langle(\epsilon_\kappa)^q\rangle$

$$\langle(\epsilon_\alpha)^q\rangle \sim \overline{\epsilon}^q Re^{-3[(\alpha-1)q+1-f(\alpha)]/(3+\alpha)},$$

$$\langle(\epsilon_\kappa)^q\rangle \sim \overline{\epsilon}^q Re^{-[(\kappa-1)q+1-\tilde{f}(\kappa)]/(\kappa+1)}.$$

Substituting the above results into Eq. (2) and comparing the exponents of Re in the expressions of $\langle(\epsilon_\alpha)^q\rangle$ and $\langle(\epsilon_\kappa)^q\rangle$, Borgas obtained a one-to-one relation between Lagrangian multifractal dimension spectrum $\tilde{f}(\kappa)$ and Eulerian multifractal dimension spectrum $f(\alpha)$

$$\tilde{f}(\kappa) = -\frac{1}{2}\kappa + \left(1 + \frac{1}{2}\kappa\right)f(\alpha),$$

$$\alpha = \frac{3\kappa}{\kappa+2}.$$

We propose that the characteristic time scale of Lagrangian velocity structure functions should be the Lagrangian characteristic time scale instead for the Eulerian characteristic time scale. Noting the Lagrangian characteristic time scale $\tau_{\mathrm{L}} = V(\nu/\overline{\epsilon}^3)^{1/4}$, we assume

$$\tau_\kappa = V(\nu/\epsilon_\kappa^3)^{1/4} \tag{7}$$

to replace Eq. (4). Equation (7) can be rewritten as the function of Re

$$\tau_\kappa = TRe^{-1/(3\kappa+1)}. \tag{8}$$

We use Eq. (8) and the saddle-point approximation to obtain

$$\langle(\epsilon_\kappa)^q\rangle \sim \overline{\epsilon}^q Re^{-[(\kappa-1)q+1-\tilde{f}(\kappa)]/(3\kappa+1)}.$$

Comparing the exponents of Re in the expressions of $\langle(\epsilon_\alpha)^q\rangle$ and $\langle(\epsilon_\kappa)^q\rangle$, we finally obtain a model that connects the Lagrangian multifractals with the Eulerian multifractals

$$\tilde{f}(\kappa) = -2\kappa + (2\kappa+1)f(\alpha), \tag{9}$$

$$\alpha = \frac{3\kappa}{2\kappa+1}. \tag{10}$$

3 A Lagrangian multifractal model for velocity increments

The Lagrangian multifractal model in last section is for dissipation rates. In this section, we will use velocity increments to develop the Lagrangian multifractal model. It is assumed that

$$\delta_r u \sim V\left(\frac{r}{L}\right)^{h_\mathrm{E}}, \qquad P(\delta_r u) \sim \left(\frac{r}{L}\right)^{3-D_\mathrm{E}(h_\mathrm{E})},$$
$$\delta_\tau v_i \sim V\left(\frac{\tau}{T}\right)^{h_\mathrm{L}}, \qquad P(\delta_\tau v_i) \sim \left(\frac{\tau}{T}\right)^{1-D_\mathrm{L}(h_\mathrm{L})}. \tag{11}$$

Here, h_E and h_L are Eulerian and Lagrangian Hölder exponents that characterize the velocity increments, $\delta_r u$ and $\delta_\tau v_i$, respectively; $D_\mathrm{E}(h_\mathrm{E})$ and $D_\mathrm{L}(h_\mathrm{L})$ are the Eulerian and Lagrangian fractal dimensions that characterize the PDFs of the Eulerian and Lagrangian velocity increments, $P(\delta_r u)$ and $P(\delta_\tau v_i)$, respectively.

In the following part, we will derive the Lagrangian multifractal model for velocity increments. Equation (10) in combination with Eqs. (1), (3), and (7) implies that $\epsilon_\alpha = \epsilon_\kappa$ in the dissipation range. Borgas [24] gives a simple physical interpretation of this identity that the dissipation rate at one point, ϵ_α (say), must occur at some instant (with value ϵ_κ) in a sufficiently long Lagrangian record (see page 397 of Ref. [24]). Following this ergodic hypothesis, we assume that, in the inertial range

$$\epsilon_r = \epsilon_\tau. \tag{12}$$

Here, ϵ_r is the local average of energy dissipation rate over a volume of typical size r [24, 32, 37], ϵ_τ is the local energy dissipation rate averaged along the trajectory over a time interval of scale τ [24, 37], r and τ are spatial and temporal separations in the inertial range. Equation (12) in combination with Kolmogorov's refined similarity hypothesis $\epsilon_r \sim (\delta_r u)^3/r$ (see page 164 of Ref. [32]) and its Lagrangian counterpart $\epsilon_\tau \sim (\delta_\tau v_i)^2/\tau$ [37] could provide a connection between Eulerian velocity increment and Lagrangian velocity increment

$$\frac{(\delta_r u)^3}{r} = \frac{(\delta_\tau v_i)^2}{\tau} \Rightarrow r^{3h_\mathrm{E}-1} \sim \tau^{2h_\mathrm{L}-1}. \tag{13}$$

Here, we still need a relationship between r and τ [33]. Instead for Biferale's assumption $\tau \sim r/\delta_r u$, we assume that

$$\frac{\tau^2}{r/\delta_r u} \sim \frac{V^2}{\epsilon_r}. \tag{14}$$

Equation (14) is based on the relationship between Lagrangian time scale $\tau_\mathrm{L} = V(\nu/\bar{\epsilon}^3)^{1/4}$ and Eulerian time scale $\tau_\mathrm{E} = (\nu/\bar{\epsilon})^{1/2}$

$$\frac{\tau_\mathrm{L}^2}{\tau_\mathrm{E}} \sim \frac{V^2}{\bar{\epsilon}}. \tag{15}$$

According to Eq. (14) and Kolmogorov's refined similarity hypothesis $\epsilon_r \sim (\delta_r u)^3/r$, we obtain

$$\tau \sim \frac{r}{(\delta_r u)^2} \sim r^{1-2h_\mathrm{E}}. \tag{16}$$

From Eqs. (13) and (16), we could obtain

$$h_\mathrm{E} = \frac{2h_\mathrm{L}}{4h_\mathrm{L}+1}, \tag{17}$$

and

$$\frac{\delta_\tau v_i}{V} \sim \left(\frac{\tau}{T}\right)^{h_\mathrm{L}} \sim \left(\frac{r}{L}\right)^{h_\mathrm{E}/2} \sim \left(\frac{\delta_r u}{V}\right)^{1/2}. \tag{18}$$

Equation (18) in combination with Eq. (16) implies that

$$\langle(\delta_\tau v_i)^p\rangle \sim \langle(\delta_r u)^{p/2}\rangle \sim \int \left(\frac{r}{L}\right)^{(p/2)h_\mathrm{E}+3-D_\mathrm{E}(h_\mathrm{E})} \mathrm{d}h_\mathrm{E} \sim \int \left(\frac{\tau}{T}\right)^{[(p/2)h_\mathrm{E}+3-D_\mathrm{E}(h_\mathrm{E})]/(1-2h_\mathrm{E})} \mathrm{d}h_\mathrm{E}. \tag{19}$$

According to Eq. (19), we could obtain $\langle(\delta_\tau v_i)^p\rangle \sim \tau^{\zeta_p^\mathrm{L}}$ with

$$\zeta_p^\mathrm{L} = \inf_{h_\mathrm{E}}\left[\frac{(p/2)h_\mathrm{E}+3-D_\mathrm{E}(h_\mathrm{E})}{1-2h_\mathrm{E}}\right]. \tag{20}$$

Equation (20) in combination with the Eulerian fractal dimension spectrum $D_\mathrm{E}(h_\mathrm{E})$ could provide a model for scaling exponents of Lagrangian velocity structure functions.

In Lagrangian multifractal theory, the scaling exponents of Lagrangian velocity structure functions, ζ_p^L, are related to the Lagrangian fractal dimension spectrum, $D_\mathrm{L}(h_\mathrm{L})$, by the Legendre transform [17]

$$\zeta_p^\mathrm{L} = \inf_{h_\mathrm{L}}[ph_\mathrm{L} + 1 - D_\mathrm{L}(h_\mathrm{L})]. \tag{21}$$

Comparing Eqs. (20) and (21), we finally obtain

$$D_\mathrm{L}(h_\mathrm{L}) = -4h_\mathrm{L} + (4h_\mathrm{L}+1)[D_\mathrm{E}(h_\mathrm{E}) - 2]. \tag{22}$$

Equations (22) and (17) provide a model for connection between Lagrangian and Eulerian multifractals for velocity increments.

4 Transformation between multifractal formalism for dissipation rates and velocity increments

The multifractal formalism for dissipation rates and velocity increments could be related by Kolmogorov's refined similarity hypothesis. The transformation between the two types of Eulerian multifractal formalism for dissipation rates and velocity increments are as follows (for details of Eulerian transformation, see page 164 of Ref. [32])

$$f(\alpha) = D_\mathrm{E}(h_\mathrm{E}) - 2, \qquad h_\mathrm{E} = \frac{\alpha}{3}. \tag{23}$$

Using Lagrangian counterpart of Kolmogorov's refined similarity hypothesis, $\epsilon_\tau \sim (\delta_\tau v_i)^2/\tau$ [37], we can obtain the following transformation between the two types of Lagrangian multifractal formalism for dissipation rates and velocity increments

$$\tilde{f}(\kappa) = D_\mathrm{L}(h_\mathrm{L}), \qquad h_\mathrm{L} = \frac{\kappa}{2}. \tag{24}$$

According to Eqs. (23) and (24), we can rewrite our model which is for dissipation rates, Eqs. (9) and (10), as

$$D_\mathrm{L}(h_\mathrm{L}) = -4h_\mathrm{L} + (4h_\mathrm{L}+1)(D_\mathrm{E}(h_\mathrm{E}) - 2),$$
$$h_\mathrm{E} = \frac{2h_\mathrm{L}}{4h_\mathrm{L}+1}.$$

While Borgas' model is transformed into

$$D_\mathrm{L}(h_\mathrm{L}) = -h_\mathrm{L} + (h_\mathrm{L}+1)(D_\mathrm{E}(h_\mathrm{E}) - 2),$$
$$h_\mathrm{E} = \frac{h_\mathrm{L}}{h_\mathrm{L}+1}.$$

We can see from the transformation that Borgas' model is equivalent to the model proposed by Boffetta et al. [17, 24–26]. Actually, both the assumption of Borgas, $\tau_\kappa = (\nu/\epsilon_\kappa)^{1/2}$, and the assumption of Boffetta et al., $\tau = r/\delta_r u$, have the same physical meaning: the Lagrangian time scale is approximately equal to the Eulerian time scale. We can also see from the transformation that our model for dissipation rates, Eqs. (9) and (10), is equivalent to the one for velocity increments, Eqs. (22) and (17).

5 Discussions

The Eulerian fractal dimension $D_\mathrm{E}(h_\mathrm{E})$ in Eq. (22) is related to the Eulerian structure functions scaling exponents ζ_p^E by the Legendre transform [32] $D_\mathrm{E}(h_\mathrm{E}) = \inf_p(ph_\mathrm{E} + 3 - \zeta_p^\mathrm{E})$, where ζ_p^E can be obtained from experimental data [4] or theoretical models, such as She–Leveque (S–L) model [5] $\zeta_p^\mathrm{E} = p/9 + 2[1 - (2/3)^{p/3}]$. Combining the S–L model with the present model, we obtain the scaling exponents of Lagrangian velocity structure functions which are listed in Table 1.

We compare the prediction of our model with the results of experiments, DNS, and previous theoretical models (see Table 1 and Fig. 1). Figure 1 shows that the DNS and experimental data support both the present model and the model proposed by Borgas and Boffetta et al. [24–26], for order $p < 6$. However, for order $p \geqslant 6$, the DNS of Benzi et al. [21] and experiment of Mordant et al. [14] support only the present model which is constructed based on the Lagrangian characteristic time scale rather than the model based on Eulerian characteristic time scale [24–26].

Table 1 Data of the scaling exponents of Lagrangian velocity structure functions from DNS, experiments and model prediction

	EXP1	EXP2	DNS1	DNS2	DNS3	M1	M2	M3
$\zeta_1^\mathrm{L}/\zeta_2^\mathrm{L}$	0.56 ± 0.01	0.56 ± 0.01			0.57 ± 0.09	0.60	0.55	
$\zeta_3^\mathrm{L}/\zeta_2^\mathrm{L}$	1.34 ± 0.02	1.34 ± 0.02			1.35 ± 0.13	1.31	1.38	
$\zeta_4^\mathrm{L}/\zeta_2^\mathrm{L}$	1.56 ± 0.06	1.58 ± 0.06	1.6 ± 0.1	1.66 ± 0.02	1.65 ± 0.17	1.58	1.71	1.66
$\zeta_5^\mathrm{L}/\zeta_2^\mathrm{L}$	1.8 ± 0.2	1.76 ± 0.1				1.80	2.00	
$\zeta_6^\mathrm{L}/\zeta_2^\mathrm{L}$		1.9 ± 0.2	2.0 ± 0.1	2.10 ± 0.10		2.00	2.26	2.14
$\zeta_7^\mathrm{L}/\zeta_2^\mathrm{L}$						2.18	2.50	
$\zeta_8^\mathrm{L}/\zeta_2^\mathrm{L}$				2.33 ± 0.17		2.35	2.72	2.45
$\zeta_9^\mathrm{L}/\zeta_2^\mathrm{L}$						2.50	2.93	
$\zeta_{10}^\mathrm{L}/\zeta_2^\mathrm{L}$				2.45 ± 0.35		2.64	3.12	2.64

EXP1 ($Re_\lambda = 740$, $10\tau_\eta < \tau < T$, Mordant et al. [13]); EXP2 ($Re_\lambda = 1\,000$, $10\tau_\eta < \tau < T$, Mordant et al. [14]); DNS1 ($Re_\lambda = 284$, $10\tau_\eta \leqslant \tau \leqslant 50\tau_\eta$, Biferale et al. [20]); DNS2 ($Re_\lambda = 600$, $10\tau_\eta < \tau < T$, Benzi et al. [21]); DNS3 ($Re_\lambda = 400$, $10\tau_\eta < \tau < 100\tau_\eta$, Huang et al. [22]); M1 (Present model combined with S–L model [5]); M2 (Model by Borgas et al. [24–26] combined with S–L model [5]); M3 (Model by Zybin et al. [30] combined with DNS data of Benzi et al. [21])

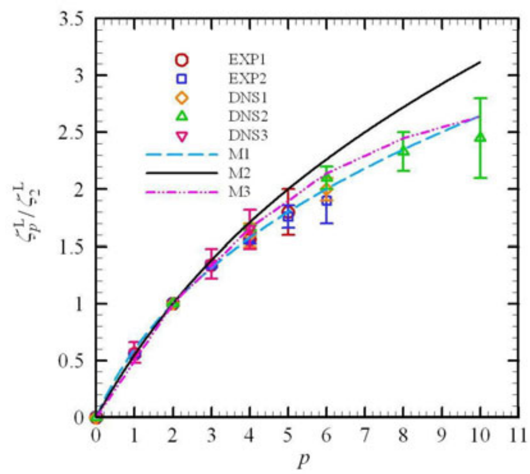

Fig. 1 Normalized scaling exponents $\zeta_p^\mathrm{L}/\zeta_2^\mathrm{L}$ of the Lagrangian velocity structure functions as a function of order p

6 Summary

In this paper, we develop a model to connect Lagrangian multifractal dimensions with Eulerian ones. Since the multifractal dimensions can be used to calculate the scaling exponents of velocity structure functions, we use the present model to calculate the scaling exponents of Lagrangian velocity structure functions from Eulerian velocity structure functions. The proposed model and assumptions are validated by comparing their prediction with experiments, DNS and previous theoretical models. The comparison shows that the scaling exponents predicted by the present model are in good agreement with the ones by experiments and DNS up to order 10, while the model based on Eulerian characteristic time scale [24–26] gives increasing deviations from experiments and DNS for orders larger than 6. The results obtained in this paper can be used to investigate the Eulerian time correlation models for turbulence-generated noise [38, 39] and Lagrangian time correlation models for particle-laden turbulence [40–42].

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