

Whole-Process Modeling of Reservoir Turbidity Currents by a Double Layer-Averaged Model

Zhixian Cao¹; Ji Li²; Gareth Pender³; and Qingquan Liu⁴

Abstract: Turbidity current is formed as subaerial open-channel sediment-laden flow plunges into a reservoir. The whole process of reservoir turbidity current, i.e., formation, propagation, and recession, is generally controlled by the water and sediment inputs from upstream and also the reservoir operation scheme specifying the downstream boundary condition. Enhanced understanding of reservoir turbidity current is critical to effective sediment management in alluvial rivers. However, until now there has been a lack of physically based and practically feasible models for resolving the whole process of reservoir turbidity current. This is because the computing cost of three-dimensional modeling is excessively high. Also, single layer-averaged models cannot resolve the formation process characterized by the transition from open-channel sediment-laden flow to subaqueous turbidity current, or the upper clear-water flow as dictated by the operation scheme of the reservoir, which has significant impacts on turbidity current. Here a new two-dimensional double layer-averaged model is proposed to facilitate for the first time whole-process modeling of reservoir turbidity current. The two hyperbolic systems of the governing equations for the two layers are solved separately and synchronously. The model is well balanced because the interlayer interactions are negligible compared with inertia and gravitation, featuring a reasonable balance between the flux gradients and the bed or interface slope source terms and thus applicable to irregular topographies. The model is benchmarked against a spectrum of experimental cases, including turbidity currents attributable to lock-exchange and sustained inflow. It is revealed that an appropriate clear-water outflow is favorable for turbidity current propagation and conducive to improving sediment flushing efficiency. This is significant for optimizing reservoir operation schemes. As applied to turbidity current in the Xiaolangdi Reservoir in the Yellow River, China, the model successfully resolves the whole process from formation to recession. The present work facilitates a viable and promising framework for whole-process modeling of turbidity currents, in support of reservoir sediment management. DOI: 10.1061/(ASCE)HY.1943-7900.0000951. © 2014 American Society of Civil Engineers.

Author keywords: Reservoir; Turbidity current; Sedimentation; Sediment flushing; Double layer-averaged model; Reservoir management.

Introduction

Reservoir turbidity current is subaqueous, sediment-laden underflow. It is formed when subaerial, open-channel sediment-laden flow plunges into a reservoir. Like other gravity currents, it is driven by the density difference from the ambient fluid. More importantly, the whole process of reservoir turbidity current, i.e., formation, propagation and recession, is generally controlled by the water and sediment inputs from upstream and also the reservoir operational scheme specifying the downstream boundary condition. In this sense, reservoir turbidity currents are distinct from self-accelerating turbidity currents in ocean environments (Parker et al. 1986). In general, turbidity currents can travel remarkable distances carrying large amounts of suspended sediment from

the plunge point to the downstream. In reservoirs, turbidity currents are often the governing processes for the transport, entrainment, and deposition of sediment (Fan and Morris 1992a). If the turbidity currents can manage to arrive at the dam, it will be possible to flush sediment out of the reservoir. Otherwise, severe sedimentation in the reservoir will generally occur. Enhanced understanding of the whole process of reservoir turbidity currents is critical to effective sediment and reservoir management.

Previous studies have focused on the threshold condition for the formation of turbidity current based on the densimetric Froude number at the plunge point, derived from laboratory experiments (Fan 1960; Singh and Shah 1971) or analytical models (Savage and Brimberg 1975; Akiyama and Stefan 1984; Parker and Toniolo 2007; Dai and Garcia 2009; Li et al. 2011). However, these cannot provide enough and effective information to quantify the whole process of reservoir turbidity currents. Also, laboratory experiments have been used to investigate the evolutionary characteristics of turbidity currents (Hürzeler et al. 1996; Lee and Yu 1997; Gladstone et al. 1998; Hallworth and Huppert 1998) and the vertical structure of the currents (Altinakar et al. 1996; Eggenhuisen and McCaffrey 2012; Nourmohammadi et al. 2011). Especially, Lee and Yu (1997) performed a series of tests to investigate the formation and propagation of turbidity currents attributable to sustained inflow and in particular demonstrated the impacts of various upstream and downstream boundary conditions. Also, they investigated the densimetric Froude number at both the incipient and stable plunge points. Yet flume experiments are constrained by the relatively small spatial scales. Interestingly, a series of large field-scale water-sediment regulation experiments has been undertaken by the Yellow River Water Resources Commission (YRCC) (2007)

¹Professor, State Key Laboratory of Water Resources and Hydropower Engineering Science, Wuhan Univ., Wuhan 430072, China; and Professor, Institute for Infrastructure and Environment, Heriot-Watt Univ., Edinburgh EH14 4AS, U.K. (corresponding author). E-mail: zxcao@whu.edu.cn

²Ph.D. Student, State Key Laboratory of Water Resources and Hydropower Engineering Science, Wuhan Univ., Wuhan 430072, China.

³Professor, Institute for Infrastructure and Environment, Heriot-Watt Univ., Edinburgh EH14 4AS, U.K.; and Visiting Professor, State Key Laboratory of Water Resources and Hydropower Engineering Science, Wuhan Univ., Wuhan 430072, China.

⁴Professor, Institute of Mechanics, Chinese Academy of Sciences, Beijing 100190, China.

Note. This manuscript was submitted on October 14, 2013; approved on August 28, 2014; published online on October 20, 2014. Discussion period open until March 20, 2015; separate discussions must be submitted for individual papers. This paper is part of the *Journal of Hydraulic Engineering*, © ASCE, ISSN 0733-9429/04014069(19)/\$25.00.

since 2002. Turbidity currents were formed in the Xiaolangdi Reservoir by plunging of the sediment-laden floods released from the Sanmenxia Reservoir at the upstream and believed to be the most important means for sediment flushing. A significant volume of hydrological data were collected, concerning the location of the front, average velocity, and sediment concentration of the turbidity currents. The field experiments are certainly essential in support of not only practical sediment management, but also development and applications of analytical and computational models.

Computational modeling is certainly attractive, because detailed processes of reservoir turbidity currents can be resolved. Generally, there have been two categories of computational models for turbidity currents, i.e., depth-resolving models and layer-averaged models. Depth-resolving models, including three-dimensional (3D) and vertical two-dimensional (2D) models, can reproduce the evolution process and the vertical structure of turbidity currents (e.g., Bournet et al. 1999; De Cesare et al. 2001; Kassem and Imran 2001; Kassem et al. 2003; Khan et al. 2005; Huang et al. 2007, 2008; Georgoulas et al. 2010; An and Julien 2014). And indeed, there have been modeling efforts to resolve the formation of turbidity currents (Kassem and Imran 2001; De Cesare et al. 2001; Georgoulas et al. 2010). Nevertheless, depth-resolving models require excessively high computational costs and thus are unrealistic for applications to large-scale prototype turbidity currents such as those in the Xiaolangdi Reservoir, Yellow River in China. This holds true for general 3D models for fluvial flow and sediment transport (Fang and Wang 2000; Wu et al. 2000). Also, the physics of turbulent turbidity currents, on which the model closures are based, is still poorly understood. In particular, it remains far from clear how to incorporate the effects of sediment into turbulence closures, even for steady and uniform sediment-laden flows in open channels. Comparatively, layer-averaged models are easier to formulate and solve. Layer-averaged refers to the fact that the physical quantities (velocity and sediment concentration) are averaged along the thickness of the turbidity currents. However, to date, most existing layer-averaged models for turbidity currents are single layer-averaged models, in which the upper clear-water flow is ignored and not modeled at all (e.g., Fukushima et al. 1985; Parker et al. 1986; Choi 1998; Bradford and Katopodes 1999a, b; Sequeiros et al. 2009; Hu et al. 2012; Lai and Wu 2013). Thus they are restricted to modeling the propagation of turbidity currents after their formation. In applications, this category of models hinges upon observed data to specify the upstream boundary conditions (e.g., Hu et al. 2012), which, however, are not generally available. Critically, single layer-averaged models cannot resolve the formation process characterized by the transition from subaerial open-channel sediment-laden flow to subaqueous turbidity current, or the upper clear-water flow dictated by the operation scheme of the reservoir. In contrast to these limitations, it is fundamental to resolve the formation process of reservoir turbidity current, not only scientifically but also practically. Equally importantly, the operation scheme of the reservoir has been found to have significant impacts on turbidity currents (Lee and Yu 1997). Until now there have been no layer-averaged models capable of modeling the whole process of reservoir turbidity currents, which is critical to effective sediment and reservoir management. In this connection, double layer-averaged models have been developed for general gravity currents (Rottman and Simpson 1983; Bonneau et al. 1993; Hallworth et al. 2003; Ungarish and Zemach 2005; Adduce et al. 2012; La Rocca et al. 2012). Two sets of layer-averaged equations are deployed to describe, respectively, the gravity current layer and the upper clear-water flow layer. It is also noted that Dallimore et al. (2004) has employed a coupled 3D hydrodynamics and 2D layer-averaged model to simulate the formation and subsequent propagation

process of reservoir saline underflows. However, most existing double layer-averaged models have not incorporated sediment transport and morphological evolution, which, however, are key features of reservoir turbidity currents over mobile bed. The 1D double layer-averaged model by Bonneau et al. (1993) takes into account sediment deposition, but ignores bed sediment entrainment and morphological evolution. Also, it is limited to lock-exchange gravity currents without inflow from the upstream or outflow at the downstream boundary. Strictly, the lock-exchange gravity current is not reservoir turbidity current that is normally dictated by the water and sediment inputs from the upstream and outflow at the downstream.

A new 2D well-balanced double layer-averaged model is developed here to facilitate, for the first time, the whole-process modeling of reservoir turbidity currents. The model is extended from the recent 1D nonwell-balanced double layer-averaged model (Li et al. 2013). The two hyperbolic systems of the governing equations for the two layers are solved separately and synchronously. Each hyperbolic system is solved by a quasiwell-balanced numerical algorithm involving drying and wetting, using a second-order accurate Godunov-type finite volume method in conjunction with the Harten-Lax-van Leer contact wave (HLLC) approximate Riemann solver. The model is benchmarked against a spectrum of experimental turbidity currents induced by lock-exchange (Bonneau et al. 1995) and sustained inflow (Lee and Yu 1997). A pilot study is presented of large-scale prototype turbidity current in the Xiaolangdi Reservoir (Li 2004; YRCC 2007), Yellow River in China to demonstrate the capability of the present model.

Mathematical Model

Governing Equations

In general, there exist interactions among the upper layer clear water flow, turbidity current and the erodible bed, which are characterized by mass and momentum exchanges. Coupled modeling is generally justified, which has been implemented in recent single layer-averaged models (Hu and Cao 2009; Hu et al. 2012). This is followed in the present 2D double layer-averaged model. The governing equations of the model are essentially 2D shallow water equations comprising the mass and momentum conservation equations, respectively, for the clear-water flow layer and the turbidity current layer, and also the mass conservation equations for sediment in the turbidity current layer and bed sediment, respectively. By analogy to a 1D case (Li et al. 2013), these are

$$\frac{\partial \mathbf{T}}{\partial t} + \frac{\partial \mathbf{E}}{\partial x} + \frac{\partial \mathbf{F}}{\partial y} = \mathbf{R}_b + \mathbf{R}_f + \mathbf{R}_e \quad (1)$$

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{G}}{\partial x} + \frac{\partial \mathbf{H}}{\partial y} = \mathbf{S}_b + \mathbf{S}_f + \mathbf{S}_e \quad (2)$$

$$\mathbf{T} = \begin{bmatrix} \eta \\ q_{wx} \\ q_{wy} \end{bmatrix} = \begin{bmatrix} \eta \\ h_w u_w \\ h_w v_w \end{bmatrix} \quad (3a)$$

$$\mathbf{E} = \begin{bmatrix} h_w u_w \\ h_w u_w^2 + 0.5g(\eta^2 - 2\eta\eta_s) \\ h_w u_w v_w \end{bmatrix} \quad (3b)$$

$$\mathbf{F} = \begin{bmatrix} h_w v_w \\ h_w u_w v_w \\ h_w v_w^2 + 0.5g(\eta^2 - 2\eta\eta_s) \end{bmatrix} \quad (3c)$$

$$\mathbf{R}_b = \begin{bmatrix} 0 \\ -g\eta \frac{\partial \eta_s}{\partial x} \\ -g\eta \frac{\partial \eta_s}{\partial y} \end{bmatrix} \quad (3d)$$

$$\mathbf{R}_f = \begin{bmatrix} 0 \\ -\frac{\tau_{wx}}{\rho_w} - E_w u_w \\ -\frac{\tau_{wy}}{\rho_w} - E_w v_w \end{bmatrix} \quad (3e)$$

$$\mathbf{R}_e = \begin{bmatrix} -E_w + \frac{\partial \eta_s}{\partial t} \\ 0 \\ 0 \end{bmatrix} \quad (3f)$$

$$\mathbf{U} = \begin{bmatrix} \eta_s \\ q_{sx} \\ q_{sy} \\ q_c \end{bmatrix} = \begin{bmatrix} \eta_s \\ h_s u_s \\ h_s v_s \\ h_s c_s \end{bmatrix} \quad (4a)$$

$$\mathbf{G} = \begin{bmatrix} h_s u_s \\ h_s u_s^2 + 0.5g(\eta_s^2 - 2\eta_s z_b) \\ h_s u_s v_s \\ h_s u_s c_s \end{bmatrix} \quad (4b)$$

$$\mathbf{H} = \begin{bmatrix} h_s v_s \\ h_s u_s v_s \\ h_s u_s^2 + 0.5g(\eta_s^2 - 2\eta_s z_b) \\ h_s v_s c_s \end{bmatrix} \quad (4c)$$

$$\mathbf{S}_b = \begin{bmatrix} 0 \\ -g\eta_s \frac{\partial z_b}{\partial x} \\ -g\eta_s \frac{\partial z_b}{\partial y} \\ 0 \end{bmatrix} \quad (4d)$$

$$\mathbf{S}_f = \begin{bmatrix} 0 \\ N_x \\ N_y \\ E - D \end{bmatrix} \quad (4e)$$

$$\mathbf{S}_e = \begin{bmatrix} E_w \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (4f)$$

$$N_x = \frac{\tau_{wx} - \tau_{bx}}{\rho_c} - \frac{\rho_w g}{\rho_c} h_s \frac{\partial h_w}{\partial x} - \frac{(\rho_0 - \rho_c)(E - D)u_s}{(1 - p)\rho_c} + \frac{(\rho_s - \rho_w)c_s u_s E_w}{\rho_c} + \frac{\rho_w E_w u_w}{\rho_c} - \frac{(\rho_s - \rho_w)gh_s^2}{2\rho_c} \frac{\partial c_s}{\partial x} \quad (4g)$$

$$N_y = \frac{\tau_{wy} - \tau_{by}}{\rho_c} - \frac{\rho_w g}{\rho_c} h_s \frac{\partial h_w}{\partial y} - \frac{(\rho_0 - \rho_c)(E - D)v_s}{(1 - p)\rho_c} + \frac{(\rho_s - \rho_w)c_s v_s E_w}{\rho_c} + \frac{\rho_w E_w v_w}{\rho_c} - \frac{(\rho_s - \rho_w)gh_s^2}{2\rho_c} \frac{\partial c_s}{\partial y} \quad (4h)$$

$$\frac{\partial z_b}{\partial t} = -\frac{E - D}{1 - p} \quad (5)$$

where \mathbf{T} and \mathbf{U} are the conserved variables; \mathbf{E} , \mathbf{F} , \mathbf{G} and \mathbf{H} are the flux variables. For the turbidity current layer, \mathbf{S}_b denotes bed slope source term, \mathbf{S}_f includes bed friction source terms and other terms related to the impacts of sediment transport and water entrainment, \mathbf{S}_e represents water entrainment source term. Similarly, for the clear-water flow layer, \mathbf{R}_b features interface slope source term, \mathbf{R}_f comprises interface friction source terms and other terms related to impacts of water entrainment, \mathbf{R}_e represents water entrainment source term and variations of interface elevation; q_{wx} and q_{wy} are conservative variables in Eq. (3); q_{sx} , q_{sy} , and q_c conservative variables in Eq. (4); t is the time; g the gravitational acceleration; x and y are the horizontal coordinates; η is the water surface elevation; η_s the elevation of interface between the clear-water layer and turbidity current layer; h_w the depth of clear-water layer; u_w and v_w are the layer-averaged velocities of clear-water layer in the x - and y -directions; h_s is the depth of turbidity current layer; u_s and v_s are the layer-averaged velocities of turbidity current layer in the x - and y -directions; c_s is the layer-averaged volumetric sediment concentration of the turbidity current layer; z_b is bed elevation; p the bed sediment porosity; ρ_w and ρ_s are the densities of water and sediment, respectively; $\rho_c = \rho_w(1 - c_s) + \rho_s c_s$ is the density of the water-sediment mixture in the turbidity current layer; $\rho_0 = \rho_w p + \rho_s(1 - p)$ is the density of the saturated bed; E and D are the sediment entrainment and deposition fluxes across the bottom boundary, respectively; τ_{wx} and τ_{wy} are the shear stresses at the interface between the clear-water flow layer and the turbidity current layer in the x - and y -directions; and τ_{bx} and τ_{by} are bed shear stresses in the x - and y -directions; and E_w is the mass flux of water entrainment across the interface between the two layers.

Model Closure

To close the governing equations of the present 2D double layer-averaged model, a set of relationships has to be introduced to determine the boundary resistance, interface shear stress, and water entrainment and sediment exchange fluxes. Generally, unsteady and nonuniform flows may experience boundary resistances substantially different from those of steady and uniform flows. This is more pronounced when sediment transport is involved, which renders the bed movable and bedforms generated. However, no generally applicable relationships are currently available to represent boundary resistance in such flows. This is also the case for the interface shear stress, for which there has been different empirical relationships (Chen and Peng 2006; Kim and LeVeque 2008; Zech et al. 2008; Lee et al. 2014). Consequently, computational studies of turbidity currents over fixed and mobile beds continue to use resistance relationships initially developed for steady and uniform flows, which are usually based on the Manning's equation.

This practice is followed for the present 2D double layer-averaged model by virtue of the conventional empirical relations

$$\tau_{wx} = \rho_w g n_w^2 (u_w - u_s) \bar{U}_{ws} / h_w^{1/3} \quad (6a)$$

$$\tau_{wy} = \rho_w g n_w^2 (v_w - v_s) \bar{U}_{ws} / h_w^{1/3} \quad (6b)$$

$$\tau_{bx} = \rho_c g n_b^2 u_s \bar{U}_s / h_s^{1/3} \quad (7a)$$

$$\tau_{by} = \rho_c g n_b^2 v_s \bar{U}_s / h_s^{1/3} \quad (7b)$$

where n_w is the roughness at the interface between the turbidity current layer and clear-water flow layer; n_b the roughness of the bed; $\bar{U}_s = \sqrt{u_s^2 + v_s^2}$, the resultant velocity of the turbidity current layer; and $\bar{U}_{ws} = \sqrt{(u_w - u_s)^2 + (v_w - v_s)^2}$, the resultant velocity difference between the two layers.

The mass flux of water entrainment E_w represents the mixing of the turbidity current with the clear water across the interface of two layers. It is determined by (Parker et al. 1986)

$$E_w = e_w \bar{U}_{ws} \quad (8)$$

where the water entrainment coefficient, e_w , is calculated empirically using the Richardson number $Ri = g' h_s / \bar{U}_{ws}^2$ and the submerged gravitational acceleration $g' = s g c_s$ with specific gravity of sediment $s = \rho_s / \rho_w - 1$

$$e_w = \frac{0.00153}{0.0204 + Ri} \quad (9)$$

The sediment deposition flux can be approximated by the product of the local near-bed sediment concentration and the sediment settling velocity. Sediment entrainment is assumed to occur at the same rate as it does under capacity conditions. Thus

$$E = \omega E_s \quad (10a)$$

$$D = \omega c_b \quad (10b)$$

where ω is the settling velocity of a single sediment particle in tranquil clear water, calculated using Zhang's formula (Zhang and Xie 1993); $c_b = r_b c_s$ the local near-bed concentration, r_b can be determined by (Parker et al. 1986)

$$r_b = 1 + 31.5 \left(\sqrt{u_*^2 + v_*^2} / \omega \right)^{-1.46} \quad (11)$$

where u_* and v_* are the bed shear velocities in the x - and y -directions. The parameter E_s is the near-bed concentration at capacity condition. Parker et al. (1986) proposed the following empirical formulation:

$$E_s = \begin{cases} 0.3 & \psi \geq 13.2 \\ 3 \times 10^{-12} \psi^{10} (1 - 5\psi^{-1}) & 5.0 < \psi < 13.2 \\ 0 & \psi \leq 5.0 \end{cases} \quad (12)$$

where $\psi = \sqrt{(u_*^2 + v_*^2)} \sqrt{sgd^3} / \nu / \omega$, d is medium sediment particle diameter, and ν is kinematic viscosity of water. In the present study, the Zhang and Xie (1993) formula is also adopted, which is well tested and widely used for suspended sediment transport of open channel flow in the Yellow River (Zhang and Xie 1993). A correction coefficient α is introduced because the

present study is concerned with turbidity currents. Following the logarithmic-matching treatment of Guo (2002), the Zhang and Xie (1993) formula reads

$$E_s = \alpha \frac{1}{20\rho_s} \frac{(\bar{U}_s^3 / gh_s \omega)^{1.5}}{1 + (\bar{U}_s^3 / 45gh_s \omega)^{1.15}} \quad (13)$$

Numerical Algorithm

Although there have been analyses of double layer-averaged models over fixed beds regarding hyperbolicity and Kelvin-Helmholtz instability, Eqs. (1)–(5) constitute a nonlinear system of eight partial differential equations, involving more equations than previously analyzed (e.g., Kim and LeVeque 2008; Castro et al. 2010). It is too complicated to be solved numerically as a single system presently, which is reserved for future studies. Therefore an alternative solution strategy is proposed. Because bed deformation is entirely determined by local entrainment and deposition fluxes under the noncapacity framework for sediment transport, Eq. (5) is separated from the remaining equations and can be readily solved. Also, from physical considerations, either the clear-water flow layer or the turbidity current layer is primarily dictated by its own inertia, gravity, and boundary resistance, whereas the interlayer interactions [primarily the second terms on the right hand side of Eqs. (4g) and (4h)] play a secondary role and can therefore be set as source terms in the momentum equations. In fact, the interlayer interactions can be confirmed to be negligible post priori, after the numerical solution of a specific case has been achieved.

Given the observations above, the two nonhomogeneous hyperbolic systems constituted by Eqs. (1) and (2) can be solved separately and synchronously (Li et al. 2013) using one of a hierarchy of numerical algorithms that can capture shock waves and contact discontinuities properly. The numerical algorithm employed in the present double layer-averaged mobile model is an extension of that in the 2D model originally developed for single-layer clear-water flow over a fixed bed (Liang 2010). Take Eq. (2) as an example. It can be solved numerically by an accurate finite volume Godunov-type approach in conjunction with the Harten-Lax-van Leer contact wave (HLLC) approximate Riemann solver (Toro 2001) on a fixed rectangular mesh. In principle, the present model is well-balanced because the interlayer interactions play secondary roles and are negligible compared with inertia and gravitation.

The following time-marching formulas are used to update flow and sediment variables to a new time step ($k + 1$)

$$\mathbf{U}_{i,j}^q = \mathbf{U}_{i,j}^k + \Delta t \mathbf{S}_{f,i,j}^{k+1} \quad (14)$$

$$\mathbf{U}_{i,j}^{k+1} = \mathbf{U}_{i,j}^r - \frac{\Delta t (\mathbf{G}_{i+1/2,j} - \mathbf{G}_{i-1/2,j})^q}{\Delta x} - \frac{\Delta t (\mathbf{H}_{i,j+1/2} - \mathbf{H}_{i,j-1/2})^q}{\Delta y} + \Delta t \mathbf{S}_{b,i,j}^k + \Delta t \mathbf{S}_{e,i,j}^k \quad (15)$$

where the subscript k represents the time level, and q indicates the state after calculating the variables from Eq. (14); subscripts i and j are the spatial node indexes; Δt is the time step; Δx and Δy are the spatial steps; $\mathbf{G}_{i+1/2,j}$, $\mathbf{G}_{i-1/2,j}$, $\mathbf{H}_{i+1/2,j}$ and $\mathbf{H}_{i-1/2,j}$ the interface fluxes.

In Eq. (14), the friction source term, \mathbf{S}_f^{k+1} , is solved by a splitting method; it is equivalent to solving the ordinary differential equations

$$\frac{d\mathbf{U}}{dt} = \mathbf{S}_f \quad (16)$$

Consider the momentum equation of Eq. (16) in the x -direction

$$\frac{dq_{sx}}{dt} = \mathbf{S}_{fx} = N_x \quad (17)$$

Eq. (17) is then discretized by a full implicit method

$$\frac{q_{sx}^q - q_{sx}^k}{\Delta t} = \mathbf{S}_{fx}^{k+1} \quad (18)$$

where the friction term is expressed using a Taylor series as

$$\mathbf{S}_{fx}^{k+1} = \mathbf{S}_{fx}^k + (\partial \mathbf{S}_{fx} / \partial q_{sx})^k \Delta q_{sx} + O(\Delta q_{sx}^2) \quad (19)$$

where $\Delta q_{sx} = q_{sx}^q - q_{sx}^k$. Neglecting the high-order terms and substituting it into Eq. (18), the following formula is attained for updating q_{sx} to the q time step

$$q_{sx}^q = q_{sx}^k + \Delta t (\mathbf{S}_{fx} / D_x)^k = q_{sx}^k + \Delta t F_x \quad (20)$$

where $D_x = 1 - \Delta t (\partial \mathbf{S}_{fx} / \partial q_{sx})^k$ is the implicit coefficient and $F_x = \mathbf{S}_{fx} / D_x$ is the friction source term including the implicit coefficient. To ensure stability, a limiting value of the implicit friction force must be identified according to the physics of the shallow flows. The maximum effect of the friction force is to constrain the flow (i.e., $q_{sx}^q q_{sx}^k \geq 0$). Then, according to Eq. (20), the limiting value of the implicit friction force can be easily derived as

$$F_x = \begin{cases} \geq -q_{sx}^k / \Delta t & \text{if } q_{sx}^k \geq 0 \\ \leq -q_{sx}^k / \Delta t & \text{if } q_{sx}^k \leq 0 \end{cases} \quad (21)$$

If F_x is computed beyond the limit, its value is replaced by the critical value $F_x = -q_{sx}^k / \Delta t$ in the actual computation. Similarly, the implicit coefficient in the y -direction can be calculated. In Eq. (17), the terms $\partial c_s / \partial x$ and $\partial h_w / \partial x$ are discretized using a central difference scheme for simplicity.

When evaluating $\mathbf{U}_{i,j}^{k+1}$ in Eq. (15), the solution of Eq. (14) is used as the initial condition. The interface fluxes $\mathbf{G}_{i+1/2,j}$, $\mathbf{G}_{i-1/2,j}$, $\mathbf{H}_{i+1/2,j}$ and $\mathbf{H}_{i-1/2,j}$ are computed using the HLLC Riemann solver (Toro 2001), which needs correct reconstruction of the Riemann states. The MUSCL method is used to achieve second-order accuracy in space. The Riemann states are defined by using the minmod slope-limited reconstruction. At the left-hand side of the cell interface, $(i + 1/2, j)$, the values are evaluated by

$$\bar{\eta}_{i+1/2,j}^L = \eta_{i,j} + 0.5\varphi(\eta_{i,j} - \eta_{i-1,j}) \quad (22a)$$

$$\bar{h}_{s\ i+1/2,j}^L = h_{s\ i,j} + 0.5\varphi(h_{s\ i,j} - h_{s\ i-1,j}) \quad (22b)$$

$$\bar{z}_{b\ i+1/2,j}^L = z_{b\ i,j} + 0.5\varphi(z_{b\ i,j} - z_{b\ i-1,j}) \quad (22c)$$

$$\bar{q}_{sx\ i+1/2,j}^L = q_{sx\ i,j} + 0.5\varphi(q_{sx\ i,j} - q_{sx\ i-1,j}) \quad (22d)$$

$$\bar{q}_{sy\ i+1/2,j}^L = q_{sy\ i,j} + 0.5\varphi(q_{sy\ i,j} - q_{sy\ i-1,j}) \quad (22e)$$

$$\bar{q}_{c\ i+1/2,j}^L = q_{c\ i,j} + 0.5\varphi(q_{c\ i,j} - q_{c\ i-1,j}) \quad (22f)$$

where φ represents the slope-limited function evaluated at cell (i, j) based on the flow and sediment data at the cell and its upwind and

downwind neighbors, and the minmod slope limiter is used for better numerical stability (Hirsch 1990)

$$\varphi(r) = \max[0, \min(r, 1)] \quad (23)$$

where r is the ratio of successive gradients of the flow and the variable under consideration; for example, e.g., for η

$$r = \frac{\eta_{i+1,j} - \eta_{i,j}}{\eta_{i,j} - \eta_{i-1,j}} \quad (24)$$

Similar expressions can be defined for q_{sx} , q_{sy} , q_c and h_s .

The interface values of the right-hand side of the cell interface $(i + 1/2, j)$ are calculated in a similar way

$$\bar{\eta}_{i+1/2,j}^R = \eta_{i,j} - 0.5\varphi(\eta_{i,j} - \eta_{i-1,j}) \quad (25a)$$

$$\bar{h}_{s\ i+1/2,j}^R = h_{s\ i,j} - 0.5\varphi(h_{s\ i,j} - h_{s\ i-1,j}) \quad (25b)$$

$$\bar{z}_{b\ i+1/2,j}^R = z_{b\ i,j} - 0.5\varphi(z_{b\ i,j} - z_{b\ i-1,j}) \quad (25c)$$

$$\bar{q}_{sx\ i+1/2,j}^R = q_{sx\ i,j} - 0.5\varphi(q_{sx\ i,j} - q_{sx\ i-1,j}) \quad (25d)$$

$$\bar{q}_{sy\ i+1/2,j}^R = q_{sy\ i,j} - 0.5\varphi(q_{sy\ i,j} - q_{sy\ i-1,j}) \quad (25e)$$

$$\bar{q}_{c\ i+1/2,j}^R = q_{c\ i,j} - 0.5\varphi(q_{c\ i,j} - q_{c\ i-1,j}) \quad (25f)$$

where φ is evaluated at cell $(i + 1, j)$. The velocity components and volumetric sediment concentration are then calculated by

$$\bar{u}_{s\ i+1/2,j}^L = \bar{q}_{sx\ i+1/2,j}^L / \bar{h}_{s\ i+1/2,j}^L \quad (26a)$$

$$\bar{u}_{s\ i+1/2,j}^R = \bar{q}_{sx\ i+1/2,j}^R / \bar{h}_{s\ i+1/2,j}^R \quad (26b)$$

$$\bar{v}_{s\ i+1/2,j}^L = \bar{q}_{sy\ i+1/2,j}^L / \bar{h}_{s\ i+1/2,j}^L \quad (26c)$$

$$\bar{v}_{s\ i+1/2,j}^R = \bar{q}_{sy\ i+1/2,j}^R / \bar{h}_{s\ i+1/2,j}^R \quad (26d)$$

$$\bar{c}_{s\ i+1/2,j}^L = \bar{q}_{c\ i+1/2,j}^L / \bar{h}_{s\ i+1/2,j}^L \quad (26e)$$

$$\bar{c}_{s\ i+1/2,j}^R = \bar{q}_{c\ i+1/2,j}^R / \bar{h}_{s\ i+1/2,j}^R \quad (26f)$$

Based on the above interface values, the Riemann states can be sought for designing the nonnegativity of water depth. As suggested by Liang (2010), a single bed elevation at the cell interface $(i + 1/2, j)$ may be defined as

$$z_{b\ i+1/2,j} = \max(\bar{z}_{b\ i+1/2,j}^L, \bar{z}_{b\ i+1/2,j}^R) \quad (27)$$

The depth components of the Riemann states are then defined by

$$h_{s\ i+1/2,j}^L = \max(0, \bar{\eta}_{s\ i+1/2,j}^L - z_{b\ i+1/2,j})$$

$$h_{s\ i+1/2,j}^R = \max(0, \bar{\eta}_{s\ i+1/2,j}^R - z_{b\ i+1/2,j}) \quad (28)$$

which preserves positive water depth. The Riemann states of other flow variables can be obtained accordingly

$$\eta_{s\ i+1/2,j}^L = h_{s\ i+1/2,j}^L + z_{b\ i+1/2,j} \quad (29a)$$

$$q_{sx\ i+1/2,j}^L = \bar{u}_{s\ i+1/2,j}^L h_{s\ i+1/2,j}^L \quad (29b)$$

$$q_{sy\ i+1/2,j}^L = \bar{v}_{s\ i+1/2,j}^L h_{s\ i+1/2,j}^L \quad (29c)$$

$$q_{c\ i+1/2,j}^L = \bar{c}_{s\ i+1/2,j}^L h_{s\ i+1/2,j}^L \quad (29d)$$

$$\eta_{s\ i+1/2,j}^R = h_{s\ i+1/2,j}^R + z_{b\ i+1/2,j} \quad (29e)$$

$$q_{sx\ i+1/2,j}^R = \bar{u}_{s\ i+1/2,j}^R h_{s\ i+1/2,j}^R \quad (29f)$$

$$q_{sy\ i+1/2,j}^R = \bar{v}_{s\ i+1/2,j}^R h_{s\ i+1/2,j}^R \quad (29g)$$

$$q_{c\ i+1/2,j}^R = \bar{c}_{s\ i+1/2,j}^R h_{s\ i+1/2,j}^R \quad (29h)$$

According to Liang (2010), for a dry-bed application, a numerical technique is needed to preserve the well-balanced solutions. For example, the bed elevation and the stage component of Riemann states are locally and instantaneously modified by subtracting Δz from the original values. And $\Delta z = \max[0, (z_{b\ i+1/2,j} - \bar{\eta}_{s\ i+1/2,j}^L)]$ denotes the difference between the actual and fake water surface level at the cell interface $(i + 1/2, j)$

$$z_{b\ i+1/2,j} \leftarrow z_{b\ i+1/2,j} - \Delta z \quad (30a)$$

$$\eta_{s\ i+1/2,j}^L \leftarrow \eta_{s\ i+1/2,j}^L - \Delta z \quad (30b)$$

$$\eta_{s\ i+1/2,j}^R \leftarrow \eta_{s\ i+1/2,j}^R - \Delta z \quad (30c)$$

The bed slope term S_b^k is discretized using the method proposed by Liang (2010). The procedure for the x -direction is outlined below; that for the y -direction is similar

$$-g\eta_s \frac{\partial z_b}{\partial x} = -g\bar{\eta}_s \left(\frac{z_{b\ i+1/2,j} - z_{b\ i-1/2,j}}{\Delta x} \right) \quad (31)$$

where $\bar{\eta}_s = (\eta_{s\ i-1/2,j}^R + \eta_{s\ i+1/2,j}^L)$.

Eq. (1) for the clear-water flow layer can be solved in a similar procedure as Eq. (2).

The current numerical scheme is explicit and its stability is controlled by the Courant-Friedrichs-Lewy condition. The time step is given by

$$\Delta t = C_r \min \left[\frac{\Delta x}{|u_{wi,j}| + (gh_{wi,j})^{1/2}}, \frac{\Delta y}{|v_{wi,j}| + (gh_{wi,j})^{1/2}}, \frac{\Delta x}{|u_{si,j}| + (gh_{si,j})^{1/2}}, \frac{\Delta y}{|v_{si,j}| + (gh_{si,j})^{1/2}} \right] \quad (32)$$

Test Cases

A series of experimental cases are solved to verify the present 2D double layer-averaged model, concerning turbidity currents attributable to lock-exchange and sustained inflow. In the present work, a fixed uniform mesh is adopted, and the spatial step is sufficiently fine to ensure mesh independence of the solution, i.e., essentially equivalent solutions are obtained with an even finer mesh. The Courant number C_r is 0.5 and bed porosity $p = 0.4$ is adopted for all the test cases.

Lock-Exchange Turbidity Currents

A series of experiments were conducted at the University of Cambridge to enhance the understanding of the depositional characteristics and front propagation of lock-exchange turbidity currents, including 1D (Hallworth and Huppert 1998) and 2D (Bonnetaze et al. 1995) cases. The present 2D double layer-averaged model is tested against some of the experiments with relatively low sediment concentrations, in which the turbidity currents can be considered as Newtonian fluids approximately. Here, the 2D experimental axisymmetric turbidity currents by Bonnetaze et al. (1995) are revisited. A plan view sketch of the flume is given in Fig. 1. The flume consists of a rectangular part (0.038 m wide and 0.306 m long) and a radial part (the width expands from 0.038 to 0.294 m within 1.83 m). A lock gate is placed at the center of the rectangular part, which separates the flume-filled water-sediment mixture on the left side and clear water on the right side. The initial thickness of the turbidity volume and clear water is 0.14 m. Turbidity currents are initiated by instantaneous vertical withdrawal of the lock gate. The median diameter of the sediment was $37 \mu\text{m}$, and its density was $3,217 \text{ kg/m}^3$. Three runs are conducted with different initial sediment concentrations: $c_{s0} = 0.019, 0.01,$ and 0.005 . E_s is calculated using Eq. (12). The spatial steps Δx and Δy are both 0.005 m. The bed roughness n_b and the interface roughness n_w are determined by fitting to a measured front location. It is found that $n_b = 0.015 \text{ m}^{-1/3}$ and $n_w = 0.005 \text{ m}^{-1/3}$ lead to satisfactory agreements with measured data.

Fig. 2 shows the turbidity current front location against time and the final deposit density plotted versus the radial distance, respectively. The radial distance is measured from the ghost origin as indicated in Fig. 1, which is set to be the intersection of the extended walls of the radial flume. From Fig. 2, the computed advance of the current front and the final deposit density by the present model agree with measured values rather well.

When the lock gate is removed, the plunging of turbidity volume leads to the formation of turbidity currents. The thickness of the current decreases sharply with the propagation of the current. Initially the turbidity current advances fast, but decelerates gradually in time [Fig. 2(a)]. The higher the initial sediment concentration, the faster the turbidity current propagates, and naturally more sediment is deposited [Fig. 2(b)]. The final deposition density has a maximum value near the ghost origin point and decreases asymptotically along the channel.

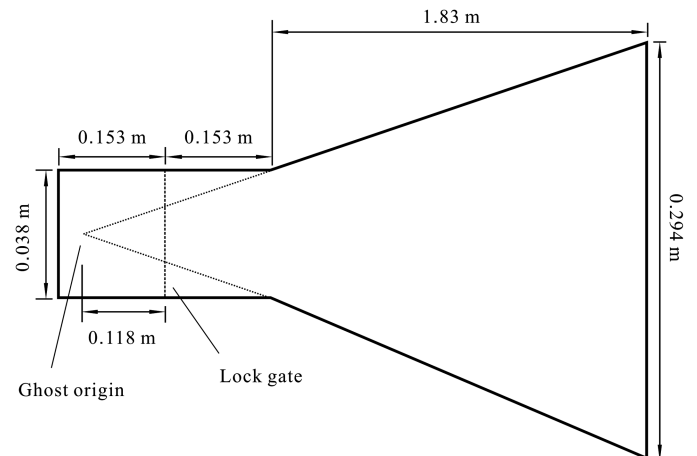


Fig. 1. Plan view sketch of experimental flume (adapted from Bonnetaze et al. 1995, reproduced with permission)

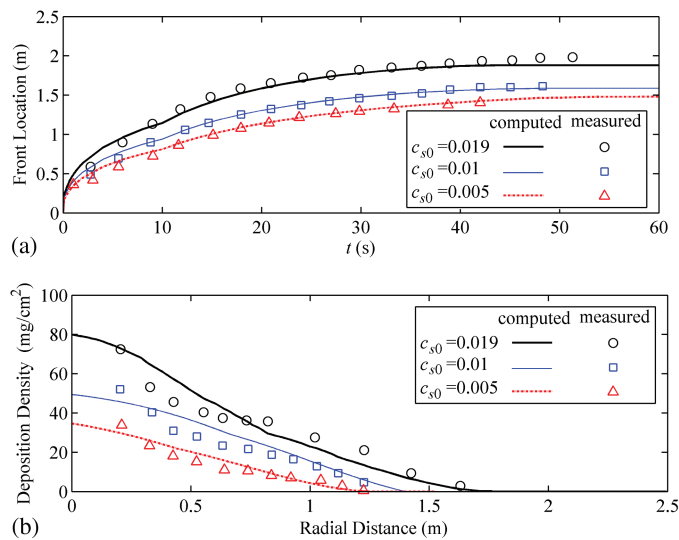


Fig. 2. (Color) Numerical solutions compared with measured data for 2D lock-exchange turbidity current: (a) front location; (b) final deposition density

Lock-exchange turbidity currents are formed by the sudden release of a fixed volume of turbid water, and driven by difference in density from the ambient water, without any inflow at the upstream or outflow at the downstream boundary. Physically, double layer-averaged models are generally applicable, though a single layer-averaged model has been suggested to be approximately workable in deep ambient water (Bonnecaze et al. 1993). Nevertheless, it is recognized that lock-exchange turbidity currents substantially differ from reservoir turbidity currents that are generally controlled by both the upstream and downstream boundary conditions. Thus the following test is warranted of the present model against reservoir turbidity currents subject to sustained inflow (and in some cases outflow) at a laboratory scale.

Turbidity Currents owing to Sustained Inflow

This subsection focuses on the turbidity currents attributable to sustained inflow from the upstream, and in some cases subject to an outflow at the downstream, in contrast to lock-exchange turbidity currents considered above. Lee and Yu (1997) carried out a series of experiments in a transparent flume of dimensions $20 \times 0.2 \times 0.6$ m and bed slope 0.02. A receiving tank was installed at the end of sloping section. The suspended material was kaolin having a specific gravity of 2.65 and a mean particle size of $6.8 \mu\text{m}$. During the experimental process, the flume was first filled with clear water to form a reservoir, and then open-channel sediment-laden flow was released from the head tank. In most runs, the outflow discharge, q_{out} , was set equal to the inflow discharge, q_{in} , whereas in some other runs, q_{out} was kept zero all the time or from some instant. The inflow discharge and its sediment concentration were kept constant in each run of the experiments.

To demonstrate the performance of the model, Series B and C are revisited. Series B was performed to investigate the migration of the plunge point and corresponding variations of the plunge criteria, whereas Series C was designed to investigate the length of the plunge region and the hydraulic characteristics of the turbidity current over a long distance. The inflow conditions for all revisited experimental runs are summarized in Table 1.

It is noted that the significant value of the systematic experiments by Lee and Yu (1997) has not been sufficiently exploited

Table 1. Summary of Inflow Conditions for All Revisited Experimental Runs

Run	Series B		Run	Series C	
	Inflow conditions			Inflow conditions	
	cm^2/s	10^{-3}		cm^2/s	10^{-3}
PP 1	23.5	3.71	TC 1	24.23	4.36
PP 2	42.25	3.71	TC 2	24.76	7.16
PP 3	70.56	2.51	TC 3	42.25	3.63
PP 4	85.10	2.00	TC 4	41.63	7.27
PP 5	86.74	3.86	TC 5	41.78	10.78
PP 6	100.21	3.27	TC 6	68.01	2.36
PP 7	86.01	5.61	TC 7	68.22	4.27
PP 8	99.64	4.98	TC 8	67.90	6.67
PP 9	101.20	6.60	TC 9	68.28	8.59
PP 10	134.46	4.85	TC 10	85.27	2.10
—	—	—	TC 11	85.45	3.88
—	—	—	TC 12	85.21	5.43
—	—	—	TC 13	84.70	7.37
—	—	—	TC 14	97.56	3.10
—	—	—	TC 15	97.52	4.73
—	—	—	TC 16	97.40	5.99
—	—	—	TC 17	96.47	7.81
—	—	—	TC 18	116.07	6.81

to support the development of analytical and computational models for reservoir turbidity currents. Only one run (Series C-TC 8) has ever been simulated with a vertical 2D model based on the Reynolds-averaged Navier-Stokes equations and κ - ϵ turbulence closure (Kassem and Imran 2001). In principle, the formation of the turbidity current could be resolved by this model, which, however, was not explicitly evaluated. Also, the impacts of the outflow as related to Series B-PP4 were not resolved at all, possibly because the flow system becomes unsteady, and the computing cost is too high. There has been a plethora of single layer-averaged models (e.g., Choi 1998; Bradford and Katopodes 1999a, b; Sequeiros et al. 2009; Hu et al. 2012; Lai and Wu 2013), but none has been verified against the observed data of Lee and Yu (1997).

For this modeling exercise, the computational domain consists of the sloping section without including the receiving tank at the far downstream end of the flume. It is assumed that the discharge at the end of the sloping section is equal to the outflow discharge because the receiving tank is rather short. At the inlet boundary, there was no clear-water flow layer; and because the inflow discharge was specified, the depth and velocity of the sediment-laden flow were determined by the method of characteristics. A downstream boundary condition is not required for the turbidity current because the computation is automatically terminated once the current reaches the downstream boundary. For the outlet boundary condition of the clear-water flow layer, the depth and velocity were determined by the method of characteristics because the outflow discharge was specified. The spatial step Δx is 0.025 m. The bed roughness n_b and the interface roughness n_w are first calibrated using measured data from Series B-PP 1, and then directly applied for the other cases. It is found that interface roughness $n_w = 0.005 \text{ m}^{-1/3} \text{ s}$ and bed roughness $n_b = 0.015 \text{ m}^{-1/3} \text{ s}$ lead to satisfactory agreements with measured data. E_s is determined according to Eq. (12) as ψ ranges between 4.0 and 20.0, which indicates partial erosion and deposition. It is also found that the maximum value of bed deformation is merely $3.4 \times 10^{-5} \text{ m}$ (not shown). Here, x_p is the distance between the plunge point and flume entrance; h_p is the current thickness at the plunge point, and $F_p = u_s / \sqrt{g_p' h_p}$ is the densimetric Froude number.

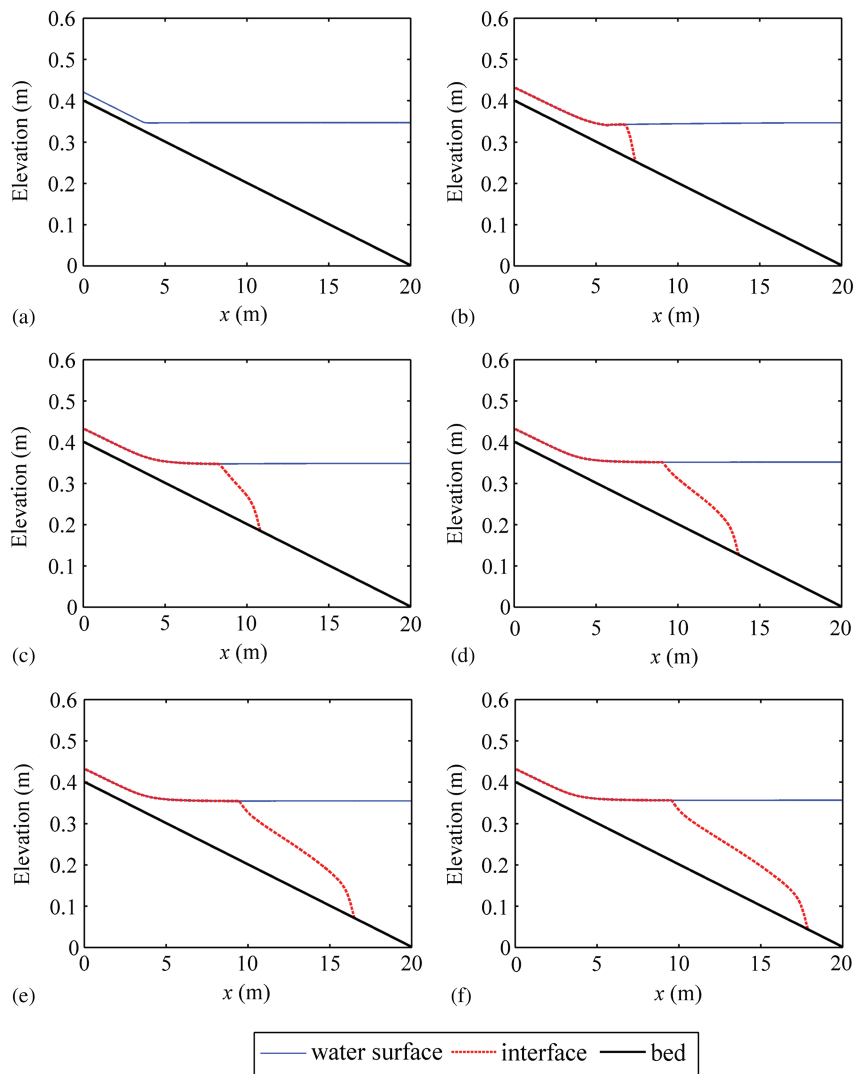


Fig. 3. (Color) Turbidity current formation and propagation (Series C-TC 15): (a) $t = 0$ s; (b) $t = 40$ s; (c) $t = 80$ s; (d) $t = 120$ s; (e) $t = 160$ s; (f) $t = 180$ s

Turbidity Current Formation and Propagation

Fig. 3 illustrates the formation and propagation processes of the turbidity current with unit-width inflow discharge, $97.52 \text{ cm}^2/\text{s}$, and volumetric sediment concentration, 4.73×10^{-3} , in relation to Run TC 15 (Table 1). It is noted that upstream from the plunge point, the interface is actually the water surface, characterizing that there is no clear water flow, and thus the flow is essentially subaerial open-channel sediment-laden flow. As the turbid water flows forward, a separation becomes pronounced from the clear water in the reservoir [Fig. 3(b)]. Then the sediment-laden flow plunges to the bottom and begins to move as an underflow, i.e., turbidity current [Figs. 3(c and d)], of which the upper boundary is indicated by the *interface*. Succinctly, the formation process of reservoir turbidity current is characterized by the transition from subaerial open-channel sediment-laden flow to subaqueous turbid flow. At this stage, the plunge point is unstable and still moves forward. By $t > 160$ s approximately, the plunge point stabilizes, and the current advances with a bulge-shaped head and elongated body.

It is noted that when the upper layer vanishes, Eq. (2) for the lower layer reduces to a system of equations of a traditional shallow water hydrodynamic and sediment model for open-channel sediment-laden flows (e.g., Cao et al. 2011), which differs from that of a single layer-averaged model for turbidity currents

(Hu et al. 2012). This exactly explains why the present double layer-averaged model can resolve the formation process of reservoir turbidity current, characterized by the transition from open-channel sediment-laden flow to subaqueous turbidity current.

To date, the authors are not aware of any previous layer-averaged models that can resolve the formation process of the turbidity currents attributable to sustained inflow, though systematic experimental observations have been available for long since Lee and Yu (1997). Most plausibly, this is because the currently available single layer-averaged models (e.g., Choi 1998; Bradford and Katopodes 1999a, b; Sequeiros et al. 2009; Hu et al. 2012; Lai and Wu 2013) simply do not have the capability of resolving the interactions between the open-channel sediment-laden flow input from the upstream and the ambient water in the reservoir, irrespective of whether the latter is static or flowing as dictated by the outflow in relation to the reservoir operation schemes. This is most telling that the present double layer-averaged model is warranted if the whole processes of reservoir turbidity currents are to be sufficiently resolved. This is further demonstrated in the following subsections, as compared against the observed data of Lee and Yu (1997) and the analytical formulations of Dai and Garcia (2009) and Li et al. (2011). Presented below are the characteristics at the plunge point, the streamwise profiles of the thickness, mean

Table 2. Parameters at Incipient and Stable Plunge Points (Series B)

Run	Incipient				Stable			
	h_p (cm)		F_p		h_p (cm)		F_p	
	Measured	Computed	Li et al. (2011)	Computed	Measured	Computed	Dai and Garcia (2009)	Computed
PP 1	5.36	5.31	0.81	0.86	6.87	6.80	0.62	0.63
PP 2	7.40	7.37	0.96	0.97	9.28	9.39	0.65	0.62
PP 3	10.25	9.97	0.96	0.96	13.85	14.12	0.69	0.68
PP 4	12.56	12.48	1.02	1.03	17.15	17.25	0.64	0.64
PP 5	10.57	10.46	0.98	0.99	13.19	12.95	0.70	0.68
PP 6	13.54	13.63	0.95	0.97	16.32	16.40	0.72	0.69
PP 7	9.68	9.45	0.97	0.98	12.63	12.52	0.64	0.64
PP 8	12.84	12.43	0.97	0.99	14.37	14.60	0.66	0.67
PP 9	10.68	10.26	1.05	1.06	13.45	13.69	0.64	0.64
PP 10	12.68	12.38	0.94	1.03	18.09	17.95	0.68	0.67

velocity, and sediment concentration of the turbidity currents, and the impacts of the outflow at the downstream.

Characteristics at the Plunge Point

Shown in Tables 2 and 3 are the parameters at the incipient and stable plunge points for Series B and Series C, corresponding to different inflow conditions. The analytical densimetric Froude number solution at the incipient plunge point developed by Li et al. (2011) is based on energy balance and includes the effects of the bed slope, sediment concentration, and the discharge of the turbidity current. Dai and Garcia (2009) analyzes the densimetric Froude number at the stable plunge point by taking into account the bed slope and inflow conditions. Tables 2 and 3 clearly illustrate that the computed results match the measured data and analytical results very well. It is shown that F_p at the incipient plunge point ranges approximately between 0.9 and 1.0, whereas F_p at the stable plunge point varies near 0.6. Thus, the incipient plunging occurs when F_p equals 0.9~1.0, and F_p reduces as the plunge point migrates downstream. The plunge point finally reaches a stable condition, where F_p equals 0.6. And the h_p and F_p at the incipient plunge point and x_p , h_p , and F_p at the stable plunge point increase with the increase of inflow discharge, but decrease with the increase of

sediment concentration. This is primarily because larger discharge or smaller sediment concentration corresponds to a smaller value of Ri and thus induces more water entrainment.

Fig. 4 shows the computed relationship between h_p and $(q_p^2/g_p')^{1/3}$ for Series B, including those not only at the incipient and stable plunge points, but also in between them. The theoretical results at the incipient plunge point according to Li et al. (2011) and at the stable plunge point according to Dai and Garcia (2009) are also included. The two dash lines represent, respectively, the incipient and stable plunge points, corresponding to $F_p = 1.0$ and 0.6 proposed by Lee and Yu (1997). Echoing Tables 2 and 3, the computed results from the present model agree with the analytical results based on Li et al. (2011) and Dai and Garcia (2009) and also the observed data of Lee and Yu (1997) rather well. Fig. 5 shows the temporal variation of the plunge point location for three typical cases in Series B. It is seen that the location of the stable plunge point and also the time from the incipient to stable plunge point are distinct as the inflow discharge and sediment concentrations vary. For a specific case (Series B-PP 4), the densimetric Froude number, F_p , decreases, and the plunge depth, h_p , increases with time, and eventually both reaches stable values (Fig. 6).

Table 3. Parameters at Incipient and Stable Plunge Points (Series C)

Run	Incipient				Stable			
	F_p		x_p		h_p		F_p	
	Li et al. (2011)	Computed	Measured	Computed	Measured	Computed	Dai and Garcia (2009)	Computed
TC 1	0.85	0.87	6.03	6.025	6.64	7.88	0.62	0.61
TC 2	0.89	0.90	5.52	5.50	5.62	6.43	0.64	0.63
TC 3	0.96	0.98	7.10	7.12	8.91	9.73	0.66	0.68
TC 4	0.95	0.94	6.29	6.31	7.17	7.94	0.65	0.67
TC 5	0.97	0.95	10.05	10.12	14.31	15.18	0.63	0.61
TC 6	0.99	0.96	10.05	10.12	14.31	15.18	0.63	0.61
TC 7	1.01	1.0	9.65	9.60	11.31	12.46	0.59	0.58
TC 8	0.98	0.99	8.05	8.03	10.53	11.13	0.65	0.62
TC 9	0.97	0.97	7.76	7.68	10.02	10.89	0.64	0.63
TC 10	0.99	0.96	11.24	11.26	17.04	17.85	0.66	0.65
TC 11	0.98	0.95	10.13	10.09	14.49	15.26	0.65	0.64
TC 12	1.02	1.03	9.46	9.56	12.97	13.69	0.64	0.66
TC 13	0.99	0.96	8.58	8.63	11.16	11.86	0.66	0.67
TC 14	0.98	0.97	10.96	10.98	16.51	17.20	0.67	0.65
TC 15	1.01	0.99	10.04	10.03	14.29	14.96	0.65	0.66
TC 16	0.99	0.98	9.55	9.68	13.18	14.29	0.67	0.64
TC 17	1.02	1.03	8.71	8.74	11.44	12.35	0.63	0.65
TC 18	0.99	0.96	9.68	9.78	9.68	10.03	0.67	0.65

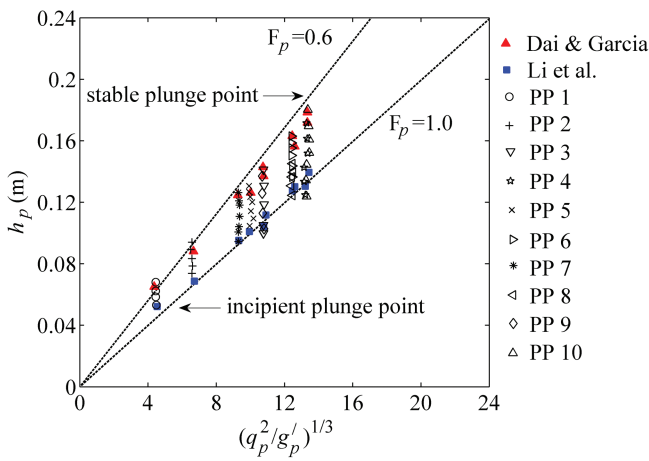


Fig. 4. (Color) Computed turbidity current thickness at plunge point compared with analytical formulations, with two dashed lines representing the incipient and stable plunge points according to Lee and Yu (1997)

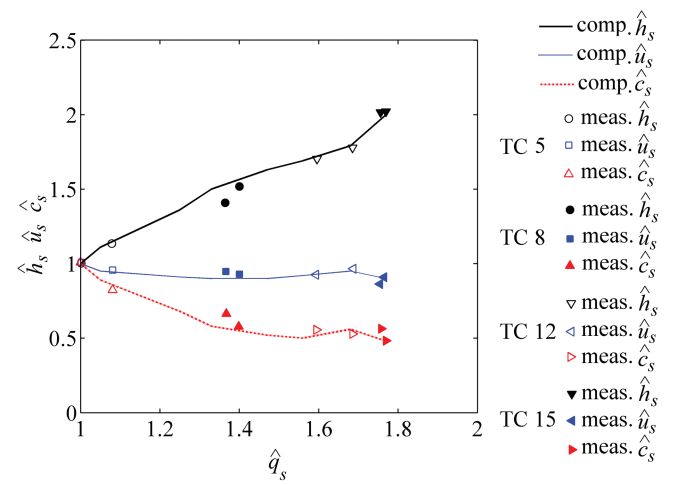


Fig. 7. (Color) Computed dimensionless turbidity current thickness, velocity, and sediment concentration compared with measured data

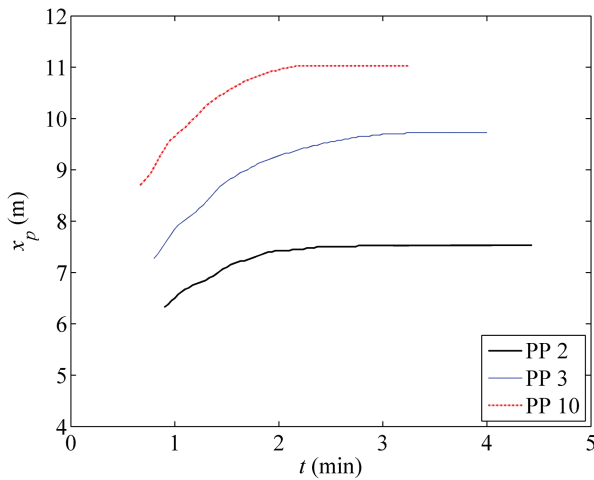


Fig. 5. (Color) Temporal variation of plunge point location

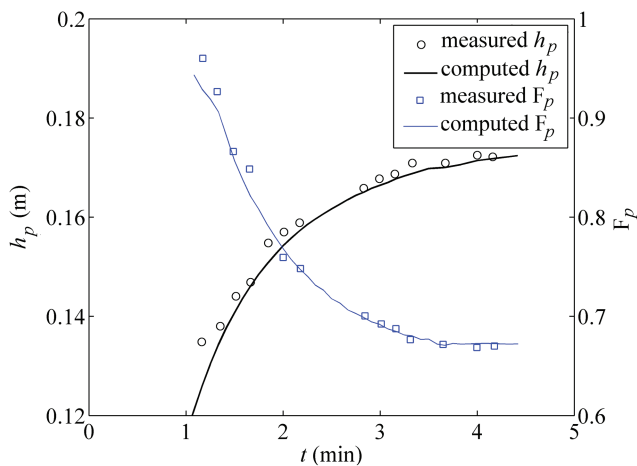


Fig. 6. (Color) Computed densimetric Froude number and turbidity current thickness at plunge point compared with measured data (Series B-PP4)

Nondimensional Profiles of Turbidity Currents

Fig. 7 shows the spatial variation of dimensionless current thickness, velocity, and sediment concentration computed by the present model along with measured data. Here, the data from Runs TC 5, TC 8, TC 12, and TC 15 at two selected cross sections located at 11.3 and 13.3 m, respectively, from the inlet of the flume are considered. Because of water entrainment, the discharge of the turbidity currents increases longitudinally and hence can be treated as an index of the distance from the plunge point. The data at $x = 11.3$ m in run TC 5 are used as the reference values to nondimensionalize the thickness, velocity, sediment concentration, and discharge of the turbidity currents, which are represented by \hat{h}_s , \hat{u}_s , \hat{c}_s and \hat{q}_s , respectively. It is seen from Fig. 7 that because of water entrainment, the thickness of turbidity currents increases longitudinally and thus the velocity and sediment concentration reduce accordingly. Overall, the observed nondimensional thickness, velocity, and sediment concentration profiles of the turbidity currents are well reproduced by the present model.

Impacts of Downstream Boundary Conditions on Turbidity Currents

Fig. 8 illustrates the impacts of downstream boundary conditions on the turbidity currents in relation to Series B-PP4. The impacts are represented by variations in the plunge flow depth, plunge location, and front location of the turbidity current. It is shown in Fig. 8 that if the outflow is cutoff from the initial state, the plunge point has not yet reached a stable state within the time period considered. Likewise, the stable plunge point becomes unstable as the outflow is terminated at 4.33 min. As the outflow increases, the water level in the reservoir will get lower. Most notably, an outflow of clear water in the upper layer generally leads to a decrease in the plunge depth [Fig. 8(a)], migration of the plunge location downstream [Fig. 8(b)], and acceleration of the propagation [Fig. 8(c)] of the turbidity current, and vice versa. Physically, a clear-water outflow facilitates a certain flow velocity of the upper layer, which leads to less interface resistance [Eq. (6)] to the turbidity current and reduced water entrainment [Eq. (8)].

From Figs. 4, 6, 7, and 8(a) and Tables 2 and 3, the computed results by the present model agree with measured values rather well, though subtle differences are spotted. This suggests that the turbidity currents are reasonably well resolved by the present model.

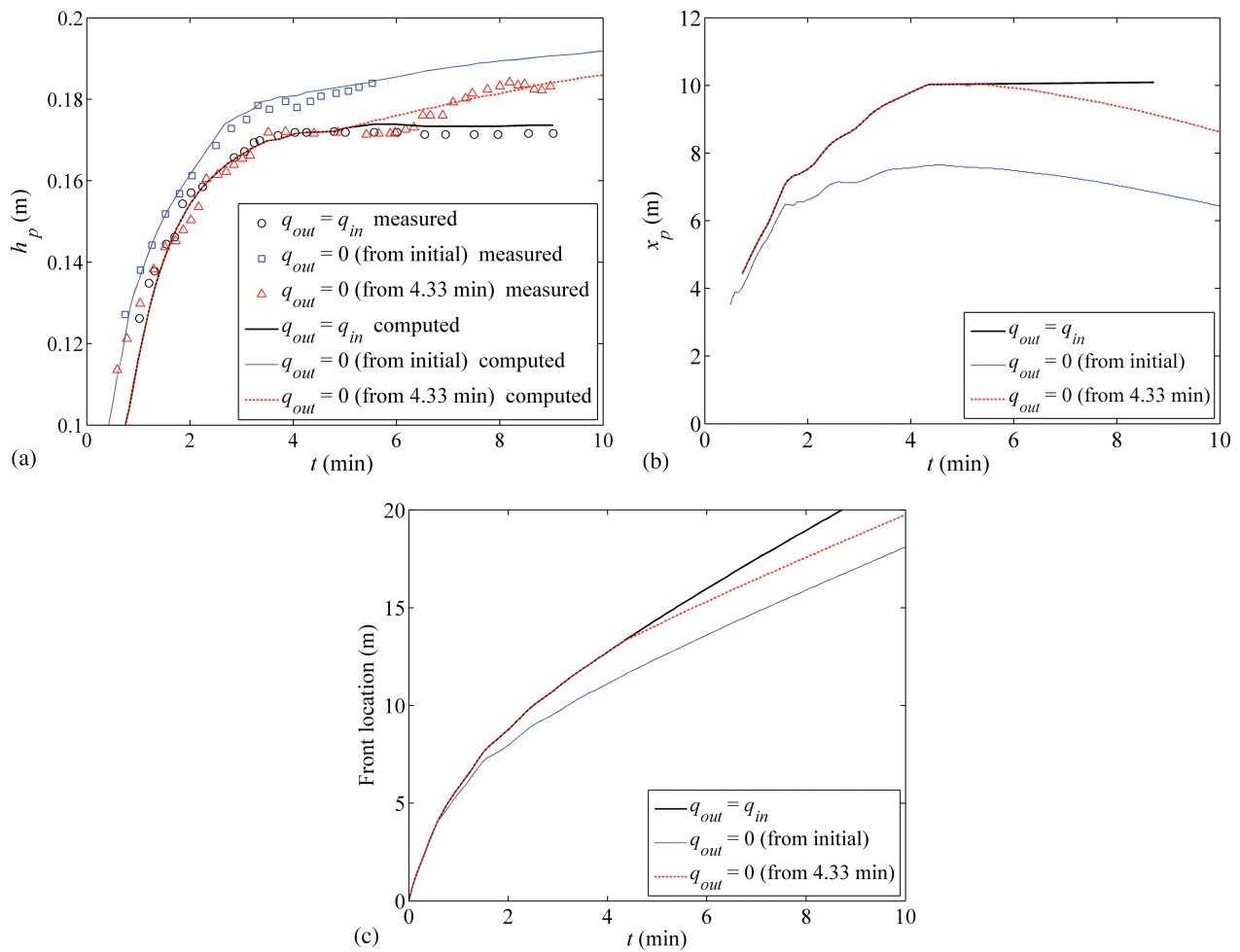


Fig. 8. (Color) Impacts of downstream boundary conditions: (a) computed turbidity current thickness at plunge point compared with measured data; (b) computed plunge location; (c) computed front location (Series B-PP 4)

Sediment Flushing Efficiency

It is interesting to evaluate sediment flushing efficiency of a reservoir. In general, it is defined as the ratio of the bulk sediment volume, V_{si} , input from the inlet of the reservoir to that (V_{so}) carried away through the downstream boundary. Here, V_{si} and V_{so} are calculated by

$$V_{si}(t) = \int_{\text{inlet}} (h_s u_s c_s) dt \quad (33a)$$

$$V_{so}(t) = \int_{\text{outlet}} (h_s u_s c_s) dt \quad (33b)$$

It is noted that Lee and Yu (1997) focused on the turbidity currents, whereas an evaluation of sediment flushing efficiency was missing. In their experiments, no bottom outlet was placed for sediment flushing at the downstream boundary (i.e., the end of the sloping section of the experimental flume). Here the experimental runs Series B-PP4 (Lee and Yu 1997) are extended to facilitate numerical investigation of the impacts of reservoir operation on sediment flushing. All the model parameters are kept the same as used in the subsection just above (Fig. 8). It is assumed that at the downstream boundary, there is a 4-cm-high bottom outlet on the bed, which has a presumed maximum (unit-width) discharge of $42.55 \text{ cm}^2/\text{s}$. When the turbidity current reaches the downstream

boundary, the bottom outlet is opened for sediment flushing, whereas the total outflow discharge (including clear-water flow and possibly turbidity current) remains the same as that in the experiments by Lee and Yu (1997). At the downstream boundary, before the arrival of the turbidity current, the depth and velocity of the clear-water flow layer are determined by the method of characteristics according to the total outflow discharge, because no downstream boundary condition is needed for the turbidity current. When the turbidity current has reached the bottom outlet, its respective discharge is equal to the maximum of the bottom outlet if the upper surface of the turbidity current is level with or higher than that of the bottom outlet; otherwise, it is equal to a proportion of the maximum discharge of the bottom outlet calculated by the turbidity current thickness relative to the height of the bottom outlet. If the turbidity current is under a subcritical regime, its depth and velocity are determined by the method of characteristics in line with its respective outflow discharge (q_s); otherwise, no downstream boundary condition is needed. Meanwhile, the depth and velocity of the clear-water flow layer are determined by the method of characteristics based on its respective discharge ($= q_{out} - q_s$).

Fig. 9 shows the sediment flushing efficiencies for the extended cases adapted from Series B-PP4 of Lee and Yu (1997). In general, sediment flushing commences as the turbidity current reaches the bottom outlet, and the flushing efficiency increases with time, which clearly requires sustained inputs of water and

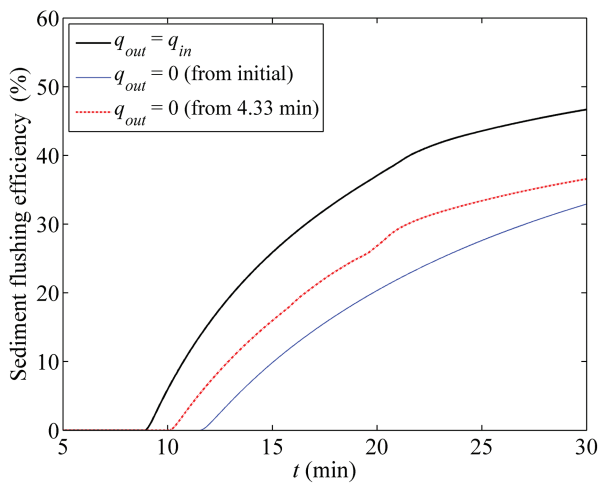


Fig. 9. (Color) Sediment flushing efficiencies for extended cases

sediment from the inlet. If the clear-water outflow is cutoff from the initial state or after 4.33 min, the sediment flushing efficiency decreases. Generally, the sooner the clear-water outflow is cutoff, the less the sediment flushed out of the reservoir. Physically, the clear-water outflow accelerates the propagation of the turbidity current [Fig. 8(c)], which leads to an increase in the amount of sediment flushed out and therefore enhanced sediment flushing efficiency.

It follows that an appropriate clear-water outflow not only favors the turbidity current propagation [Fig. 8(c)], but also is conducive to improving sediment flushing efficiency (Fig. 9). This is undoubtedly of significance for developing optimal sediment management schemes for reservoirs on the Yellow River in China and others worldwide, which suffer from severe sedimentation problems. It adds to the current understanding of the effects of distinct reservoir operation schemes on sedimentation mitigation (Fan and Morris 1992a, b). Nevertheless, more detailed investigations are necessary to delimit quantitatively the impacts of reservoir operation schemes on sediment flushing by means of turbidity currents. This certainly holds for the Xiaolangdi Reservoir, for which a case study is presented below. Particularly, it remains to be unraveled if an excessive clear-water outflow would spoil the turbidity currents that have already formed, as a result of Kelvin-Helmholtz instability.

Equally importantly, the impacts of the downstream boundary conditions on turbidity currents are in essence substantiated by the clear-water outflow through spillway and flood diversion, which is usually determined by the reservoir operation scheme. These significant impacts clearly tell that previous single layer-averaged models (e.g., Choi 1998; Bradford and Katopodes 1999a, b; Sequeiros et al. 2009; Hu et al. 2012; Lai and Wu 2013) are inadequate for applications in such cases because the clear-water flow is not modeled at all. The advantage (enhanced capability) of the present double layer-averaged model is evident.

Pilot Study of Large-Scale ProtoType Turbidity Currents

In this subsection, the present model was applied in a pilot study of the large-scale prototype turbidity currents in the Xiaolangdi Reservoir, Yellow River in China. From June 19 to July 13, 2004, a water-sediment regulation experiment was conducted to make full use of the extra water storage to reduce sedimentation in

the Yellow River. During the whole experiment, two separate turbidity current events were formed in the Xiaolangdi Reservoir by releasing sediment-laden flows from the Sanmenxia and Wanjiashai Reservoirs. Here, the second event is revisited, because field data show that the first turbidity current dissipated at approximately 6.5 km upstream of the Xiaolangdi Dam (Li 2004; YRCC 2007) and thus is unsuitable for whole-process modeling. The present pilot study covers the whole process of the turbidity currents, from formation and propagation to recession, which is distinct from work by Hu et al. (2012), which only modeled the propagation of turbidity currents while neglecting the impacts of the clear-water flow in the upper layer.

Case Description

This turbidity current occurred in the afternoon of July 7, approximately 1 day after the end of the first event. Though indirectly, it was generated because of the water release from the Wanjiashai Reservoir between July 2 and 7. When this water flow entered the Sanmenxia Reservoir and thus increased its water storage, the flow discharge at Sanmenxia increased to approximately 5,200 m³/s, which induced a second sediment-laden flood and thus the second turbidity current in the Xiaolangdi Reservoir. There are 56 cross sections in the 130-km-long main stream between Sanmenxia and Xiaolangdi dams. And the impacts of tributaries in between the two dams are tentatively neglected because they play a secondary role. The inflow discharge and its sediment concentration, which are actually released from the Sanmenxia Reservoir, are shown in Fig. 10 along with the outflow discharge through the Xiaolangdi Dam. The inflow essentially relates to a hyperconcentrated flood modulated by the Sanmenxia Reservoir. The time $t = 0$ h corresponds to 1:00 p.m., July 7, when the flow discharge at Sanmenxia reached its peak value. The computational time is 108 h (from 1:00 p.m., July 7 to 3:00 a.m., July 12). The total outflow discharge, Q_{out} , comprised of a clear water discharge, Q_w , and when appropriate a discharge, Q_s , of the turbidity current, was kept constant and amounted to 2,700 m³/s. A total of 18 bottom outlets with the depth of 5 m are distributed at different locations under the dam. The maximum discharge for sediment flushing through the bottom outlets under the Xiaolangdi Dam was 1,500 m³/s. At the upstream boundary, the discharge was specified, and the velocity and depth of the sub-aerial sediment-laden flow or clear-water flow were determined by the method of characteristics. The downstream boundary

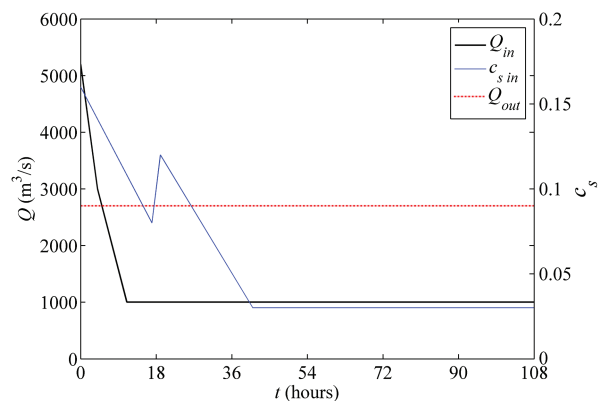


Fig. 10. (Color) Inflow discharge and sediment concentration along with outflow discharge

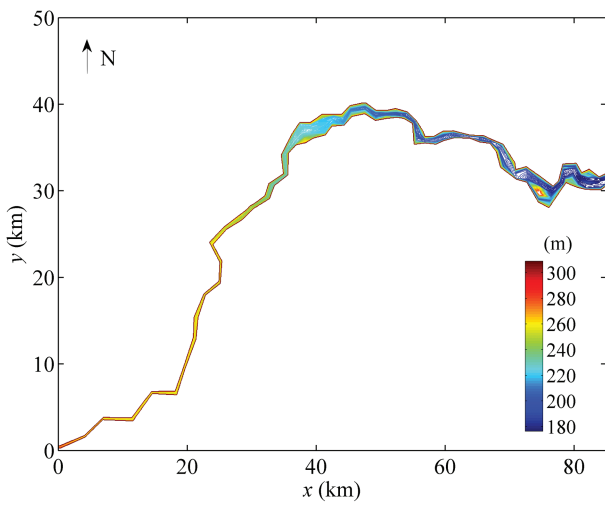


Fig. 11. (Color) Contour of initial bed topography based on the survey in May 2004

conditions are implemented in a similar manner to those in the subsection entitled “Sediment Flushing Efficiency.”

The initial bed topography is interpolated from 56 cross sections surveyed in May 2004 (Fig. 11), because the bed morphology immediately prior to the occurrence of the turbidity current was not surveyed. Initially, there is no turbidity current on the river bed. The following parameters are specified with reference to the background of the Xiaolangdi Reservoir: $p = 0.4$, $\rho_s = 2,650 \text{ kg/m}^3$, and $d = 20 \text{ }\mu\text{m}$. Here, the spatial step of 25 m is adopted. E_s is calculated through Eq. (13) following Hu et al. (2012). The Courant number, C_r , is set to be 0.4. The interface roughness, n_w , adopted is set to be $0.005 \text{ m}^{-1/3} \text{ s}$, following the calibrated value for the test cases related to the laboratory experiments described in the previous section. The bed roughness, n_b , and the correction coefficient, α , need to be calibrated by fitting to the measured front location. It is found that bed roughness $n_b = 0.035 \text{ m}^{-1/3} \text{ s}$ and correction coefficient $\alpha = 30$ lead to satisfactory agreement with measured data. These calibrated parameters are appreciably different from those calibrated by Hu et al. (2012), which mostly can be ascribed to the limitation that the motion of the upper clear-water is not taken into account in the single layer-averaged by Hu et al. (2012).

Advance of Turbidity Currents

Information on current advance can facilitate timely operation of the bottom outlets under the dam so that sediment can be flushed out of the reservoir (Fan and Morris 1992a). If the bottom outlets are closed upon the arrival of the current, the turbidity currents may lead to severe sedimentation, or alternatively, if the bottom outlets are opened too early, stored water may be wasted. Shown in Fig. 12 is the computed front location of the sediment-laden flow (i.e., open-channel sediment-laden flow upstream from the plunging point or the turbidity current downstream from the plunging), measured along the course of the river. The measured data shown in Fig. 12 correspond to the arrival time (approximately $t = 20 \text{ h}$) of the turbidity current at Xiaolangdi Dam. In Fig. 12, the computed results with the fine grid (25 m) and a coarser grid (50 m) are included. The difference between the two mesh resolutions is just marginally discernible, which demonstrates that the 25-m-mesh resolution is sufficiently fine (i.e., mesh independence of the numerical solution is attained). From Fig. 12, the computed current

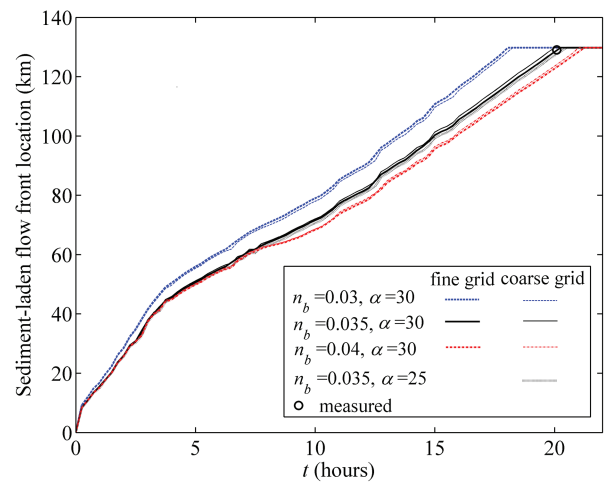


Fig. 12. (Color) Computed front location of sediment-laden flow compared with measured data

propagation with $n_b = 0.035$ and $\alpha = 30$ compares best with the measured data. The advance of the turbidity current front is primarily affected by the two parameters, n_b and α . A faster advance is generally obtained with a larger correction coefficient, α , and a smaller bed roughness, n_b . Physically, in relation to a larger correction coefficient, α , bed sediment entrainment increases [Eq. (13)], which leads to a higher sediment concentration and thus a larger driving force for the turbidity current. Therefore, it propagates faster. The larger the bed resistance, as represented by n_b , the more energy the turbidity current dissipates, and thus it propagates slower.

Whole Process of Turbidity Currents

Figs. 13 and 14 demonstrate the formation, propagation, and recession process of the turbidity current by virtue of the clear-water layer thickness, h_w , turbidity current thickness h_s , and sediment concentrations, c_s , respectively, at four instants ($t = 4, 8, 20$, and 108 h). Also, Fig. 15 shows the whole process of the turbidity current by means of the water surface, interface, and bed along the thalweg.

As illustrated in Figs. 13(a and b), 14(a) and 15(b), the flow at $t = 4 \text{ h}$ is divided into two distinct sections, i.e., the open-channel sediment-laden flow upstream and the clear-water flow downstream. At approximately $t = 6.6 \text{ h}$, the open-channel sediment-laden flow advances to the cross section at approximately $x = 38 \text{ km}$, characterized by an abrupt increase in longitudinal bed slope (Fig. 11). Here, the turbid water plunges into the clear water and begins to propagate as underflow, which marks the formation of turbidity current. Alternatively, the plunge point is located at roughly 56.5 km from the inlet along the course of the river, as shown in Fig. 15(c). From Fig. 14(b), a current with relatively high sediment concentration is formed, and the highest sediment concentration occurs at the front.

After traveling approximately 13.5 h since its formation, the turbidity current arrives at the Xiaolangdi Dam [Figs. 12, 13(e and f), 14(c) and 15(d)] and begins to be drained out through the bottom outlets; the largest turbidity current thickness occurs at the narrowest cross section (at $x = 65 \text{ km}$ and $y = 35 \text{ km}$ approximately); and the plunge point has hardly migrated downstream from the plunging point at $x = 38 \text{ km}$ [Fig. 15(d)]. As shown in Fig. 14(c), the sediment concentration starts to reduce gradually except in the

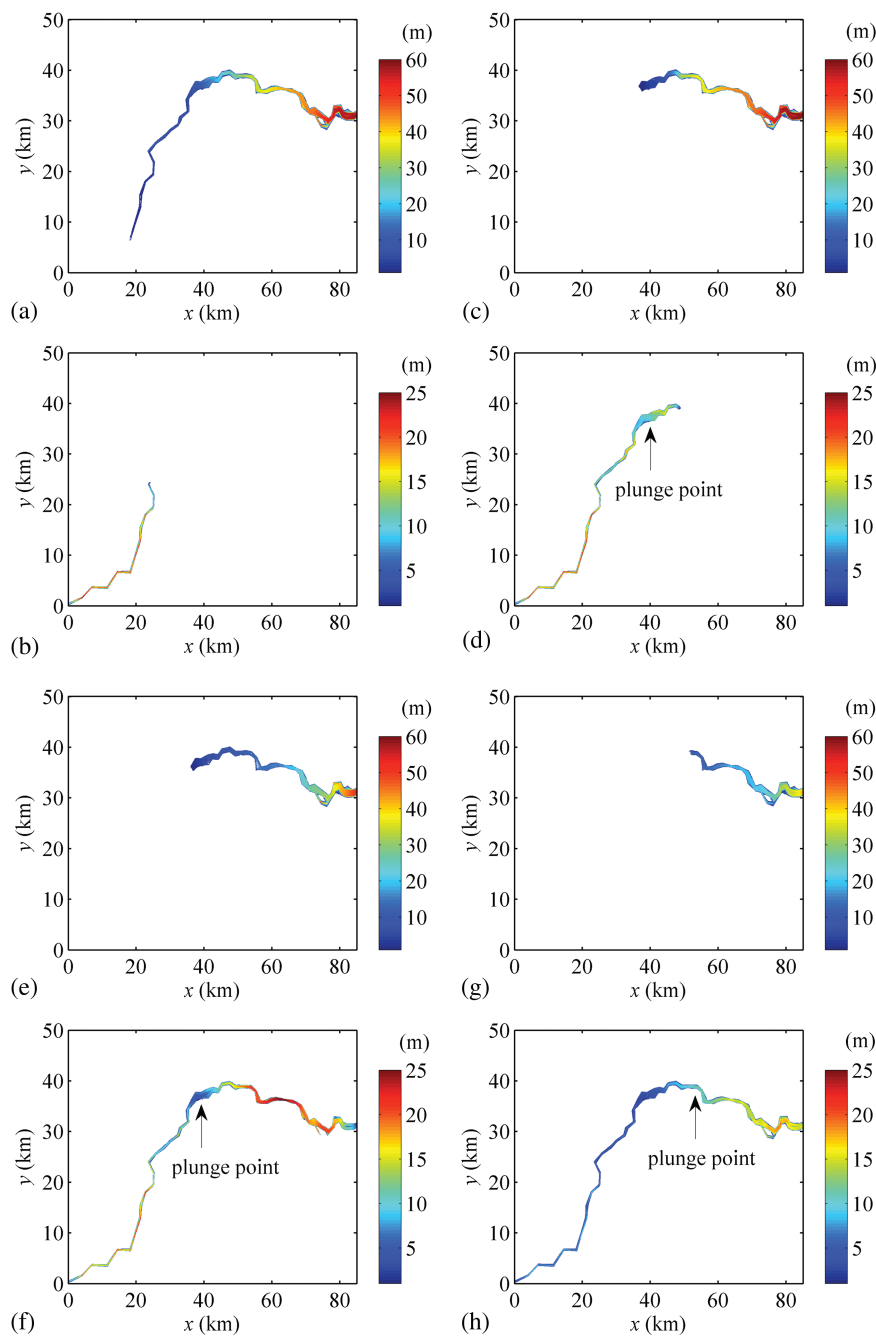


Fig. 13. (Color) Distributions of turbidity, current thickness, and clear-water thickness: (a) h_w at $t = 4$ h; (b) h_s at $t = 4$ h; (c) h_w at $t = 8$ h; (d) h_s at $t = 8$ h; (e) h_w at $t = 20$ h; (f) h_s at $t = 20$ h; (g) h_w at $t = 108$ h; (h) h_s at $t = 108$ h

section close to the inlet where sediment-laden flow continues to be released (Fig. 10).

After being vented through the bottom outlets for approximately 88 h, the thickness of the turbidity current [Fig. 13(h)] and sediment concentration [Fig. 14(d)] have decreased considerably, along with a significant movement of the plunge point to the downstream [Fig. 15(f)]. This clearly reflects a state of recession of the turbidity current. It is rational to anticipate that the turbidity current would finally recede if clear water continues to be released at the Sanmenxia Reservoir, and the turbidity current is allowed to flush through the bottom sluice gates under the Xiaolangdi Dam.

These observations, along with the reasonable agreement with observed data for the turbidity current advance, suggest that the present

model with properly specified parameters can properly resolve the whole process of turbidity currents in Xiaolangdi Reservoir.

Bed Deformation and Sediment Mass Conservation

Fig. 16 shows the bed scouring depth, defined as $z_b(x, y, 0) - z_b(x, y, t)$. The turbidity current appears to be erosive during the early stage, as indicated by the positive values of the bed scouring depth [Figs. 16(a and b)]. This is a clear manifestation of the occurrence of bed scour. Physically, this is attributable to the rather high discharge and sediment input released at the Sanmenxia Reservoir combined with the discharge through the Xiaolangdi Dam (Fig. 10). As the inflow discharge and sediment concentration decrease gradually, bed aggradation occurs during the course of the

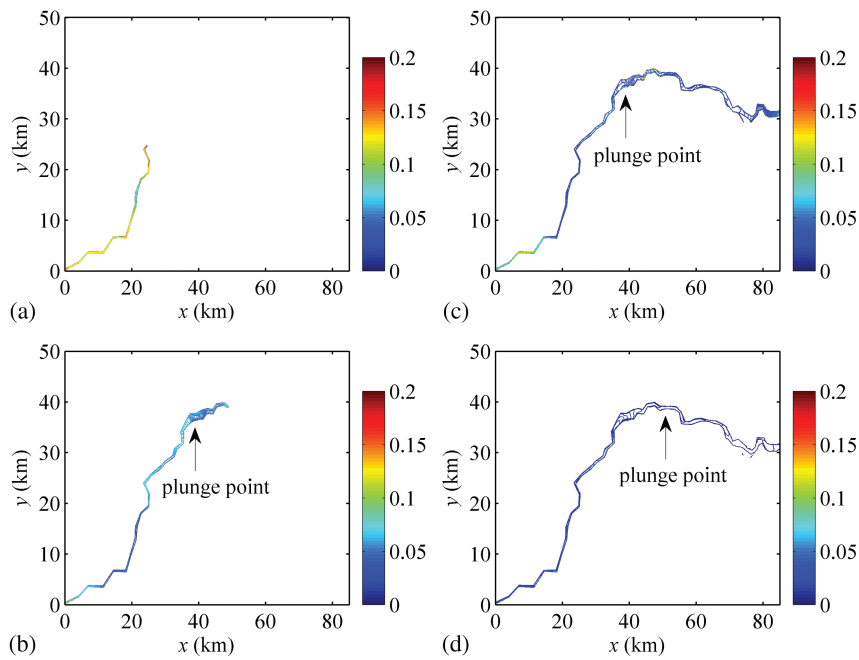


Fig. 14. (Color) Distributions of volumetric sediment concentration: (a) $t = 4$ h; (b) $t = 8$ h; (c) $t = 20$ h; (d) $t = 108$ h

turbidity current propagation toward the dam [Figs. 16(c and d)]. By $t = 108$ h, the whole domain of the Xiaolangdi Reservoir sees bed aggradation, except locally narrow reaches [Fig. 16(d)].

For turbidity currents in an ocean environment, it has been suggested that self-accelerating mechanism exists (Parker et al. 1986). Specifically, the current entrains sediment from the bed, which leads to a higher sediment concentration, and thus a larger difference in its density from the ambient water. This essentially corresponds to an increase in the driving force for the current. Then, the current accelerates, and picks up more sediment from the bed, which further accelerates the current. Therefore, a self-reinforcing cycle is established. This mechanism cannot be precluded for the reservoir turbidity current in question, because the influence of the coefficient α in relation to bed sediment entrainment [Eq. (13)] is marginally detectable (Fig. 12). Nevertheless, it is hard to isolate the effect of the self-accelerating mechanism from the control of the up- and downstream boundary conditions in the reservoir. This is also the case for the effect of sediment deposition out from the turbidity currents, which is opposite to the influence of the self-accelerating mechanism. Fortunately, all of those are properly incorporated in the present fully coupled model.

An evaluation of the global sediment mass conservation is certainly warranted because it can reveal not only the bulk aggradation or degradation in the Xiaolangdi Reservoir along with sediment flushing efficiency, but the performance of the present model. It accounts for the budget of the sediment that enters the Xiaolangdi Reservoir from the release at the Sanmenxia Reservoir, is flushed out at the Xiaolangdi Reservoir by means of turbidity current, and also the sediment content within the flow. For this purpose, the volume, $V_{sc}(t)$, of the sediment contained within the flow is calculated by

$$V_{sc}(t) = \iint h_s c_s dx dy \quad (34)$$

and the volume of the sediment attributable to bed erosion or deposition by $V_{sb}(t)$

$$V_{sb}(t) = (1 - p) \iint [z_b(x, y, 0) - z_b(x, y, t)] dx dy \quad (35)$$

The volume, $V_{si}(t)$, of the sediment input from the upstream, and that, $V_{so}(t)$, carried away through the bottom outlets are, respectively

$$V_{si}(t) = \iint_{\text{inlet}} (h_s u_s c_s) dy dt \quad (36a)$$

$$V_{so}(t) = \iint_{\text{outlet}} (h_s u_s c_s) dy dt \quad (36b)$$

The bulk mass conservation of the sediment phase provides that the residual sediment volume

$$R_s \equiv V_{si}(t) - V_{so}(t) + V_{sb}(t) - V_{sc}(t) + V_{sc}(0) = 0 \quad (37)$$

In a perfect case, R_s should vanish, but in general it does not because of numerical errors. It is an important indicator of the performance of a numerical model in the sense of mass conservation. A positive value of $V_{sb}(t)$ indicates bulk degradation, and the reverse demonstrates a bulk aggradation.

The volumes of sediment input from the upstream boundary (V_{si}), carried away through the bottom outlets (V_{so}), scoured from or deposited at the bed (V_{sb}), and contained within the flow (V_{sc}) along with their residuals (R_s) are illustrated in Fig. 17. It is demonstrated that by $t = 18$ h, there is no sediment flushed out from the reservoir ($V_{so} = 0$) because the turbidity current has not yet arrived at the bottom outlets. Before $t = 18$ h, the sediment volume contained within the flow (V_{sc}) exceeds the sediment input from upstream (V_{si}), thus the sediment scoured from bed is considerable ($V_{sb} > 0$), echoing the occurrence of bed scour as illustrated in Figs. 16(a and b). Along with the gradual reduction of the sediment input rate, both V_{sc} and V_{sb} decrease, and especially V_{sb} reduces to be negative, which indicates a shift from bulk degradation to bulk aggradation in the Xiaolangdi Reservoir. At approximately $t = 20.5$ h, because of the arrival of the turbidity current, the bottom sluicing gates under Xiaolangdi Dam are opened for sediment

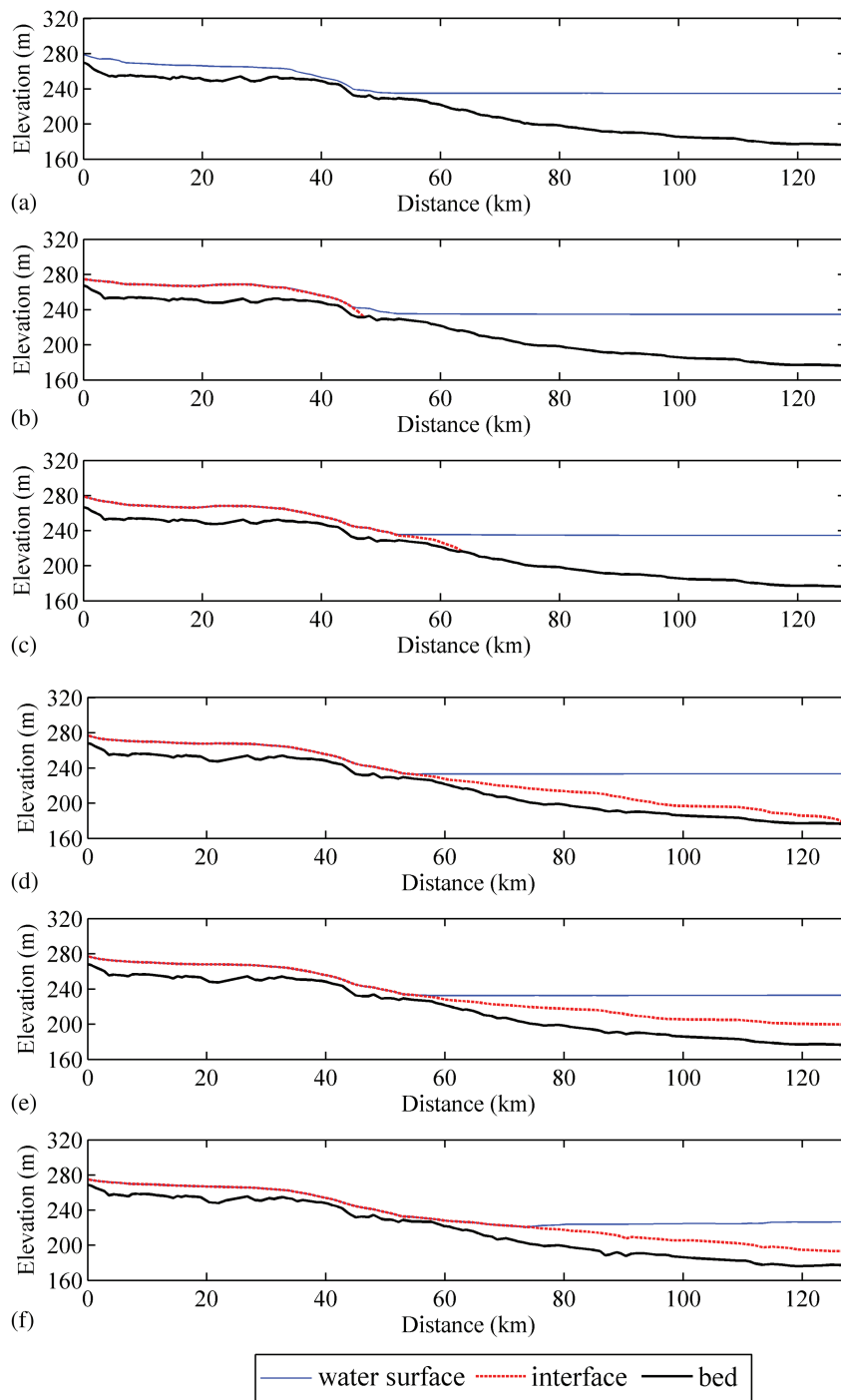


Fig. 15. (Color) Water surface, interface and bed profiles along the thalweg: (a) $t = 0$ h; (b) $t = 4$ h; (c) $t = 8$ h; (d) $t = 20$ h; (e) $t = 36$ h; (f) $t = 108$ h

flushing, inducing sediment output. After $t = 60$ h, the sediment input rate becomes stable, V_{so} increases gradually, V_{sc} is little changed, whereas V_{sb} decreases further, characterizing continuous bulk aggradation. At $t = 108$ h, approximately 1.173×10^7 m³ of sediment has been flushed downstream by means of the turbidity current, whereas the bulk aggradation in the Xiaolangdi Reservoir amounts to 1.196×10^7 m³, which are, respectively, equivalent to 37.47 and 38.2% of the sediment input from the Sanmenxia Reservoir. The sediment flushing efficiency (37.47%) of this particular turbidity current is consistent with the empirical range (18–36%) for the Sanmenxia Reservoir (Fan and Morris 1992b) immediately

upstream the Xiaolangdi Reservoir. Yet this sediment flushing efficiency is considerably lower than others that can be more than 60% (Fan and Morris 1992b). Echoing the observations (Figs. 8 and 9) derived from the computational tests in relation to the laboratory experiments by Lee and Yu (1997), further investigations are necessary to optimize the reservoir operation scheme in line with differing inflow and sediment inputs so that the sediment flushing efficiency can be maximized.

There has been no estimate of the bulk aggradation during the period of this particular turbidity current to confirm the present modeling in this regard. Yet according to YRCC (2007), the bulk

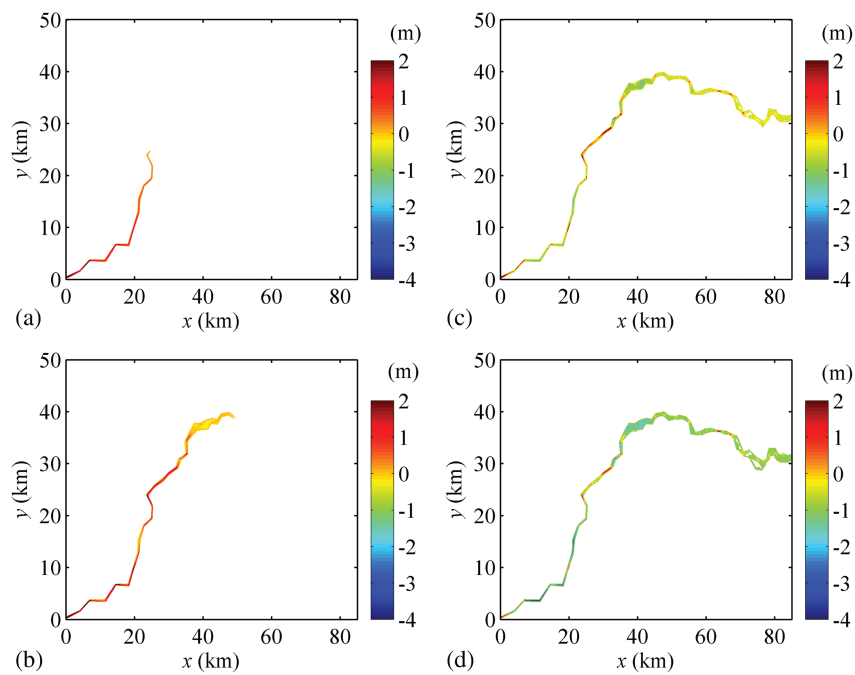


Fig. 16. (Color) Distributions of bed scouring depth: (a) $t = 4$ h; (b) $t = 8$ h; (c) $t = 20$ h; (d) $t = 108$ h

aggradation was approximately $6.0 \times 10^7 \text{ m}^3$ over two months, during which two turbidity current events occurred, and the first (from 6:00 p.m., July 5 to 7:00 p.m., July 6, 2004) did not manage to reach the Xiaolangdi Dam, and, therefore, the sediment released from the Sanmenxia Reservoir entirely deposited on the bed. Because this information and also that tributary contributions have been neglected in the present modeling, the amount of bulk aggradation ($1.196 \times 10^7 \text{ m}^3$) in connection with the particular turbidity current studied herewith is reasonable. Equally notably, the bulk residual (R_s) of sediment in the Xiaolangdi Reservoir is very small (Fig. 17) compared with the sediment volumes scoured from or deposited at the bed, input from the upstream or output through the bottom outlets. More specifically, the ratio of the residual, R_s , to V_{si} is only 1.61%. This further confirms the excellent

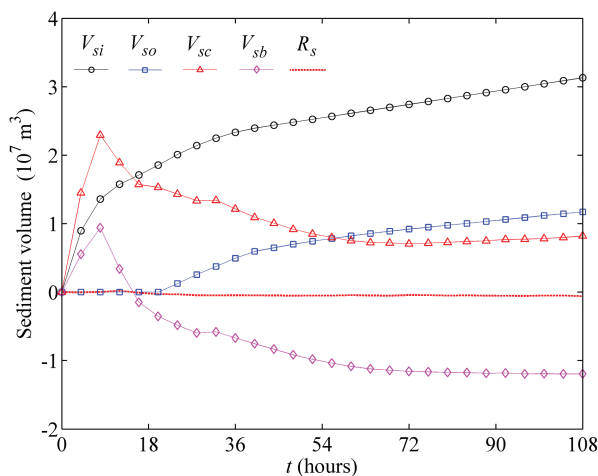


Fig. 17. (Color) Sediment volumes input from the upstream, output through bottom outlets, scoured from or deposited at the bed, contained within the flow and their residual

performance of the present model to resolve reservoir turbidity currents.

Conclusions and Remarks

A new 2D double layer-averaged model is developed, facilitating for the first time whole-process modeling of reservoir turbidity currents over erodible, irregular bed. Unsteady inflow discharge and sediment inputs and outflow hydrographs can be readily incorporated. It has been demonstrated to perform very well compared with a spectrum of laboratory experiments and rather satisfactorily in comparison with field data collected in the Xiaolangdi Reservoir, Yellow River in China. This work facilitates a viable and promising tool for the whole-process modeling of reservoir turbidity currents, in support of reservoir sediment management.

Computational tests using the present model reveal that an appropriate clear-water outflow is favorable for the propagation of turbidity currents, and also conducive to enhancing sediment flushing efficiency. This is significant for optimal operations of reservoirs suffering from sedimentation problems. However, it remains to be determined if an excessive clear-water outflow would spoil the turbidity currents that have already formed as a result of Kelvin-Helmholtz instability. Further investigations are necessary to delimit quantitatively the impacts of reservoir operation schemes on sediment flushing by means of turbidity currents.

Uncertainty of the model primarily arises from the estimations of the interface and bed resistances and bed sediment entrainment, which certainly warrant systematic fundamental investigations of the mechanisms of turbidity currents. Uncertainty can be dealt with by empiricism that can be accrued through practice using more measured datasets. In its present form, the double layer-averaged model is confined to single sized sediment transport, whereas practically sediments in turbidity currents may be heterogeneous. Also, turbidity currents with high contents of fine sediments may behave as nonNewtonian fluids, which necessitate physically improved

constitutive relationships to be incorporated in the model. These are reserved for future studies.

Acknowledgments

The research is funded by Natural Science Foundation of China (Grant Nos. 10932012 and 10972164).

Notation

The following symbols are used in this paper:

- C_r = Courant number (-);
 c_b = local near-bed concentration (-);
 c_s = volumetric sediment concentration of the turbidity current layer (-);
 d = medium sediment particle diameter (m);
 \mathbf{E} = vector defined in Eq. (3);
 E, D = sediment entrainment and deposition fluxes respectively (m s^{-1});
 E_s = near-bed concentration at capacity condition;
 E_w = mass exchange flux of clear water between the two layers (m s^{-1});
 e_w = coefficient for mass exchange of clear water (-);
 \mathbf{F} = vector defined in Eq. (3);
 F_p = densimetric Froude number at plunge point (-);
 \mathbf{G} = vector defined in Eq. (4);
 g = gravitational acceleration (m s^{-2});
 g' = submerged gravitational acceleration (m s^{-2});
 \mathbf{H} = vector defined in Eq. (4);
 h_p = plunge depth (m);
 h_s = depth of turbidity current layer (m);
 h_w = depth of clear-water flow layer (m);
 i, j = spatial node indexes;
 k = index denoting the time step;
 n_b = bed roughness ($\text{m}^{-1/3}$ s);
 n_w = interface roughness ($\text{m}^{-1/3}$ s);
 p = bed sediment porosity (-);
 q = index denoting the auxiliary time step;
 q_{sx}, q_{sy}, q_c = conservative variables in Eq. (4);
 q_{wx}, q_{wy} = conservative variables in Eq. (3);
 $\mathbf{R}_b, \mathbf{R}_f, \mathbf{R}_e$ = source terms for the clear-water flow layer;
 Ri = Richardson number (-);
 R_s = residuals of sediment conservation (m^3);
 r_b = empirical coefficient (-);
 $\mathbf{S}_b, \mathbf{S}_f, \mathbf{S}_e$ = source terms for the turbidity current layer;
 s = specific gravity of sediment (-);
 \mathbf{T} = vector of conserved variables of the clear-water flow layer;
 t = time (s);
 \mathbf{U} = vector of conserved variables of the turbidity current layer;
 \bar{U}_s = resultant velocity of the turbidity current layer (m s^{-1});
 \bar{U}_{ws} = resultant velocity difference between two layers (m s^{-1});
 u_s, v_s = mean velocities of the turbidity current layer in x - and y -directions (m s^{-1});
 u_w, v_w = mean velocities of the clear-water flow layer in x - and y -directions (m s^{-1});
 u_*, v_* = friction velocity in x - and y -directions (m s^{-1});

- $V_{si}(t), V_{so}(t), V_{sb}(t), V_{sc}(t)$ = sediment volumes input from the upstream, output through bottom outlets, scoured from or deposited at the bed, contained within the flow, respectively (m^3);
 ν = kinematic viscosity of water ($\text{m}^2 \text{s}^{-1}$);
 x, y = horizontal coordinates (m);
 x_p = distance between the plunge point and flume entrance (m);
 z_b = bed elevation (m);
 α = correction coefficient (-);
 Δt = time step (s);
 $\Delta x, \Delta y$ = spatial step in x - and y -directions (m);
 η = water surface elevation (m);
 η_s = interface elevation (m);
 ρ_c, ρ_0 = densities of water-sediment mixture and saturated bed (kg m^{-3});
 ρ_w, ρ_s = densities of water and sediment, respectively (kg m^{-3});
 τ_{bx}, τ_{by} = bed shear stress in x - and y -directions ($\text{kg m}^{-1} \text{s}^{-2}$);
 τ_{wx}, τ_{wy} = interface shear stress in x - and y -directions ($\text{kg m}^{-1} \text{s}^{-2}$);
 ψ = parameter in Eq. (12) (-); and
 ω = settling velocity of a single sediment particle in tranquil clear water (m s^{-1}).

References

- Adduce, C., Sciortino, G., and Proietti, S. (2012). "Gravity currents produced by lock exchanges: Experiments and simulations with a two-layer shallow-water model with entrainment." *J. Hydraul. Eng.*, 10.1061/(ASCE)HY.1943-7900.0000484, 111–121.
 Akiyama, J., and Stefan, H. G. (1984). "Plunging flow into a reservoir: Theory." *J. Hydraul. Eng.*, 10.1061/(ASCE)0733-9429(1984)110:4(484), 484–499.
 Altinakar, M., Graf, W., and Hopfinger, E. (1996). "Flow structure of turbidity currents." *J. Hydraul. Res.*, 34(5), 713–718.
 An, S., and Julien, P. (2014). "Three-dimensional modeling of turbid density currents in Imha reservoir, South Korea." *J. Hydraul. Eng.*, 10.1061/(ASCE)HY.1943-7900.0000851, 05014004.
 Bonnetcaze, R. T., Hallworth, M. A., Huppert, H. E., and Lister, J. R. (1995). "Axisymmetric particle-driven gravity currents." *J. Fluid Mech.*, 294, 93–121.
 Bonnetcaze, R. T., Huppert, H. E., and Lister, J. R. (1993). "Particle-driven gravity currents." *J. Fluid Mech.*, 250, 339–369.
 Bournet, P. E., Dartus, D., Tassin, B., and Vincon-Leite, B. (1999). "Numerical investigation of plunging density current." *J. Hydraul. Eng.*, 10.1061/(ASCE)0733-9429(1999)125:6(584), 584–594.
 Bradford, S. F., and Katopodes, N. D. (1999a). "Hydrodynamics of turbid underflows. I: Formulation and numerical analysis." *J. Hydraul. Eng.*, 10.1061/(ASCE)0733-9429(1999)125:10(1006), 1006–1015.
 Bradford, S. F., and Katopodes, N. D. (1999b). "Hydrodynamics of turbid underflows. II: Aggradation, avulsion, and channelization." *J. Hydraul. Eng.*, 10.1061/(ASCE)0733-9429(1999)125:10(1016), 1016–1028.
 Cao, Z., Yue, Z. Y., and Pender, G. (2011). "Landslide dam failure and flood hydraulics. Part II: Coupled mathematical modelling." *Nat. Hazard.*, 59(2), 1021–1045.
 Castro, M., Frings, J., Noelle, S., Pares, C., and Puppo, G. (2010). "On the hyperbolicity of two- and three-layer shallow water equations." *Proc., Int. Conf. Hyperbolic Problems*, American Mathematical Society, MI, 657–664.
 Chen, S. C., and Peng, S. H. (2006). "Two-dimensional numerical model of two-layer shallow water equations for confluence simulation." *Adv. Water Res.*, 29(11), 1608–1617.
 Choi, S. U. (1998). "Layer-averaged modelling of two-dimensional turbidity currents with a dissipative-Galerkin finite element method. Part I: Formulation and application example." *J. Hydraul. Res.*, 36(3), 339–362.

- Dai, A., and Garcia, M. H. (2009). "Analysis of plunging phenomena." *J. Hydraul. Res.*, 47(5), 638–642.
- Dallimore, C. J., Imberger, J., and Hodges, B. R. (2004). "Modeling a plunging underflow." *J. Hydraul. Res.*, 10.1061/(ASCE)0733-9429(2004)130:11(1068), 1068–1076.
- De Cesare, G., Schleiss, A., and Hermann, F. (2001). "Impact of turbidity currents on reservoir sedimentation." *J. Hydraul. Eng.*, 10.1061/(ASCE)0733-9429(2001)127:1(6), 6–16.
- Eggenhuisen, J. T., and McCaffrey, W. D. (2012). "The vertical turbulence structure of experimental turbidity currents encountering basal obstructions: Implications for vertical suspended sediment distribution in non-equilibrium currents." *Sedimentology*, 59(3), 1101–1120.
- Fan, J. (1960). "Experimental studies on density currents." *Scientia Sinica*, 9(2), 275–303.
- Fan, J., and Morris, G. L. (1992a). "Reservoir sedimentation. I: Delta and density current deposits." *J. Hydraul. Eng.*, 10.1061/(ASCE)0733-9429(1992)118:3(354), 354–369.
- Fan, J., and Morris, G. L. (1992b). "Reservoir sedimentation. II: Reservoir desiltation and long-term storage capacity." *J. Hydraul. Eng.*, 10.1061/(ASCE)0733-9429(1992)118:3(370), 370–384.
- Fang, H. W., and Wang, G. Q. (2000). "Three-dimensional mathematical model of suspended-sediment transport." *J. Hydraul. Eng.*, 10.1061/(ASCE)0733-9429(2000)126:8(578), 578–592.
- Fukushima, Y., Parker, G., and Pantin, H. M. (1985). "Prediction of ignitive turbidity currents in Scripps submarine canyon." *Mar. Geol.*, 67(1–2), 55–81.
- Georgoulas, A. N., Angelidis, P. B., Panagiotidis, T. G., and Kotsovinos, N. E. (2010). "3D numerical modelling of turbidity currents." *Environ. Fluid Mech.*, 10(6), 603–635.
- Gladstone, C., Phillips, J. C., and Sparks, R. S. J. (1998). "Experiments on bidisperse, constant-volume gravity currents: Propagation and sediment deposition." *Sedimentology*, 45(5), 833–843.
- Guo, J. (2002). "Logarithmic matching and its application in computational hydraulics and sediment transport." *J. Hydraul. Res.*, 40(5), 555–565.
- Hallworth, M., Huppert, H. E., and Ungarish, M. (2003). "On inwardly propagating high-Reynolds-number axisymmetric gravity currents." *J. Fluid Mech.*, 494, 255–274.
- Hallworth, M. A., and Huppert, H. E. (1998). "Abrupt transitions in high-concentration, particle-driven gravity currents." *Phys. Fluids*, 10(5), 1083–1087.
- Hirsch, C. (1990). *Computational methods for inviscid and viscous flows: Numerical computation of internal and external flows*, Wiley, New York.
- Hu, P., and Cao, Z. (2009). "Fully coupled mathematical modeling of turbidity currents over erodible bed." *Adv. Water Resour.*, 32(1), 1–15.
- Hu, P., Cao, Z., Pender, G., and Tan, G. (2012). "Numerical modelling of turbidity currents in the Xiaolangdi reservoir, Yellow River, China." *J. Hydrol.*, 464, 41–53.
- Huang, H., Imran, J., and Pirmez, C. (2007). "Numerical modeling of poorly sorted depositional turbidity currents." *J. Geophys. Res.*, 112(C1), 1–15.
- Huang, H., Imran, J., and Pirmez, C. (2008). "Numerical study of turbidity currents with sudden-release and sustained-inflow mechanisms." *J. Hydraul. Eng.*, 10.1061/(ASCE)0733-9429(2008)134:9(1199), 1199–1209.
- Hürzeler, B. E., Imberger, J., and Ivey, G. N. (1996). "Dynamics of turbidity current with reversing buoyancy." *J. Hydraul. Eng.*, 10.1061/(ASCE)0733-9429(1996)122:5(230), 230–236.
- Kassem, A., and Imran, J. (2001). "Simulation of turbid underflows generated by the plunging of a river." *Geology*, 29(7), 655–658.
- Kassem, A., Imran, J., and Khan, J. A. (2003). "Three-dimensional modeling of negatively buoyant flow in diverging channels." *J. Hydraul. Eng.*, 10.1061/(ASCE)0733-9429(2003)129:12(936), 936–947.
- Khan, S. M., Imran, J., Bradford, S., and Syvitski, J. (2005). "Numerical modeling of hyperpycnal plume." *Mar. Geol.*, 222, 193–211.
- Kim, J., and LeVeque, R. J. (2008). "Two-layer shallow water system and its applications." *Proc., Int. Conf. Hyperbolic Problems*, American Mathematical Society, MI, 737–743.
- Lai, Y. G., and Wu, K. (2013). "Modeling of turbidity current and evaluation of diversion plans at Shihmen Reservoir in Taiwan." *World Environment and Water Resources Congress 2013: Showcasing the future*, ASCE, Reston, VA, 1736–1746.
- La Rocca, M., Adduce, C., Sciortino, G., Pinzon, A. B., and Boniforti, M. A. (2012). "A two-layer shallow-water model for 3D gravity currents." *J. Hydraul. Res.*, 50(2), 208–217.
- Lee, H. Y., and Yu, W. S. (1997). "Experimental study of reservoir turbidity current." *J. Hydraul. Eng.*, 10.1061/(ASCE)0733-9429(1997)123:6(520), 520–528.
- Lee, W. K., Borthwick, A. G., and Taylor, P. H. (2014). "Tracer dynamics in two-layer density-stratified estuarine flow." *Proc., ICE Eng. Comput. Mech.*, 167(1), 41–49.
- Li, G. Y. (2004). "The third test of water and sediment regulation conducted on the Yellow River." *Yellow River*, 26(10), 1–8 (in Chinese).
- Li, J., Cao, Z., Pender, G., and Liu, Q. (2013). "A double layer-averaged model for dam-break floods over erodible bed." *J. Hydraul. Res.*, 51(5), 518–534.
- Li, Y., Zhang, J., and Ma, H. (2011). "Analytical Froude number solution for reservoir density inflows." *J. Hydraul. Res.*, 49(5), 693–696.
- Liang, Q. (2010). "Flood simulation using a well-balanced shallow flow model." *J. Hydraul. Eng.*, 10.1061/(ASCE)HY.1943-7900.0000219, 669–675.
- Nourmohammadi, Z., Afshin, H., and Firoozabadi, B. (2011). "Experimental observation of the flow structure of turbidity currents." *J. Hydraul. Res.*, 49(2), 168–177.
- Parker, G., Fukushima, Y., and Pantin, H. M. (1986). "Self-accelerating turbidity currents." *J. Fluid Mech.*, 171, 145–181.
- Parker, G., and Toniolo, H. (2007). "Note on the analysis of plunging of density flows." *J. Hydraul. Eng.*, 10.1061/(ASCE)0733-9429(2007)133:6(690), 690–694.
- Rottman, J. W., and Simpson, J. E. (1983). "Gravity currents produced by instantaneous releases of a heavy fluid in a rectangular channel." *J. Fluid Mech.*, 135, 95–110.
- Savage, S. B., and Brimberg, J. (1975). "Analysis of plunging phenomena in water reservoirs." *J. Hydraul. Res.*, 13(2), 187–205.
- Sequeiros, O. E., Cantero, M. I., and Garcia, M. H. (2009). "Sediment management by jets and turbidity currents with application to a reservoir for flood and pollution control in Chicago, Illinois." *J. Hydraul. Res.*, 47(3), 340–348.
- Singh, B., and Shah, C. R. (1971). "Plunging phenomenon of density currents in reservoirs." *Int. Water J.*, 26(1), 59–64.
- Toro, E. F. (2001). *Shock-capturing methods for free-surface shallow flows*, Wiley, Chichester, U.K.
- Ungarish, M., and Zemach, T. (2005). "On the slumping of high Reynolds number gravity currents in two-dimensional and axisymmetric configurations." *Eur. J. Mech. B. Fluids*, 24(1), 71–90.
- Wu, W., Rodi, W., and Wenka, T. (2000). "3D numerical modeling of flow and sediment transport in open channels." *J. Hydraul. Eng.*, 10.1061/(ASCE)0733-9429(2000)126:1(4), 4–15.
- Yellow River Conservancy Commission (YRCC). (2007). *The third test of water and sediment regulation conducted on the Yellow River*, Yellow River Conservancy Press, Zhengzhou, China (in Chinese).
- Zech, Y., Soares-Frazaõ, S., Spinewine, B., and Grelle, N. (2008). "Dam-break induced sediment movement: Experimental approaches and numerical modelling." *J. Hydraul. Res.*, 46(2), 176–190.
- Zhang, R., and Xie, J. (1993). *Sedimentation research in China—Systematic selections*, China Water Power Press, Beijing (in Chinese).