

## Peeling behavior of a thin-film on a corrugated surface



Zhilong Peng, Shaohua Chen\*

LNM, Institute of Mechanics, Chinese Academy of Sciences, Beijing 100190, China

### ARTICLE INFO

#### Article history:

Received 13 September 2014

Received in revised form 14 November 2014

Available online 11 February 2015

#### Keywords:

Thin-film

Corrugated substrate

Peeling behavior

Interface strength

Mode-mixity

### ABSTRACT

The peeling behavior of a thin-film perfectly adhering on a corrugated substrate is investigated theoretically. Unlike the usually adopted average method of introducing an effective adhesion energy, the effect of substrate roughness is considered directly in this paper and an accurate closed-form solution to the peel-off force under quasi-static peeling process is achieved. Comparing to the results obtained by the average method and those of a smooth substrate case shows that the peel-off force in the present model varies periodically, similar to the roughness of substrates. Furthermore, it is interesting to find that the peeling strength (defined by the maximal peel-off force) of the corrugated interface can be significantly improved with the increase of substrate roughness, while the peel-off force obtained by the average method was found to decrease monotonically or increase first and then decrease with the increasing surface roughness. Spontaneous detachment happens locally at the valley or crest of each asperity when the substrate roughness is large enough, but it does not influence the enhanced trend of the maximal peel-off force. The effect of mode-mixity dependent interface adhesion energy on the peel-off force is also considered, by which the interface peeling strength is further improved. The results in this paper should be helpful for deep understanding of the interface behavior between a film and a rough substrate and be useful for the design of film/substrate interfaces with high interface quality in nano-devices.

© 2015 Elsevier Ltd. All rights reserved.

### 1. Introduction

Adhesion and peeling mechanisms of thin-films on substrates have been attracting considerable attention because film/substrate systems are ubiquitous in many applications and other technologies, such as automobiles, micro-electromechanical systems, coating technology as well as biological adhesion (Kim et al., 1989; Peng and Chen, 2011, 2012; Peng et al., 2010; Pesika et al., 2007; Sauer, 2011; Thouless and Jensen, 1992; Tian et al., 2006). The interface adhesion strength and adhesion energy are two important properties for materials protecting, connecting and strengthening as well as designing of high-quality interfaces (Wei and Hutchinson, 1998). Peel-test, as a classical technique, is one of the efficient method for assessing the interface mechanical properties (Spies, 1953).

The peeling behaviors of thin-films on substrates have been widely investigated experimentally and theoretically (Chen et al., 2013; Kim et al., 1989; Kinloch et al., 1994; Sauer, 2011; Thouless and Jensen, 1992; Wei and Hutchinson, 1998). One of the most widely used theoretical models for elastic thin-films is the classical Kendall's peeling theory (Kendall, 1975), which shows

the peel-off force depends not only on the interface adhesion energy but also on the elastic deformation of films as well as the peeling angle. It has provided a direct measuring method to achieve the interfacial properties with the help of the peel-off force, for example, the adhesion strength and adhesion energy. Based on such a pioneer work, extensive studies including the adhesion mechanism of elastic-plastic thin-film (Kim and Kim, 1988; Kinloch et al., 1994; Wei and Hutchinson, 1998), visco-elastic thin-film (Chen et al., 2013; Loukis and Aravas, 1991), heterogeneous thin-film (Xia et al., 2012, 2013) have been carried out. Recently, the peeling model has also been applied in the field of bio-inspired study on gecko adhesion (Peng and Chen, 2012; Peng et al., 2010; Pesika et al., 2007; Sauer, 2011; Tian et al., 2006).

However, most of the works focused on thin-films adhering on smooth and flat substrates. As we know, natural surfaces, even highly polished ones, possess roughness in many different length scales. Significant influence of surface roughness on adhesion between thin-films and substrates has been found (DelRio et al., 2007; Fuller and Tabor, 1975; Persson, 2002). The pioneering study was carried out by Fuller and Tabor (1975), in which a theoretical model was established based on a Gaussian distribution assumption of surface asperity and the whole contact force was obtained using the JKR model (Johnson et al., 1971) for each individual asperity. It was found that relatively small surface roughness could

\* Corresponding author. Tel.: +86 10 82543960.

E-mail address: [chenshaohua72@hotmail.com](mailto:chenshaohua72@hotmail.com) (S. Chen).

reduce or even remove adhesion. Later, an average method by introducing an effective adhesion energy  $\Delta\gamma_{\text{eff}}$  was proposed by Persson (2002), Persson and Gorb (2003), Persson and Tosatti (2001) and Palasantzas and De Hosson (2003a,b) with  $\Delta\gamma_{\text{eff}}A_0 = \Delta\gamma A - U_{\text{el}}$ , where  $A_0$  is defined as a nominal contact area,  $A$  the true atomic contact area,  $U_{\text{el}}$  the elastic bending energy and  $\Delta\gamma$  the adhesion energy of a smooth and flat surface. It was shown that whether the surface roughness increases the adhesion force depends on the competition between the increasing adhesion energy  $\Delta\gamma(A - A_0)$  and the bending elastic energy  $U_{\text{el}}$  stored in films. Inspired by gecko adhesion on rough surface, the present authors studied a model of nano-thin films with finite length in adhesive contact with a sinusoidal surface and the results consist well with the experimental findings (Peng and Chen, 2011). From above, one can see that the average method actually equalizes a rough surface to a flat one by adopting the effective adhesion energy  $\Delta\gamma_{\text{eff}}$ . Although the average method provides valuable insights on the adhesion of thin-films on rough surfaces, detailed peeling process is neglected. Recently, numerical calculations have been carried out to simulate the peeling behaviors of thin-films on rough substrates. For example, molecular dynamics simulation was used to study the peeling process of a graphene sheet on a corrugated surface (Chen and Chen, 2013), in which the peeling force varies with surface roughness during the peeling process and the maximum of the peeling force is much larger than the average one. Finite element calculation was carried out to simulate the peeling behavior of thin-films bonded on substrates of different surface morphologies, including a flat surface, a sinusoidal one and a wavy surface with two-level sinusoidal characteristics (Zhao et al., 2013), where the interface strength (maximum of the peeling force) can be significantly improved by the substrate roughness, especially in the hierarchical case. Another noticeable field related to thin-films on corrugated substrates is the stretchable electronics (Feng et al., 2007; Jiang et al., 2007; Khang et al., 2008; Meitl et al., 2006; Wu et al., 2011). When a thin-film is deposited onto a pre-strained elastomeric substrate or compress a structure that consist of a thin-film on a compliant substrate, this can create, through nonlinear buckling process, well-defined sinusoidal distributions of surface topography with the thin-film completely adhering on the substrate (Jiang et al., 2007; Khang et al., 2008). Most of the recent experimental and theoretical researches have focused on understanding and controlling thin-film buckling to form a corrugated interface (Bowden et al., 1998; Chung et al., 2011; Hendricks et al., 2010; Jiang et al., 2007). Few studies focus on the adhesion behaviors between the buckling thin-film and substrate. Chan et al. (2008) and Lin et al. (2008) respectively measured the adhesion force between a corrugated compliant substrate and a glass probe, but opposite results are derived. Therefore, systematically theoretical analysis of the detailed peeling process for such an interface is still lacking.

In this paper, a theoretical model of a thin-film in adhesive contact with a sinusoidal rough surface is established and the effect of substrate roughness on the peeling behavior is considered directly with different peeling angles. Influence of the mode-mixity dependent interface adhesion energy on the peel-off force is also investigated in the corrugated model. The results derived in the present paper are further compared with those predicted by the average method and those in a flat substrate case.

## 2. Theoretical model of a thin-film peeling from a corrugated substrate

Considering an elastic thin-film peeled quasi-statically from a rough substrate with sinusoidal surface morphology as shown in Fig. 1. The film adheres perfectly on the rough substrate with a

length  $l$  and the length  $L - l$  of the film is peeled-off at a peeling angle  $\theta_F$  under a peeling force  $F$  acted at the left end of the film. The length of the thin-film  $L$  is assumed to be long enough so that the tangential angle at the left end of the film is always equal to the peeling angle, i.e.,  $\theta_L = \theta_F$ . The tangential angle of each point on the film is defined as  $\theta$  with respect to the horizontal plane.  $E$  and  $h$  denote the Young's modulus and thickness of the thin-film, respectively. Without loss of generality (Chen and Chen, 2013; Liu et al., 2007; Peng and Chen, 2011; Zhao et al., 2013), the surface roughness is assumed to abide by a sinusoidal function  $y = a - \cos(kx)$ , where  $a$  is the amplitude of the roughness,  $k = 2\pi/\lambda$  the wave number and  $\lambda$  the wavelength.

The potential energy of the film/substrate system at the state shown in Fig. 1 can be expressed as,

$$E = \int_0^l \frac{D}{2} \theta'^2 ds + \int_l^L \frac{D}{2} \theta'^2 ds + \int_l^L \frac{1}{2} E \varepsilon^2 h ds - \vec{F} \cdot \vec{u}_F - \int_l^L F \varepsilon ds - \int_0^l \Delta\gamma ds \quad (1)$$

where the first and second terms on the right hand side of Eq. (1) are the bending elastic energy, and  $D$  is the bending stiffness of the film,  $s$  is the arc-length of the film from the origin  $o$ . The third term on the right hand side is the tension strain energy, where the elastic strain of the film is  $\varepsilon = F \cos(\theta - \theta_F)/(Eh)$ . The fourth and fifth terms are the potential of the external applied force  $\vec{F}$ , and the last term is the interfacial adhesion energy. The corresponding displacement  $\vec{u}_F$  of the loading point can be given as (Xia et al., 2013),

$$\vec{u}_F = \int_0^L \begin{pmatrix} \cos \theta - \cos \theta_F \\ \sin \theta - \sin \theta_F \end{pmatrix} ds \quad (2)$$

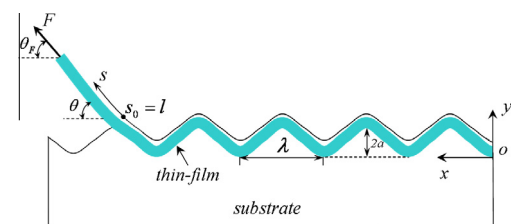
which is measured relative to a reference position at

$$\begin{pmatrix} L \cos \theta_F \\ L \sin \theta_F \end{pmatrix} \quad (3)$$

The elastic bending energy of the film in the perfectly bonding region  $(0, l)$  can be obtained explicitly as,

$$\begin{aligned} U(l) &= \int_0^l \frac{D}{2} \theta'^2 ds = \int_0^{x(l)} \frac{D}{2} \left( \frac{\partial^2 y}{\partial x^2} \right)^2 dx \\ &= \frac{Eh^3}{24} \int_0^{x(l)} a^2 k^4 \cos^2(kx) dx \\ &= \frac{Eh^3}{48} a^2 k^4 \left\{ x(l) + \frac{1}{2k} \sin[2kx(l)] \right\} \end{aligned} \quad (4)$$

where the bonding length  $l$  is a variable with  $l = \int_0^{x(l)} \sqrt{1 + a^2 k^2 \sin^2(kx)} dx$ . Substituting Eqs. (2) and (4) into Eq. (1) yields,



**Fig. 1.** Schematic of an elastic thin-film peeled from a sinusoidal rough substrate. The segment  $(0, l)$  of the film adheres perfectly on the rough substrate, and the part  $(l, L)$  is peeled-off under a peeling force  $F$  at a peeling angle  $\theta_F$ .  $\theta$  is the tangential angle of each point on the film with respect to a flat referred surface,  $\lambda$  is the wavelength and  $a$  is the amplitude of the roughness.

$$\begin{aligned}
E &= \int_0^l \frac{D}{2} \theta'^2 ds + \int_l^L \frac{D}{2} \theta'^2 ds + \int_l^L \frac{1}{2} E \varepsilon^2 h ds - \vec{F} \cdot \vec{u}_F - \int_l^L F \varepsilon ds \\
&\quad - \int_0^l \Delta \gamma ds \\
&= \int_l^L \frac{D}{2} \theta'^2 ds + \int_l^L \frac{F^2}{2Eh} \cos^2(\theta - \theta_F) ds \\
&\quad - \int_0^l F [\cos(\theta - \theta_F) - 1] ds - \int_l^L \frac{F^2}{Eh} \cos(\theta - \theta_F) ds + U(l) \\
&\quad - \Delta \gamma l \tag{5}
\end{aligned}$$

Let the first variation of the potential energy in Eq. (5) with respect to  $\theta$  and  $l$  equal zero,

$$\delta E = \delta E_1 + \delta E_2 = 0 \tag{6}$$

where

$$\begin{aligned}
\delta E_1 &= - \int_l^L \left[ D \theta'' + \frac{F^2}{Eh} \cos(\theta - \theta_F) \sin(\theta - \theta_F) - F \sin(\theta - \theta_F) \right. \\
&\quad \left. - \frac{F^2}{Eh} \sin(\theta - \theta_F) \right] \delta \theta ds + D \theta' \delta \theta \Big|_l^L \tag{7}
\end{aligned}$$

and

$$\begin{aligned}
\delta E_2 &= \left[ \frac{1}{2} D \theta'^2 - \frac{F^2}{2Eh} \cos^2(\theta - \theta_F) + \frac{F^2}{Eh} \cos(\theta - \theta_F) \cos(\theta_L - \theta_F) \right] \Big|_{s=l} \delta l \\
&\quad + \frac{dU}{dl} \delta l - \Delta \gamma \delta l \tag{8}
\end{aligned}$$

here  $\frac{dU}{dl} = \frac{dU}{dx} \frac{dx}{dl} = \frac{Eh^3}{48\sqrt{1+a^2k^2} \sin^2[kx(l)]} a^2 k^4 \{1 + \cos[2kx(l)]\}$ .

Combining the boundary conditions  $\theta(l) = \varphi$ ,  $\theta(L) = \theta_F$ ,  $\theta'(L) = 0$ , where  $\varphi = \arctan\{ak \sin[kx(l)]\}$ , leads to the quasi-statically governing equations

$$\begin{aligned}
D \theta'' + \frac{F^2}{Eh} \cos(\theta - \theta_F) \sin(\theta - \theta_F) - F \sin(\theta - \theta_F) \\
- \frac{F^2}{Eh} \sin(\theta - \theta_F) = 0 \tag{9}
\end{aligned}$$

and

$$\begin{aligned}
\frac{1}{2} D \theta'^2 - \frac{F^2}{2Eh} \cos^2(\theta_F - \varphi) + \frac{F^2}{Eh} \cos(\theta_F - \varphi) \\
+ \frac{Eh^3 a^2 k^4 \{1 + \cos[2kx(l)]\}}{48\sqrt{1+a^2k^2} \sin^2[kx(l)]} = \Delta \gamma \tag{10}
\end{aligned}$$

Multiplying  $\theta'$  on both sides of Eq. (9) and then integrating it from  $\theta(l)$  to  $\theta(L)$ , we can simplify Eq. (9) with the help of Eq. (10) as

$$\frac{F^2}{2Eh} + F[1 - \cos(\theta_F - \varphi)] - \Delta \gamma + \frac{Eh^3 a^2 k^4 [1 + \cos(2kx)]}{48\sqrt{1+a^2k^2} \sin^2(kx)} = 0 \tag{11}$$

Introducing dimensionless parameters  $\bar{F} = F/Eh$ ,  $\bar{\Delta \gamma} = \Delta \gamma/Eh$ ,  $\bar{a} = a/\lambda$ ,  $\bar{h} = h/\lambda$  and  $\bar{x} = x/\lambda$ . Eq. (11) can be rewritten as

$$\frac{1}{2} \bar{F}^2 + \bar{F}[1 - \cos(\theta_F - \varphi)] - \bar{\Delta \gamma} + \frac{(2\pi)^4 \bar{a}^2 \bar{h}^2 [1 + \cos(4\pi \bar{x})]}{48\sqrt{1+(2\pi)^2 \bar{a}^2 \sin^2(2\pi \bar{x})}} = 0 \tag{12}$$

Solving Eq. (12) yields a closed-form solution to the peel-off force

$$\begin{aligned}
\bar{F} &= -[1 - \cos(\theta_F - \varphi)] \\
&\quad + \sqrt{[1 - \cos(\theta_F - \varphi)]^2 + 2\bar{\Delta \gamma} - \frac{(2\pi)^4 \bar{a}^2 \bar{h}^2 [1 + \cos(4\pi \bar{x})]}{24\sqrt{1+(2\pi)^2 \bar{a}^2 \sin^2(2\pi \bar{x})}}} \tag{13}
\end{aligned}$$

One may note that, for the special case of a thin-film adhering on a flat substrate, i.e.,  $\varphi = 0$ ,  $a/\lambda = 0$ , Eq. (13) can be reduced to

$$\bar{F} = -(1 - \cos \theta_F) + \sqrt{(1 - \cos \theta_F)^2 + 2\bar{\Delta \gamma}} \tag{14}$$

which is well consistent with the Kendall's state-steady peeling model (Kendall, 1975).

The peel-off force  $\bar{F}$  in Eq. (13) is a function of the adhesion length  $\bar{x}$ . Using the following conditions

$$\begin{cases} \frac{\partial \bar{F}}{\partial \bar{x}} = 0 \\ \frac{\partial^2 \bar{F}}{\partial \bar{x}^2} < 0 \\ \theta_F - \varphi = \min(\theta_F - \varphi) \end{cases} \tag{15}$$

leads to the maximum one,

$$\begin{cases} \bar{F}_{\max} = -[1 - \cos(\theta_F - \varphi_{\bar{x}})] + \sqrt{[1 - \cos(\theta_F - \varphi_{\bar{x}})]^2 + 2\bar{\Delta \gamma}} \\ \bar{x} = n + 1/4, n = 0, 1, 2, \dots \end{cases} \tag{16}$$

The peel-off force obtained with the average method proposed by Persson (2002), Persson and Gorb (2003), Persson and Tosatti (2001) for a thin-film peeled off from a corrugated substrate is also given for comparison

$$\bar{F} = -(1 - \cos \theta_F) + \sqrt{(1 - \cos \theta_F)^2 + 2\bar{\Delta \gamma}_{\text{eff}}} \tag{17}$$

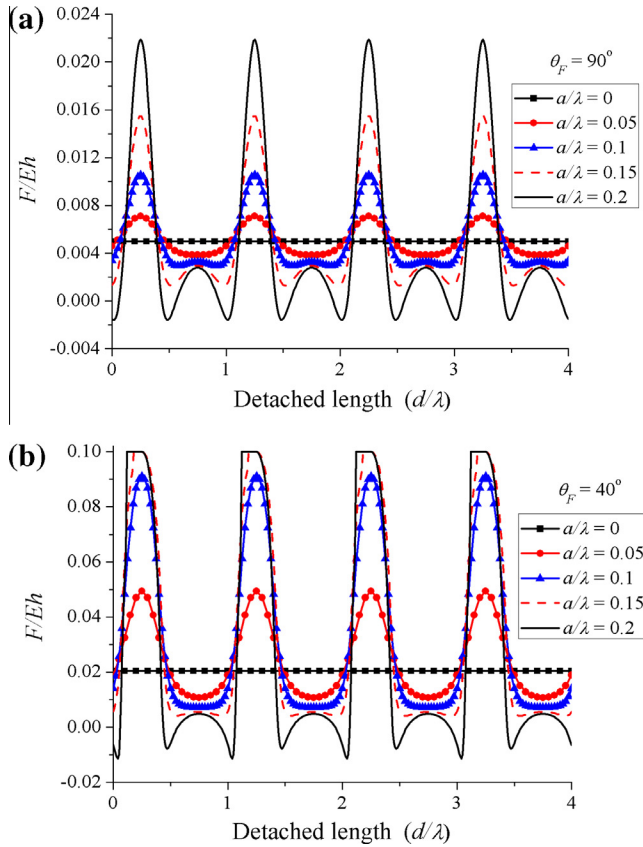
where the effective adhesion energy  $\bar{\Delta \gamma}_{\text{eff}}$  is expressed as

$$\bar{\Delta \gamma}_{\text{eff}} = \bar{\Delta \gamma} \frac{\bar{L}}{\bar{L}_0} - \frac{(2\pi)^4 \bar{a}^2 \bar{h}^2}{48} \left[ 1 + \frac{\sin(4\pi \bar{L}_0)}{4\pi \bar{L}_0} \right] \tag{18}$$

and  $\bar{L} = \int_0^{\bar{L}_0} \sqrt{1 + (2\pi)^2 \bar{a}^2 \sin^2(2\pi \bar{x})} d\bar{x}$  is the non-dimensionally true length of the film,  $\bar{L}_0$  is the nominal length of the film.

### 3. Results and discussion

The peel-off force varying as a function of the detached length for cases of different surface roughness is shown in Fig. 2(a) and (b) with the peeling angle  $\theta_F = 90^\circ$  and  $40^\circ$ , respectively. The other non-dimensional parameters take values of  $\Delta \gamma/Eh = 0.005$ ,  $h/\lambda = 0.05$ . It is found that the peel-off force varies with the same periodicity of the surface roughness, while it keeps a constant for the case of flat substrate ( $a/\lambda = 0$ ). However, the varying function of the peel-off force of the present model is not a sinusoidal or cosine one, which depends on the locally varying peeling angle and bending energy at each point. As a result, the minimal peeling-off force always emerges at the valley or crest of each asperity, where the bending energy and the local peeling angle  $\theta_F - \varphi$  take the maximum simultaneously. Interestingly, the value of the minimal peel-off force is zero or even negative in the case of relatively large surface roughness, which suggests that spontaneous detachment would happen at these locations and is well consistent with the MD simulation results (Chen and Chen, 2013). The maximal peel-off force increases significantly with the increasing surface roughness, and it is always larger than that on flat substrates ( $a/\lambda = 0$ ), which means that the peeling strength of perfectly adhering interface (often defined by the maximal peeling force) can be significantly improved by introduction of the interface rough morphology. The results are well consistent with the numerical ones obtained by Zhao et al. (2013), in which the peeling behavior of a thin-film bonded to a rough substrate was simulated and three kinds of film/substrate interfaces were considered, including a flat interface, a wavy sinusoidal one and a wavy one with two-level sinusoidal hierarchy. A periodically varying peeling force was found and the interface strength could be significantly



**Fig. 2.** The non-dimensional peel-off force  $F/Eh$  varying along the detached length during the peeling process with different surface roughness and peeling angles. (a) For  $\theta_F = 90^\circ$ ; (b) for  $\theta_F = 40^\circ$ .

improved by the wavy interface, especially by a hierarchically wavy one. Furthermore, similar improvement of peel-off force on rough substrates was also found experimentally in a visco-elastic film case (Peng et al., 2014).

According to the Kendall's model, the maximal peel-off force always emerges at the location with the smallest true peeling angle of  $\theta_F - \varphi$ . As mentioned above, the angle  $\varphi$  ( $\varphi = \arctan[ak \sin(kx)]$ ) on a rough surface varies at each point. For a fixed small peeling angle  $\theta_F$ , a region of  $\theta_F - \varphi \leq 0$  cannot be avoided, especially for larger surface roughness. If the tangential friction along the interface between the thin-film and rough substrate is not considered, the peeling behavior of the film at the region  $\theta_F - \varphi \leq 0$  is similar to that of a thin-film peeled from a flat substrate at  $0^\circ$  peeling angle. As a result, the maximal peel-off force will keep a constant ( $\bar{F}_{\max} = \sqrt{2\Delta\gamma}$ ) in the regions ( $\theta_F - \varphi \leq 0$ ) as shown in Fig. 2(b).

Fig. 3 shows the relation between the maximal peel-off force and surface roughness. The maximal peel-off force increases monotonically with the increase of surface roughness for the case of relatively large peeling angles  $\theta_F$ ; while it increases first and then keeps a constant for the case of relatively small peeling angles  $\theta_F$ , which is consistent with the results in Fig. 2(b). With a given surface roughness, the peeling strength (maximal peel-off force) decreases with the increasing peeling angle. The detailed relationship between the maximal peel-off force and the peeling angle can be found in Fig. 4, where the peel-off force varying with the peeling angle for the case of flat substrate ( $a/\lambda = 0$ ) is also given for comparison. It shows that the peeling strength on rough substrates is always larger than that on flat one at a given peeling angle.

For comparison, the peel-off force obtained by the average method of Eq. (17) is also shown in Fig. 5 as a function of the

surface roughness  $a/\lambda$  for different adhesion energies with fixed peeling angle  $\theta_F = 90^\circ$  and  $h/\lambda = 0.05$ . The peel-off force exhibits significant dependence on the non-dimensional adhesion energy of smooth surfaces  $\Delta\gamma/Eh$ . For a relatively small adhesion energy  $\Delta\gamma/Eh$ , the peel-off force decreases monotonically with the increase of surface roughness. While for a large adhesion energy, the peel-off force increases first and then decreases after reaching a maximum with the increase of surface roughness, which mainly results from the competition between the bending energy stored in the film and the interface adhesion energy. The characteristic of the results in Fig. 5 is typical for the average method (Persson, 2002; Persson and Gorb, 2003; Persson and Tosatti, 2001), in which an effective adhesion energy was adopted to simplify the effect of surface roughness on interface strength. The trends of interface strength varying with surface roughness are obviously different for the average method and the present model, which is mainly due to the effective adhesion energy  $\Delta\gamma_{\text{eff}}/Eh$  in Eq. (18) depending nonlinearly on the value of adhesion energy of smooth surfaces  $\Delta\gamma/Eh$  and surface roughness  $a/\lambda$ .

One may note that the adhesion energy  $\Delta\gamma$  in the above analysis is assumed to be a material constant. Actually, it is strictly not a constant, but depends on the interface mode-mixity (Chen et al., 2009; Evans et al., 1990). A commonly used expression of the mode-mixity dependent interface adhesion energy is (Hutchinson and Suo, 1992)

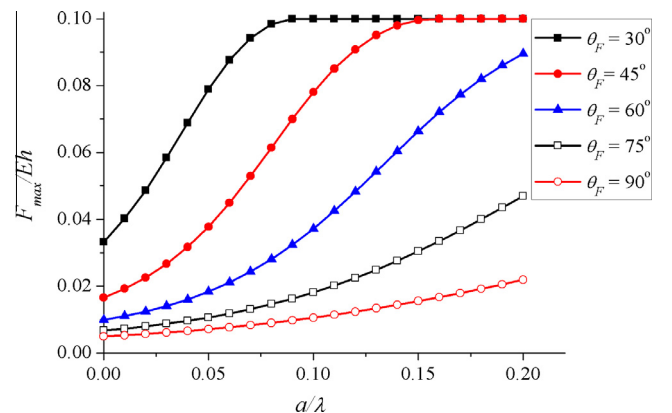
$$\Delta\gamma(\psi) = \frac{\Delta\gamma}{1 - (1 - \lambda') \sin^2 \psi} \quad (19)$$

where  $\psi$  is the phase angle,  $\lambda'$  is a parameter adjusting the influence of mode II contribution, ranging from 0 to 1.

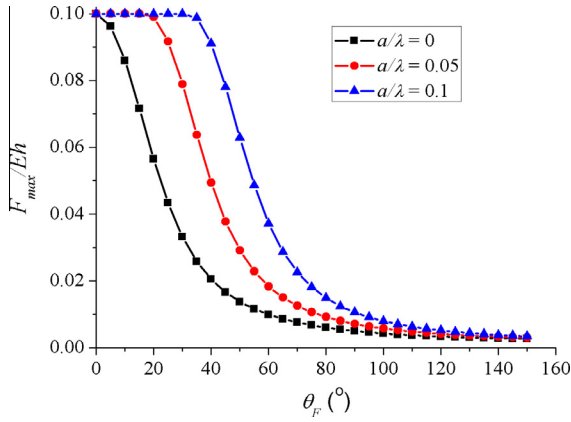
The phase angle was defined as  $\psi = \tan^{-1}(K_{II}/K_I)$  (Hutchinson and Suo, 1992), where  $K_I$  and  $K_{II}$  are mode I and II stress intensity factors induced by the normal and tangential interface tractions, respectively. Thouless and Jensen (1992) have studied the effect of phase angle on the peeling behaviors of a thin-film contact with a substrate. Later, Li et al. (2004) further derived a general expression of the phase angle, considering the effect of transverse shear on the delamination in layered materials. In the present work, the model proposed by Thouless and Jensen (1992) is adopted for simplicity and the expression for the phase angle is

$$\psi = \arctan \frac{\sqrt{\sin^2 \phi + 2(1 - \cos \phi)/(F/Eh) + \tan \omega \cos \phi}}{-\tan \omega \sqrt{\sin^2 \phi + 2(1 - \cos \phi)/(F/Eh) + \cos \phi}} \quad (20)$$

where  $\phi = \theta_F - \varphi$  is the local (true) peeling angle in the present model,  $\omega$  is a scalar function depending on Dundur's parameters



**Fig. 3.** The maximal peel-off force (peeling strength) as a function of the substrate roughness at different peeling angles.



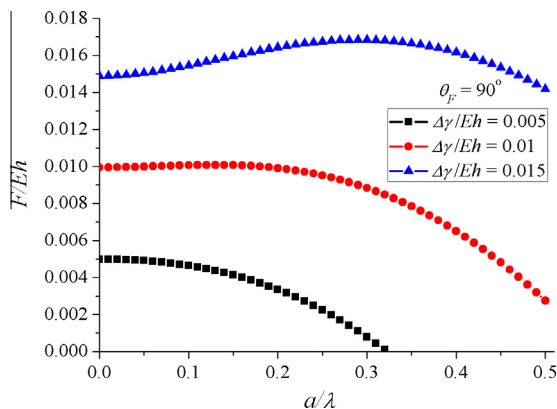
**Fig. 4.** Variation of the maximal peel-off force at different peeling angles in cases of different substrate roughness.  $a/\lambda = 0$  corresponds to the flat substrate case, which is given for comparison.

due to material mismatch between thin-films and substrates. [Suo and Hutchinson \(1990\)](#) has found that  $\omega$  is not sensitive to Dundur's parameters and the value of  $\omega$  varies only between about 44–66°. In this paper, we adopt  $\omega$  to be a constant 55°.

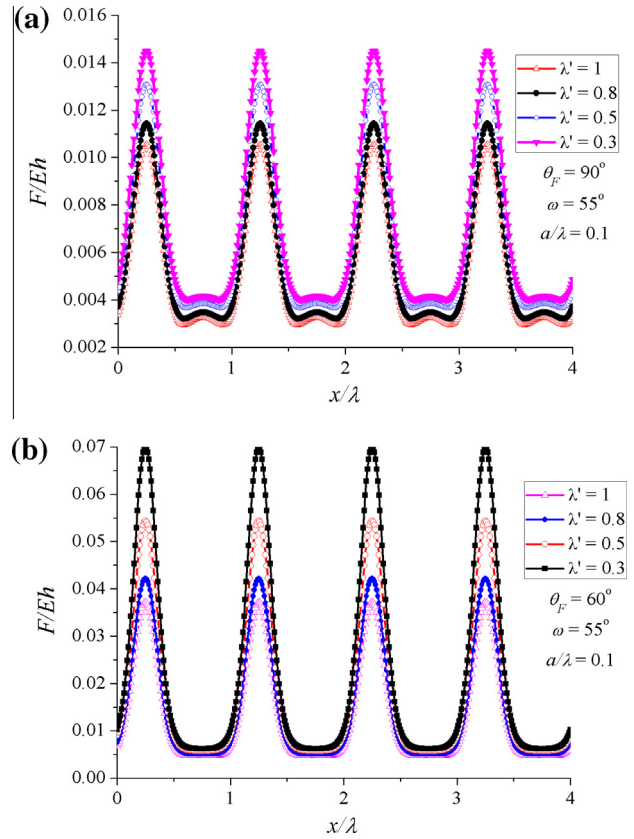
Considering the effect of mode-mixity on the interface adhesion energy, we can rewrite Eq. (12) as

$$\frac{1}{2}\bar{F}^2 + \bar{F}[1 - \cos(\theta_F - \varphi)] - \overline{\Delta\gamma}(\psi) + \frac{(2\pi)^4 \bar{a}^2 \bar{h}^2 [1 + \cos(4\pi\bar{x})]}{48\sqrt{1 + (2\pi)^2 \bar{a}^2 \sin^2(2\pi\bar{x})}} = 0 \quad (21)$$

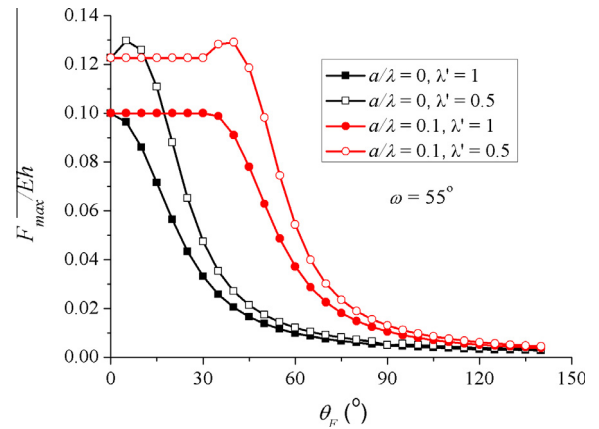
The peel-off force varying with the detached length is shown in [Fig. 6](#) for different values of  $\lambda'$  and determined surface roughness. The peeling angle in [Fig. 6\(a\)](#) is 90° and that is in [Fig. 6\(b\)](#) is 60°. Both [Fig. 6\(a\)](#) and (b) show that the varying trend of the peel-off force is not influenced qualitatively, but quantitatively and the peel-off force is entirely increased with the decrease of values of  $\lambda'$ . Furthermore, the maximal peel-off force is further enhanced by the mode-mixity dependent adhesion energy, comparing to the case of a constant adhesion energy. The maximal peel-off force as a function of peeling angles are shown in [Fig. 7](#), where cases of a flat substrate and a rough substrate are involved and two kinds of values of  $\lambda'$  are considered, respectively. We find that when the effect of mode-mixity is taken into account, the maximal peel-off is not occurred at 0° peeling angle, but shift to a small peeling angle. The peeling strength in the case of mode-mixity dependent adhesion energy is always larger than that of a constant one for both flat and rough substrates.



**Fig. 5.** The peel-off force obtained by the average method varying with surface roughness with different adhesion energies.



**Fig. 6.** The peel-off force varying along the detached length when the mode-mixity dependent adhesion energy is considered with different values of  $\lambda'$ . (a) For  $\theta_F = 90^\circ$ ; (b) for  $\theta_F = 60^\circ$ .



**Fig. 7.** The effect of mode-mixity dependent adhesion energy on the relation between the maximal peel-off force and the peeling angle with different surface roughness.

#### 4. Conclusions

Quasi-statically peeling behavior of an elastic thin-film perfectly adhering on a rough substrate with sinusoidal morphology is theoretically considered. The detailed peeling process of the thin-film from the rough substrate is analyzed. Closed-form solutions to the peel-off force and the maximal one are achieved, and compared with the results obtain by the previous average method and that on flat substrate. It is shown that the peel-off force undergoes a periodic variation during the peeling process and the maximal

peel-off force is significantly improved by the surface roughness, which increases with the increase of surface roughness. An interesting finding is that the thin-film can spontaneously detach locally from the substrate for relative large roughness, which is well consistent with the existing numerical results. The effect of mode-mixity dependent adhesion energy on the peel-off force is further considered, which can improve the peel-off force during the peeling process, comparing with the case of a constant adhesion energy. The results in this paper should be helpful not only for deep understanding of detailed peeling behaviors of films adhering on rough substrates but also for the design of micro and nano-devices related to film/substrate systems, such as stretchable electronics.

### Acknowledgments

The work reported here is supported by NSFC through Grants #11302228, #11125211, #11372317 and the 973 Nano-project (2012CB937500).

### References

- Bowden, N., Brittain, S., Evans, A.G., Hutchinson, J.W., Whitesides, G.M., 1998. Spontaneous formation of ordered structures in thin films of metals supported on an elastomeric polymer. *Nature* 393, 146–149.
- Chan, E.P., Smith, E.J., Hayward, R.C., Crosby, A.J., 2008. Surface wrinkles for smart adhesion. *Adv. Mater.* 20, 711–716.
- Chen, H., Chen, S.H., 2013. The peeling behaviour of a graphene sheet on a nano-scale corrugated surface. *J. Phys. D – Appl. Phys.* 46, 435305.
- Chen, B., Wu, P.D., Gao, H.J., 2009. Pre-tension generates strongly reversible adhesion of a spatula pad on substrate. *J. R. Soc. Interface* 6, 529–537.
- Chen, H., Feng, X., Huang, Y., Huang, Y.G., Rogers, J.A., 2013. Experiments and viscoelastic analysis of peel test with patterned strips for applications to transfer printing. *J. Mech. Phys. Solids* 61, 1737–1752.
- Chung, J.Y., Nolte, A.J., Stafford, C.M., 2011. Surface wrinkling: a versatile platform for measuring thin-film properties. *Adv. Mater.* 23, 349–368.
- DelRio, F.W., Dunn, M.L., Phinney, L.M., Bourdon, C.J., De Boer, M.P., 2007. Rough surface adhesion in the presence of capillary condensation. *Appl. Phys. Lett.* 90, 163104.
- Evans, A.G., Ruhle, M., Dalgleish, B.J., Charalambides, P.G., 1990. The fracture energy of bimaterial interfaces. *Mater. Sci. Eng. A – Struct. Mater. Prop. Microstruct. Process.* 126, 53–64.
- Feng, X., Meitl, M.A., Bowen, A.M., Huang, Y., Nuzzo, R.G., Rogers, J.A., 2007. Competing fracture in kinetically controlled transfer printing. *Langmuir* 23, 12555–12560.
- Fuller, K.N.G., Tabor, D., 1975. Effect of surface-roughness on adhesion of elastic solids. *Proc. R. Soc. Lond. A* 345, 327–342.
- Hendricks, T.R., Wang, W., Lee, I., 2010. Buckling in nanomechanical films. *Soft Matter* 6, 3701–3706.
- Hutchinson, J.W., Suo, Z., 1992. Mixed-mode cracking in layered materials. *Adv. Appl. Mech.* 29, 63–191.
- Jiang, H.Q., Khang, D.Y., Song, J.Z., Sun, Y.G., Huang, Y.G., Rogers, J.A., 2007. Finite deformation mechanics in buckled thin films on compliant supports. *Proc. Natl. Acad. Sci. USA* 104, 15607–15612.
- Johnson, K.L., Kendall, K., Roberts, A.D., 1971. Surface energy and contact of elastic solids. *Proc. R. Soc. Lond. A* 324, 301–313.
- Kendall, K., 1975. Thin-film peeling – elastic term. *J. Phys. D – Appl. Phys.* 8, 1449–1452.
- Khang, D.Y., Xiao, J.L., Kocabas, C., Maclaren, S., Banks, T., Jiang, H.Q., Huang, Y.Y.G., Rogers, J.A., 2008. Molecular scale buckling mechanics on individual aligned single-wall carbon nanotubes on elastomeric substrates. *Nano Lett.* 8, 124–130.
- Kim, K.S., Kim, J., 1988. Elasto-plastic analysis of the peel test for thin-film adhesion. *J. Eng. Mater. Technol. – Trans. ASME* 110, 266–273.
- Kim, J., Kim, K.S., Kim, Y.H., 1989. Mechanical effects in peel adhesion test. *J. Adhes. Sci. Technol.* 3, 175–187.
- Kinloch, A.J., Lau, C.C., Williams, J.G., 1994. The peeling of flexible laminates. *Int. J. Fract.* 66, 45–70.
- Li, S., Wang, J., Thouless, M.D., 2004. The effects of shear on delamination in layered materials. *J. Mech. Phys. Solids* 52, 193–214.
- Lin, P.C., Vajpayee, S., Jagota, A., Hui, C.Y., Yang, S., 2008. Mechanically tunable dry adhesive from wrinkled elastomers. *Soft Matter* 4, 1830–1835.
- Liu, J.L., Feng, X.Q., Wang, G.F., Yu, S.W., 2007. Mechanisms of superhydrophobicity on hydrophilic substrates. *In: J. Phys. – Condes. Matter* 19, 356002.
- Loukis, M.J., Aravas, N., 1991. The effects of viscoelasticity in the peeling of polymeric films. *J. Adhes.* 35, 7–22.
- Meitl, M.A., Zhu, Z.T., Kumar, V., Lee, K.J., Feng, X., Huang, Y.Y., Adesida, I., Nuzzo, R.G., Rogers, J.A., 2006. Transfer printing by kinetic control of adhesion to an elastomeric stamp. *Nat. Mater.* 5, 33–38.
- Palasantzas, G., De Hosson, J.T.M., 2003a. Influence of roughness on the detachment force of elastic films from self-affine rough surfaces. *J. Appl. Phys.* 94, 3041–3044.
- Palasantzas, G., De Hosson, J.T.M., 2003b. Influence of surface roughness on the adhesion of elastic films. *Phys. Rev. E* 67, 021604.
- Peng, Z.L., Chen, S.H., 2011. Effects of surface roughness and film thickness on the adhesion of a bioinspired nanofilm. *Phys. Rev. E* 83, 051915.
- Peng, Z.L., Chen, S.H., 2012. Effect of pre-tension on the peeling behavior of a bio-inspired nano-film and a hierarchical adhesive structure. *Appl. Phys. Lett.* 101, 163702.
- Peng, Z.L., Chen, S.H., Soh, A.K., 2010. Peeling behavior of a bio-inspired nano-film on a substrate. *Int. J. Solids Struct.* 47, 1952–1960.
- Peng, Z.L., Wang, C., Chen, L., Chen, S.H., 2014. Peeling behavior of a viscoelastic thin-film on a rigid substrate. *Int. J. Solids Struct.* 51, 4596–4603.
- Persson, B.N.J., 2002. Adhesion between elastic bodies with randomly rough surfaces. *Phys. Rev. Lett.* 89, 245502.
- Persson, B.N.J., Gorb, S., 2003. The effect of surface roughness on the adhesion of elastic plates with application to biological systems. *J. Chem. Phys.* 119, 11437–11444.
- Persson, B.N.J., Tosatti, E., 2001. The effect of surface roughness on the adhesion of elastic solids. *J. Chem. Phys.* 115, 5597–5610.
- Pesika, N.S., Tian, Y., Zhao, B.X., Rosenberg, K., Zeng, H.B., McGuiggan, P., Autumn, K., Israelachvili, J.N., 2007. Peel-zone model of tape peeling based on the gecko adhesive system. *J. Adhes.* 83, 383–401.
- Sauer, R.A., 2011. The peeling behavior of thin films with finite bending stiffness and the implications on gecko adhesion. *J. Adhes.* 87, 624–643.
- Spies, G.J., 1953. The peeling test on redux-bonded joints: a theoretical analysis of the test devised by aero research limited. *Aircr. Eng. Aerosp. Technol.* 25, 64–70.
- Suo, Z.G., Hutchinson, J.W., 1990. Interface crack between two elastic layers. *Int. J. Fract.* 43, 1–18.
- Thouless, M.D., Jensen, H.M., 1992. Elastic fracture-mechanics of the peel-test geometry. *J. Adhes.* 38, 185–197.
- Tian, Y., Pesika, N., Zeng, H.B., Rosenberg, K., Zhao, B.X., McGuiggan, P., Autumn, K., Israelachvili, J., 2006. Adhesion and friction in gecko toe attachment and detachment. *Proc. Natl. Acad. Sci. USA* 103, 19320–19325.
- Wei, Y., Hutchinson, J.W., 1998. Interface strength, work of adhesion and plasticity in the peel test. *Int. J. Fract.* 93, 315–333.
- Wu, J., Kim, S., Chen, W.Q., Carlson, A., Hwang, K.C., Huang, Y.G., Rogers, J.A., 2011. Mechanics of reversible adhesion. *Soft Matter* 7, 8657–8662.
- Xia, S., Ponson, L., Ravichandran, G., Bhattacharya, K., 2012. Toughening and asymmetry in peeling of heterogeneous adhesives. *Phys. Rev. Lett.* 108, 196101.
- Xia, S.M., Ponson, L., Ravichandran, G., Bhattacharya, K., 2013. Adhesion of heterogeneous thin films – I: elastic heterogeneity. *J. Mech. Phys. Solids* 61, 838–851.
- Zhao, H.P., Wang, Y.C., Li, B.W., Feng, X.Q., 2013. Improvement of the peeling strength of thin films by a bioinspired hierarchical interface. *Int. J. Appl. Mech.* 5, 1350012.