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The instability of water-mud interface in viscous two-layer flow with large viscosity contrast

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Abstract The temporal instability of parallel viscous two-phase mixing layers is extended to current-fluid mud by considering a composite error function velocity profile. The influence of viscosity ratio, Reynolds number, and Froude number on the instability of the system are discussed and a new phenomenon never discussed is investigated based on our numerical results. It is shown that viscosity can enlarge the unstable wave number range, cause new instability modes, and certainly reduce the growth rate of Kelvin–Helmholtz (K–H) instability.
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Keywords fluid mud, Kelvin–Helmholtz (K–H) instability, large viscosity contrast, Chebyshev collocation algorithm

Two-layer flow of water and fluid mud with clear interface pervades coastal areas. Fluid mud can be characterized as a fluid with complex behavior. It can be found in estuaries with high suspended-sediment concentrations, or on muddy coasts. Behaviors of fluid mud, its deposition and especially its resuspension, are of significant importance for waterway construction.

Fluid mud typically exhibits bulk densities between 1.05–1.20 kg/m^3. It can be considered as Newtonian fluid at a sufficient low density. Fluid mud is often associated with a sharp vertical density gradient. Clear interface waves can be formed under low speed current forcing. The interface waves break up if the current is strong enough, and the fluid mud is brought into the water column. This mechanism is similar to that of classical Kelvin–Helmholtz (K–H) instability, except for the viscosity being considered here.

The classical K–H instability is associated with the steady, parallel, two-dimensional, in-viscid, uniform, stratified shear layer. It appears at interface of different layers when the velocity difference is high enough, the density difference is small enough, or the wave is short enough.[1] The K–H instability proved to be a generic instability of shear flows at large Reynolds number. The interest of most researches has been focused on the growth rate, rather than the critical Reynolds number, because the flow becomes unstable at a relatively low Reynolds number.

It is natural to enquire how viscous effects, inevitably present in real fluids, modify the results obtained from an in-viscid analysis. It is well known that viscosity has two opposite effects on the hydrodynamics stability:[2] the expected stabilizing dissipative effect and a destabilizing effect. In comparison with in-viscid theory, viscous stability theory for two-phase flow introduces additional different mechanisms for instability, such as "interfacial mode"[3,4] at small Reynolds numbers and

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"shear mode"[5,6] at sufficiently large Reynolds numbers. For fluids with large viscosity contrast, the viscosity should be included in instability analysis.

There is a large amount of literature[7–11] on the stability of parallel viscous two-phase fluid flow. The formulation of the two-layer flow stability problem involves more than six dimensionless parameters. For this complexity, studies in this field usually restrict to a small parameter region. It is generally very hard to apply these results directly to current-fluid mud.

This work has been motivated by the need to calculate the transient growth rate of instability of fluid mud under current forcing. Linear spectra is calculated numerically and the effects of Reynolds number, Froude number, viscosity ratio, and density ratio are investigated. We will restrict ourselves to the study of parallel two-layer flow at large Reynolds numbers.

According to the generalized Squire theorem,[12] it is sufficient to solve the stability problem in two dimensions. The flow configuration is shown schematically in Fig. 1. The coordinates x and y are along and perpendicular to the undisturbed interface, respectively, with the origin of y located at the interface.

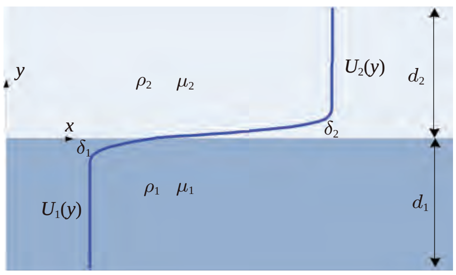

Fig. 1. Basic velocity profiles in the current and fluid mud.

The base flow for the viscous two-layer flow is composed of error-function profiles in each layer. The argument of the error function is scaled with its thickness in each phase. The base flow with time-dependent boundary layer thicknesses is exact solution of the first Stokes problem, and it serves well as the base flow for the viscous problem as $U_1(y) = U_1^*\mathrm{erf}(y/\delta_1)$ $(y < 0)$, $U_2(y) = U_2^*\mathrm{erf}(y/\delta_2)$ $(y > 0)$, where $\mathrm{erf}(y) = (2/\sqrt{\pi})\int_0^y \exp(-\xi^2)\,\mathrm{d}\xi$ and U_j^* $(j = 1,2)$ represent the asymptotic velocities. The first Stokes solution is time-dependent, its boundary layer thickness being given by $\delta_j = 3.6\sqrt{\nu_j t}$,[13] where $\nu_j = \mu_j/\rho_j$, and μ_j, ρ_j denote viscosities, densities. We introduce the density ratio $r = \rho_1/\rho_2$ and viscosity ratio $m = \mu_1/\mu_2$. Continuity of tangential stress on the interface requires $\mu_2\partial U_2/\partial y = \mu_1\partial U_1/\partial y$ at $y = 0$. Assuming that the base flow evolves from an initial K–H state, stress continuity implies $U_2^*/U_1^* = \sqrt{mr}$. We carry out an analysis of this profile in the laminar mixing layer at a particular snapshot, considering that the instability will develop faster than the boundary layers.

The thicknesses of calculation domain in the y direction are d_1, d_2 respectively, therefore we have two additional dimensionless parameters $n_1 = d_1/\delta_1, n_2 = d_2/\delta_1$.

The stability of parallel two-phase flow is investigated by disturbing the base flow infinitesimally. We assume that the flow is incompressible, and introduce the stream functions to represent the disturbance velocities (u_j, v_j) in the fluids, so that $(u_j, v_j) = (\partial\Psi_j/\partial y, -\partial\Psi_j/\partial x)$. We assume $\Psi_j(x,y,t) = \psi_j(y)\mathrm{e}^{\mathrm{i}k(x-ct)}$, where i is the imaginary unit, k is a real wave number and c is the complex wave velocity. The real part of c determines the phase velocities, and the imaginary part c_i gives the growth rates $\omega_i = c_i k$.

Substitution of the stream functions into the linearized Navier–Stokes equations results in the well-known Orr–Sommerfeld equations in dimensionless form, in which the length is scaled by the boundary thickness of fluid mud δ_1, the velocity by the characteristic velocity U_0 (here U_0 is defined as $U_0 = U_1^*$), and the pressure by $\rho_1 U_0^2$. We have $\psi_1'''' - 2k^2\psi_1'' + k^4\psi_1 + \mathrm{i}kRe[U_1(k^2\psi_1 - \psi_1'') + U_1''] = \mathrm{i}ckRe(k^2\psi_1 - \psi_1'')$, for the fluid mud $-n_1 < y < 0$, and $\psi_2'''' - 2k^2\psi_2'' + k^4\psi_2 + \mathrm{i}kRe\{(m/r)[U_2(k^2\psi_2 - \psi_2'') + U_2'']\} = \mathrm{i}ckRe[(m/r)(k^2\psi_2 - \psi_2'')]$ for the water $0 < y < n_2$.

Primes indicate differentiation with respect to y. The mud flow Reynolds number is defined as $Re = \rho_1 U_0 \delta_1/\mu_1$. Hence, the current Reynolds number is $Re_2 = \rho_2 U_2^* \delta_2/\mu_2 = mRe$.

The continuity[14] of the velocity components and the stress components at the interface $y = 0$ give four conditions as $\psi_1 = \psi_2$, $\psi_1' + U_1'\psi_1/(c-U) = \psi_2' + U_2'\psi_2/(c-U)$, $\psi_1'' + k^2\psi_1 + U_1''\psi_1/(c-U) = [\psi_2'' + k^2\psi_2 + U_2''\psi_2/(c-U)]/m$, $(\psi_1''' - 3k^2\psi_1') + \mathrm{i}kRe[(c-U)\psi_1' + U_1'\psi_1] = (\psi_2''' - 3k^2\psi_2')/m + \mathrm{i}kRe[(c-U)\psi_2' + U_2'\psi_2]/r + \mathrm{i}kRe(Fr^{-1} + k^2 S^{-1})(\psi_1' - \psi_2')/(U_2' - U_1')$ where $S = \delta_1\rho_1 U_0^2/T$ is the Weber number, T is the interfacial tension and $Fr = (\rho_1 U_0^2)/[g(\rho_1 - \rho_2)\delta_1]$ is dimensionless buoyancy, or Froude number.

The boundary conditions at the lower wall are no-penetration and no-slip, which read $\psi_1 = \psi_1' = 0$ at $y = -n_1$ and the disturbances can be ignored far from the interface, which leads to $\psi_2 = \psi_2' = 0$ for $y \to \infty$.

The calculations show that the thicknesses of fluid mud layer and depth of water layer have little influence on the stability with $n_1, n_2 \geqslant 8$. For $n_1, n_2 \leqslant 3$, the growth rate of long wave mode resulting from viscosity mismatch is sensitive to the change of n_1, n_2.

The stability problem described by the governing differential equations together with the conditions at the boundaries and the interface represents a generalized eigenvalue problem, in which the wave velocity c is the eigenvalue. A Chebyshev collocation algorithm[14] is used to evaluate the eigenvalue problem.

Our code was validated by comparing our result with that of Betchov and Szewczyk,[15] as shown in Fig. 2 with excellent agreement. We choose $\rho_2 = 1\,000$ kg/m^3, $\mu_2 = 0.97$ mPa·s, and adopt the following parameter value ranges: $r \in [1, 1.2]$, $m \in [62.5, 16\,000]$, $Re \in [1, 100]$, $Fr^{-1} \in [60, 160]$, spanning a broad range of current-fluid mud combinations. If the density and viscosity of fluid mud are specified as $\rho_1 = 1\,200$ kg/m^3, $\mu_1 = 1$ Pa·s, the above Re and Fr correspond to U_2^*=0.5–4.5 m/s, δ_1=0.01–0.50 m. Surface tension usually shows a stabilizing effect.[4,9] But to study the least stable cases, the surface tension is not included in the following calculation for simplicity.

The K–H instability results from the destabilizing effect of shear, which overcomes the stabilizing effect of stratification.[1] Its growth rate increases with wave numbers but is limited because of viscosity. There is a critical wave number k at which the growth rate is maximized. Figure 3 shows growth rates as a function of the wave number for different Fr. We see two peaks in growth rate curve over different intervals of the wave number k. The one at small k is clearly caused by the K–H mechanism. The other is caused by viscosity contrast, because the growth rate of short-wave disturbance will decay to zero due to viscosity for flow with equal viscosity between the interface, and its growth rate decreases with Fr. It can be seen in Fig. 3 that the unstable wave number range gets larger and larger as Fr decreases.

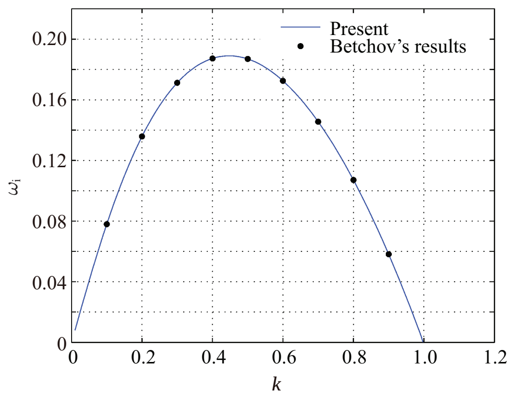

Fig. 2. Growth rate vs. wave number k for erf function velocity profile for Re=5 000, Fr^{-1}=0, S=0, m=1, r=1, n_1=8, n_2=8.

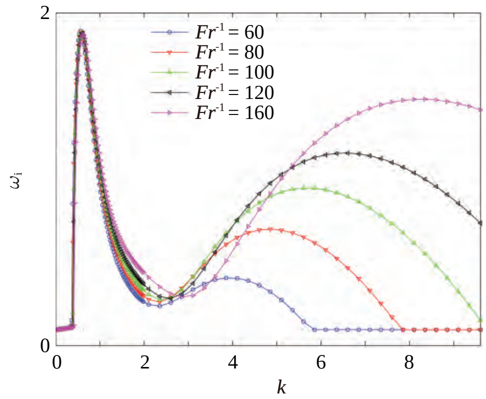

Fig. 3. The growth rate as a function of the wave number for different Fr. The flow parameters are $Re = 100, S = 0, m = 1\,000, r = 1.2, n_1 = 3, n_2 = 8$.

Similar conclusions can be drawn for Reynolds number, which is shown in Fig. 4. The dimensional growth rate ω_i^* can be expressed as $\omega_i^* = \omega_i U_0/\delta_1 = \omega_{i2} U_2^*/\delta_2$, where ω_{i2} denotes the growth rate in dimensionless form by scaling length with δ_2, velocity with U_2^*. It shows that ω_{i2} relates to ω_i by $\omega_{i2} = \omega_i/m$. In Fig. 4, the growth rate ω_i of K–H mode, and therefore ω_{i2}, is nearly independent of the Reynolds number for Reynolds numbers greater than about 10, which suggests that the mechanism is essentially governed by inertial effects rather than viscosity. Since we have $\omega_i^* = \omega_i U_0/\delta_1 = \omega_i \sqrt{1/(mr)} U_2^*/\delta_1$, if m, r are given, the dimensional growth rate is proportional to the velocity of free stream U_2^* and inversely proportional to the boundary thickness of fluid mud δ_1.

In most cases, the parameters of free stream are available, while those of fluid mud are not. In order to investigate the response of fluid mud with different viscosities to the current with the same Reynolds number of free stream, we calculated the growth rate as a function of the wave number for different viscosity ratio m, as shown in Fig. 5.

In Fig. 5, the maximum growth rate of K–H mode decreases with viscosity ratio m, and it is far below that of K–H instability in-viscid limit, which has a maximum growth rate about 0.22. If U_2^*, and hence U_2^*/δ_2 are fixed (for Re_2, ρ_2, μ_2 are fixed), the dimensional growth rate ω_i^* has a fixed ratio to the dimensionless growth rate ω_{i2}. We can infer that the viscosity has greatly reduced the growth rate of K–H instability in current-fluid mud.

It can also be seen from Fig. 5 that there are two instability modes over different intervals of wave number. One is the K–H mode already discussed above, the other is caused by viscosity mismatch, denoted as Mode II in Fig. 5. If the viscosity of the fluid mud is very high, for example, 8 000 times that of water, the growth rate of Mode II dramatically exceeds that of the K–H mode with viscosity of fluid mud 1 000 times that of water, which suggests that the growth rate does not decrease monotonously with viscosity of fluid mud.

In summary, viscosity should be included in two-layer fluid flow with large viscosity contrast such as fluid mud and current. Viscosity is found to enlarge the unstable wave number range, cause

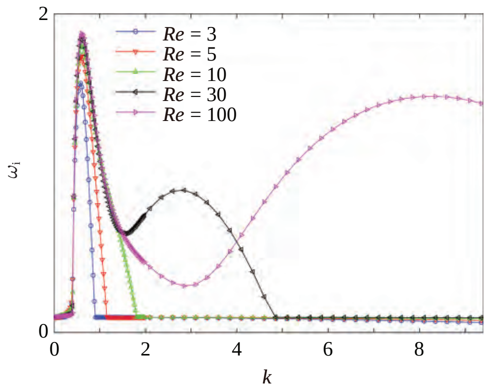

Fig. 4. The growth rate versus the wave number for different *Re*. The flow parameters are Fr^{-1}=160, S=0, m=1 000, r=1.2, n_1=3, n_2=8.

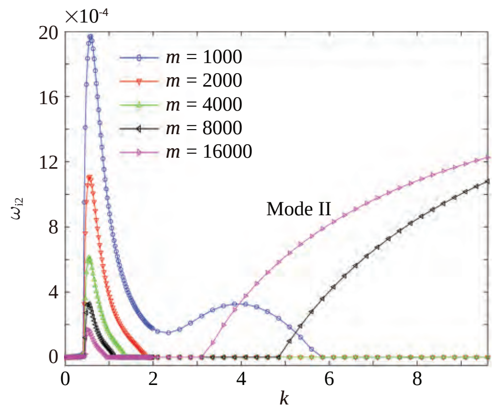

Fig. 5. The growth rate of different modes as a function of the wave number for different m. The flow parameters are Fr^{-1}=60, Re_2=100 000, S=0, r=1.2, n_1=8, n_2=8.

new instability modes, and reduce the growth rate of K–H instability. We find that a mode resulting from viscosity difference between water and fluid mud will arise at the interface, compete with, even overwhelm the K–H mode when the viscosity difference is large enough. The growth rate of K–H instability, which is nearly independent of Reynolds number of free stream, is greatly reduced by the viscosity.

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