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Mass Flow Rate and Pressure Distribution of Gas through Three-dimensional Micro-Channels

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Abstract. An effective method to predict the mass flow rate and pressure distribution of gas through three dimensional micro-channels with different cross-section shapes has been proposed. For rectangular cross sections often employed in experiment, the present solutions versus measured data of Zohar et al. (2002) show that the side walls significantly affect the mass flow rates as the aspect ratio is smaller than 10, whereas the non-dimensional pressure distributions, mainly determined by the inlet-to-outlet pressure ratio, are insensitive to the aspect ratio.

Keywords: micro-channel, rarefied gas, mass flow rate, pressure distribution, three dimensional effect
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KINETIC EQUATION OF MICROCHANNEL GAS FLOWS

When gas flowing through microchannels, it is well known that the Navier-Stokes equations with the nonslip boundary conditions can no longer be expected to be valid as the Knudsen number (Kn) based on the characteristic length of the microchannels is larger than 0.01. The crude approximation is to introduce the slip velocity and temperature jump at the surfaces of microchannels, but it also breaks down as Kn increases. It is desirable to establish a kinetic equation suitable to microchannel gas flows over the entire Knudsen regime. Shen [1] firstly made an attempt and achieved the success. He proposed that the generalized Reynolds equation used in the gas film lubrication problem [2], where the flow rates of the Poiseuille flow are calculated from the Boltzmann equation, can be degenerated for solving the microchannel gas flow problem (see also [3] for comprehensive discussion on Shen's approach by C. Cercignani). The theory proposed by Shen are mainly considered for two dimensional flow, however, the cross section of the channel may have significant effect on gas flow in micro-channels due to the high surface-area ratio. To explore the role of cross section, the three dimensional gas flows in micro-channel with different aspect ratio should be investigated.

Based on the mass conservation law, Fan, Xie & Jiang [4] provided a universal basis for Shen's approach. If $\dot{M}$ is the mass flow rate in the streamwise direction through any cross section of a micro-channel, then

$$d\dot{M}/dx = 0 \tag{1}$$

For the Poiseuille flows with various cross-section shapes in the entire Knudsen regime, the corresponding mass flow rates can be obtained using the information preservation method and the direct simulation Monte Carlo method, and have been fitted as follows [5]

$$\dot{M}/\dot{M}_* = \frac{A}{Kn} + B + C\ln(1+DKn)\,. \tag{2}$$

The values of parameters A, B, C and D are summarized in Table 1, with

$$\dot{M}_* = -S^{3/2}(dp/dx)/v_m\,, \tag{3}$$

where S the area of the cross section, dp/dx the streamwise pressure gradient, $v_m = \sqrt{2RT}$ most probable thermal speed, R the universal gas constant, T the gas temperature, $Kn = \lambda/\sqrt{S}$, and λ the molecular mean free path.

Substitution of Eqs. (2) and (3) into Eq. (1) yields the kinetic equation of gas through three dimensional microchannels with various cross-section shapes, i.e.

$$-\frac{S^{3/2}}{V_m}d[\frac{A}{Kn}\frac{dp}{dx} + B\frac{dp}{dx} + C\ln(1+DKn)\frac{dp}{dx}]/dx{=}0\,. \tag{4}$$

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TABLE 1. Values of the parameters in the fitting relation (2) [5].

Shape	A	B	C	D
circle	7.14×10^{-2}	4.28×10^{-1}	4.20×10^{-2}	8.50×10^{2}
hexagon	6.21×10^{-2}	5.18×10^{-1}	3.20×10^{-2}	5.20×10^{2}
semicircle	4.78×10^{-2}	4.75×10^{-1}	3.58×10^{-2}	3.39×10^{2}
square	6.23×10^{-2}	4.89×10^{-1}	3.53×10^{-2}	4.00×10^{2}
rectangle (ϕ=2)	5.09×10^{-2}	4.71×10^{-1}	3.78×10^{-2}	2.23×10^{2}
rectangle (ϕ=5)	2.63×10^{-2}	3.27×10^{-1}	4.42×10^{-2}	2.15×10^{2}
rectangle (ϕ=10)	1.45×10^{-2}	2.16×10^{-1}	4.75×10^{-2}	1.90×10^{2}
rectangle (ϕ=20)	7.80×10^{-3}	1.30×10^{-1}	4.58×10^{-2}	1.70×10^{2}
rectangle (ϕ=50)	3.10×10^{-3}	1.17×10^{-1}	3.85×10^{-2}	4.42×10^{1}
rectangle (ϕ=100)	1.48×10^{-3}	1.01×10^{-1}	3.18×10^{-2}	2.20×10^{1}

SOLUTIONS TO PRESSURE DISTRIBUTION AND MASS FLOW RATE

If the normalized factors are the microchannel length L and the outlet pressure p_o, then

$$X = x/L\,,\ \ P = p/p_o\,. \tag{5}$$

For the hard-sphere model, the mean free path has the following expression

$$\lambda = \frac{16}{5\sqrt{\pi}} \times \frac{\mu}{\rho v_m}\,, \tag{6}$$

ρ and μ are the gas density and viscosity coefficient, respectively.

According to Eq. (3), the local Knudsen number is related to the outlet Knudsen number Kn_o as

$$Kn \equiv \frac{\lambda}{\sqrt{S}} = \frac{\lambda_o}{\sqrt{S}} \times \frac{\lambda}{\lambda_o} = \frac{Kn_o}{P}\,. \tag{7}$$

Substituting Eqs. (5) and (7) into Eq. (4), with a derivation process similar to that described in detail in [4], we obtain the solutions to the pressure distribution and mass flow rate of gas through a micro-channel

$$\frac{1}{2}\left(P^2 - P_i^2\right) + BKn_o\left(P - P_i\right) + CKn_o \times \left[P \ln\left(1 + \frac{DKn_o}{P}\right) - P_i \ln\left(1 + \frac{DKn_o}{P_i}\right) + DKn_o \ln\left(\frac{P + DKn_o}{P_i + DKn_o}\right)\right] = C_A X\,, \tag{8}$$

$$\dot{M} = -\frac{5\sqrt{\pi}A}{16}\frac{S^2}{\mu(RT)}\frac{p_o^2}{L}C_A\,, \tag{9}$$

where P_i is the normalized inlet pressure, C_A a constant determined by the cross-section shape and the inlet and outlet pressures

$$C_A = \frac{1}{2}(1 - P_i^2) + BKn_o(1 - P_i) + CKn_o \times [\log(1 + DKn_o) - P_i \log(1 + \frac{DKn_o}{P_i}) + DKn_o \log(\frac{1 + DKn_o}{P_i + DKn_o})\,. \tag{10}$$

THREE-DIMENSIONAL EFFECTS

Take rectangular micro-channels often used in experiments [6-8] as an example. As shown in Fig. 1, three-dimensional effects in such situation result from the width-to-height ratio ϕ.

Figure 2 presents the theoretical and experimental relations of the mass flow rates to the difference between the inlet and outlet pressures Δp. In the experiments carried out by Zohar et al [8], the micro-channel was 0.53μm by height, 40μm by width, and 4000μm by length. The corresponding value of ϕ = 40/0.53, about 75. From the three theoretical profiles predicted by Eq. (9) with the different width-to-height ratios, it is clear seen that the theoretical and experimental results agree under the same width-to-height ratio (ϕ = 75), but the side wall effects become significantly as ϕ is smaller than 10 and make the theoretical profile with ϕ = 1, i.e. the square cross section, quite different from that with ϕ = 75, e.g. the former mass flow rate only about 30% of the latter when Δp = 300kPa.

Figure 3 compares the theoretical and experimental pressure distributions, which agree well each other. The experimental data came from Zohar et al. [8]. There were two cases. Both of them had the same outlet pressure of

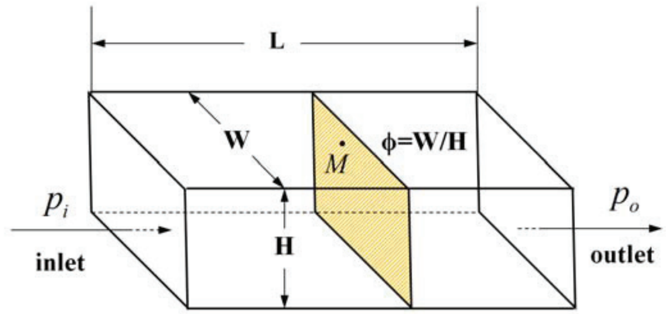


FIGURE 1. Schematic of micro-channel with rectangular cross section.

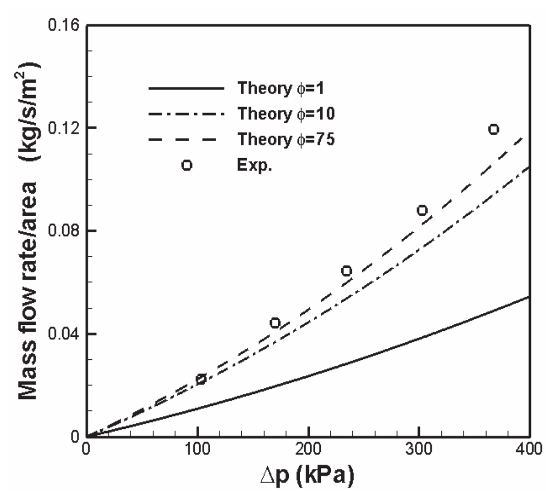
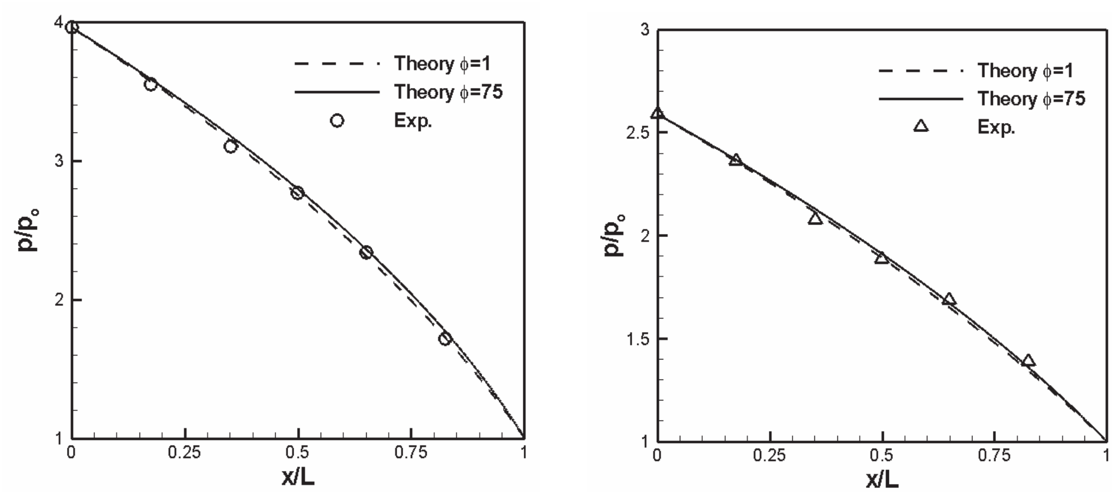


FIGURE 2. Mass flow rate versus inlet-to-outlet pressure difference of gas through rectangular microchannels.



(a) Δp＝300kPa (b) Δp＝116kPa

FIGURE 3. Normalized streamwise pressure distributions of rectangular microchannel gas flows.

1atm, but the inlet pressures were different corresponding to the values of Δp = 300kPa and 161kPa, respectively. The theoretical distributions are easily obtained from Eq. (8). Contrary to the mass flow rate, the side walls of the microchannels have negligible effect on the pressure distributions.

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