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Influence of Particle Gradation Curve on Granular Packing Characteristics

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Abstract

In this article, ten kinds of granular materials with different particle gradation curves based on the regularized incomplete Beta function are produced and packed by discrete element method. Then the influences of particle gradation on granular packing and frictional characteristics are examined. Merely adjusting the two parameters in the regularized incomplete Beta function, we are able to generate a numerical granular material with all kinds of granular packing, either well or poorly graded now. The model is proved to be a very useful tool for conventional discrete element method to simulate mechanical behavior of realistic granular materials.

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Keywords: Particle Gradation Curve, BetaBeta Function, Granular Materials, Packing, Discrete Element Method

1. Introduction

Granular material, such as sand and gravel, is composed of an assemblage of particles [1]. Their mechanical characteristics are primarily dependent on the geometry and physical characteristics of particles, such as mineral type, particle size/shape, particle gradation and their arrangement, in which, particle gradation is an important factor when evaluating granular materials. It seems to be considerably hard to control all geometric and physical characteristics in indoor soil tests. In triaxial compression test, a soil sample is generally regarded as a macroscopic volume element without any insight into the micro-scale or particle level structure and physics. So it can only provide global stress strain response for constructing phenomenological constitutive law [2]. In contrast, discrete element method (DEM) [3] is an approach treating the intrinsic mechanism of granular material at particle level, which has already found wide applications in both fundamental and applied research in soil/rock mechanics and geotechnical engineering [4-8]. It is a promising numerical scheme to deal with the influences of particle's geometrical and physical characteristics on the mechanical characteristics of granular materials. However, it is still challenging regarding how to simulate real granular materials well. For example, dilatancy and friction angle are often found larger or less than those in real experiments. People have developed rolling resistance model [9-11], clump model [12] to upgrade spherical particle model capability in the simulation of the mechanical response of real granular materials. However, the influence of particle gradation on mechanical behavior of granular materials

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in DEM is usually neglected up to now, which motivated us to discuss the mathematical modeling of particle gradation curve and its influences on the mechanical behavior of granular materials.

2. Mathematical Modeling of Particle Gradation Curve

In real granular materials, particle gradation curves are different remarkably, in which some of them exhibit wide distributions, others are narrowly spanned. Due to the computational limitation, only those particle gradation curves with limited particle size span could be modeled in DEM. The mathematical model of particle gradation curve should be capable of producing varieties of distribution, including simple distribution, power distribution, even distribution with controlling parameters as less as possible. The regularized incomplete Beta function as shown in Eq.(1) proposed by Voivret [13] is found to be able to satisfy most requirements,

$$\beta(x;a,b) = \frac{1}{B(a,b)}\int_0^x t^{a-1}(1-t)^{b-1}dt \tag{1}$$

, where complete Beta function is defined as $B(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$, and $\Gamma(x)$ denotes conventional Gamma function.

Substitute x in Eq.(1) with the relative diameter $d_r = \frac{d - d_{\min}}{d_{\max} - d_{\min}}$, where $d_{\max}$ and $d_{\min}$ mean the maximum and minimum diameters, and d is the diameter of particles to be generated. Then, we have the volume accumulated function as shown in Eq.(2).

$$h(d_r;a,b) = \beta(x = d_r;a,b) \tag{2}$$

Fig.1 depicts five particle gradation curves generated by different combinations of parameters a and b.

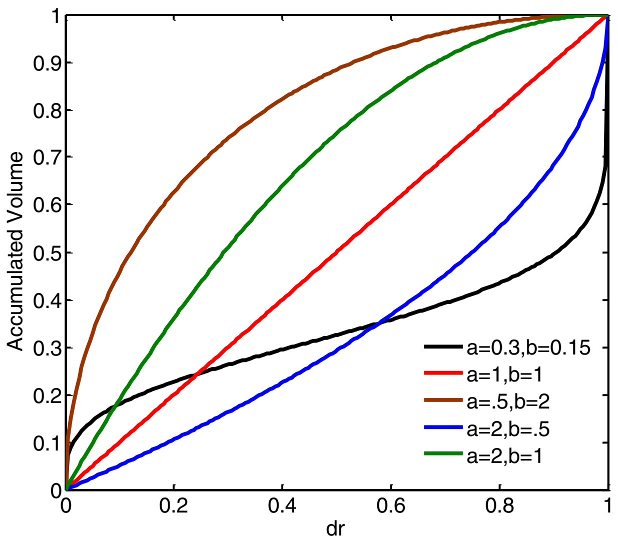
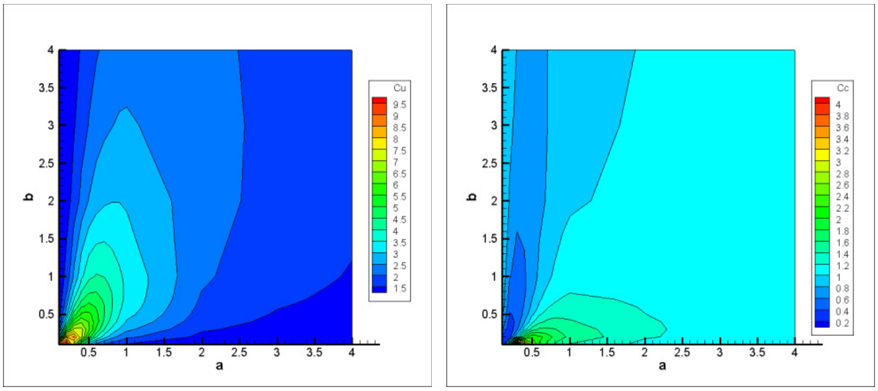


Fig. 1. Particle gradation curves generated by five couples of parameters a and b

Two fundamental parameters describing particle gradation curve are the coefficient of non -uniformity $C_u = \frac{d_{60}}{d_{10}}$ and the coefficient of convexity $C_c = \frac{d_{30}^2}{d_{10}d_{60}}$. If C_u is large, particle size distribution shows polydispersity and is prone to dense packing, such as the black line shown in Fig.1. If C_c is large, the intermediate particle size dominates and the convexity of particle size distribution curve is obvious like the brown line in Fig.1. Two coefficients C_u and C_c for the proposed mathematical model in Eq.(2) are primarily dependent on three parameters: particle size span defined as ratio of maximum to minimum size of particles, a and b. For example, when diameter span is 8, the contours of C_u and C_c vary with a and b, as shown Fig.2.



(a) (b)

Fig. 2. Contour of C_u (a) and C_c (b) of different couples (a,b). Diameter span is 8.

More mathematical analysis of Eq. (2) states that the mathematical model exhibits several useful properties when generating particle gradation curve.

- If a=1 or b=1, particle gradation curve obeys power distribution. In particular, particle gradation curve is uniformly distributed as a=b=1.
- If a=b, the particle gradation curve is symmetric about (0.5, 0.5).
- If a<1 and b<1, particles mostly approach to both ends of maximum and minimum sizes. Furthermore, particles only have two sizes as a <<1 and b<<1.
- If a>1 and b>1, particle distributes in the intermediate size zone. Furthermore, particles turn out of mono-size distribution as a>>1 and b>>1.
- Parameter a controls the ratio of fine particles and parameter b coordinates the ratio of coarse particles.
- When the diameter span is larger, C_u and C_c may vary more diversely.

3. Packing Characteristics

Based on the above analysis, ten couples of a and b as seen in Fig.3 are used to generate particle gradation curves under particle size span of 8. All C_u and C_c of the ten samples of different particle gradation curves are shown in Table 1.

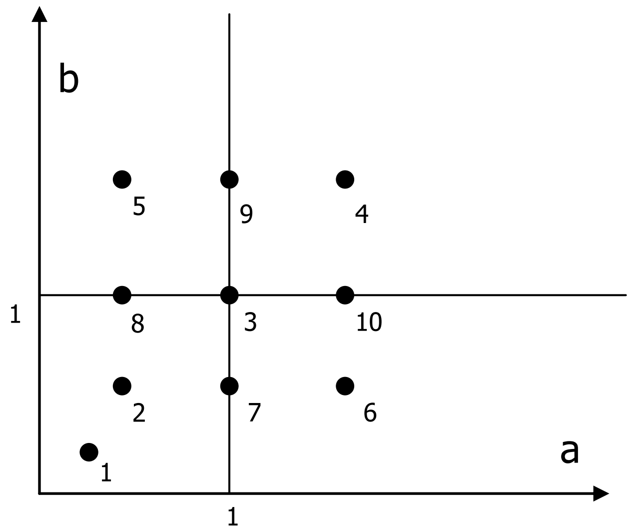


Fig. 3. Ten couples of parameters a and b used to generate ten particle gradation curves.

Table 1 C_u and C_c of ten samples of different particle gradation curves with particle size span of 8

Item	PSD#	a	b	C_u	C_c
1	1	0.3	0.15	7.1	1.8
2	2	0.5	0.5	4.8	0.9
3	3	1.0	1.0	3.1	1.1
4	4	2.0	2.0	2.1	1.1
5	5	0.5	2.0	2.2	0.7
6	6	2.0	0.5	1.7	1.2
7	7	1.0	0.5	3.0	1.3
8	8	0.5	1.0	3.3	0.7
9	9	1.0	2.0	2.6	0.9
10	10	2.0	1.0	2.0	1.1

Usually, triaxial or biaxial compression tests are conducted to examine the mechanical characteristics of granular materials in the laboratory. In digital world, however, numerical simulation of triaxial compression is implemented to test the mechanical behavior of granular materials. Similarly, a sample should be prepared appropriately at first. There are several sample generation scheme, such as multilayer with under-compaction method [14]. However, the radius expansion method [14, 15] is preferred in this study since the main target test is to examine the effect of particle gradation curves on the packing. Firstly, particles with radius smaller than those specified particle gradation curves are generated with one of the four generation techniques in Yade [16]. Then, radius of particles is expanded step by step by relaxing the unbalanced force between particles until a specified target stress at the wall is reached. Inter-particle friction angle is used to control the initial density. A larger inter-particle friction produces a looser sample. For each of the ten particle gradation curves, four inter-particle friction angles, 0.5^0, 10^0, 20^0 and 30^0 are selected to generate four samples of dense, medium dense, loose and very loose states. The indicators of packing structure such as porosity and coordination number generated by the ten particle gradation curves with inter-particle friction angle of 0.5^0 are shown in Table 2. In contrast, Table 3 exhibits packing characteristics generated by the first particle gradation curve with four different inter-particle friction angles.

Table 2 Packing structure characteristics generated by ten particle gradation curves with inter-particle angle 0.5^0

Item	PSD#	Porosity	Coordination Number
1	1	0.244	6.04
2	2	0.272	6.18
3	3	0.291	6.19
4	4	0.325	6.21
5	5	0.326	6.21
6	6	0.347	6.22
7	7	0.298	6.20
8	8	0.300	6.19
9	9	0.307	6.20
10	10	0.335	6.22

Table 3 Packing structure characteristics by 1st particle gradation curve with different inter-particle angles

Item	Inter-particle Friction angle(^{0})	Porosity	Coordination Number
1	0.5	0.244	6.04
2	10	0.277	5.01
3	20	0.305	4.53
4	30	0.325	4.19

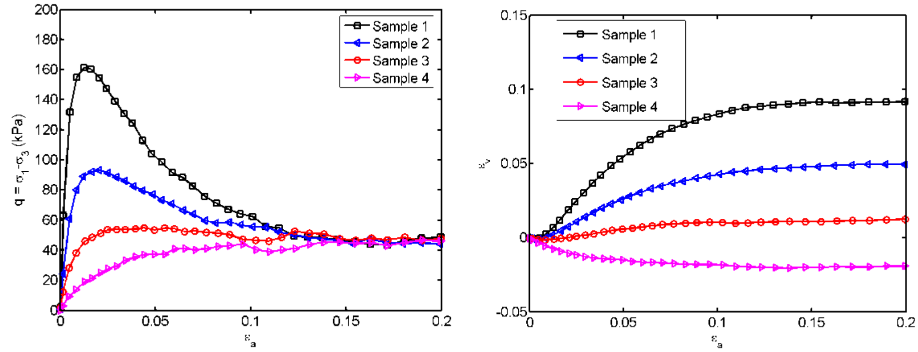
For cases with the same inter-particle friction angle during sample generation in Table 2, a particle gradation curve with larger C_u generates a denser packing sample, but there is no obvious characteristics for coordination number. However, for cases with the same particle gradation curve, Table 3 shows that a smaller inter-particle friction angle generates a denser packing and the coordination number is larger. In particular, packing density of the generated samples is better than others when a<1 and b<1.

4. Mechanical Characteristics

Conventional triaxial compression tests over the 40 samples generated above were simulated by DEM on the Yade platform [17] with the conventional contact law. Contact parameters such as contact modulus, stiffness ratio and inter-particle friction angle are chosen as 100MPa, 0.4 and 30^0, respectively. Figure 4 plots the stress strain response and volume strain variation for the four samples generated by the first particle gradation curve. Figure 5 describes the variation of coordination number Z and fabric deviator F during triaxial loading. Following Thornton [18], coordination number, representing number of contacts per particle is defined in Eq.(3), in which C is the total number of contacts, N, N_0 and N_1 are total particle number, particle numbers with zero and only one contact. Fabric tensor F_{ij} is defined following Satake [19] as shown in Eq.(4), where n_i is the contact direction, C contact number. Fabric tensor reflects the orientation distribution of contacts. The fabric deviator F is defined as $\sqrt{\frac{3}{2}F_{ij}'F_{ij}'}$, which equals to $F_1 - F_3$ in axial symmetric case, where F_1 and F_3 are principal values of F_{ij} in axial and lateral direction, and F_{ij}' is the deviatoric fabric defined as $F_{ij}' = F_{ij} - \frac{1}{3}F_{kk}\delta_{ij}$. The fabric deviator represents the anisotropy of the whole sample.

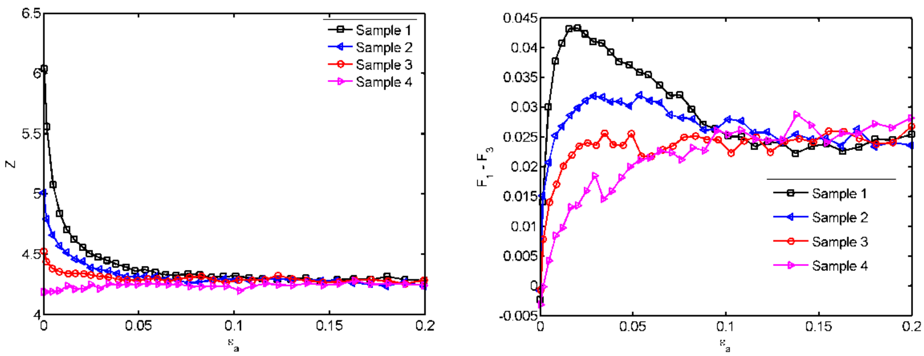
$$Z = \frac{2C - N_1}{(N - N_0 - N_1)} \tag{3}$$

$$F_{ij} = \frac{1}{C}\sum n_i n_j \tag{4}$$



(a) (b)

Fig. 4. (a) Stress strain response and (b) volume strain variation for the four samples generated by particle gradation curve #1.



(a) (b)

Fig. 5. (a) Coordination number and (b) fabric deviator variation for the four samples generated by particle gradation curve #1.

Figure 4 indicates that most mechanical behaviours of both dense and loose granular materials such as hardening, softening and dilatancy are simulated qualitatively. Loose sample exhibits only hardening behaviour, while dense sample demonstrates strong softening and dilatancy behaviours. All the samples generated by the same particle gradation curve approach to the same critical state at large deformation. Figure 5 gives the variation of micromechanical indicators during loading: Coordination number of dense sample is decreasing, while that of loose sample is increasing. And finally both of them are approaching to the same number at critical state. The variation of fabric deviator shown in Fig.5(b) shares similar tendency as stress-strain response in Fig.4(a).

Table 4 provides all the peak friction angles of ten samples from dense to loose states with inter-particle angle 0.5^0 by ten particle gradation curves. We can see that the friction angle of dense sample produced by the 1st and 2nd particle gradation curves can reach as high as 35^0 to 38^0, which lies in the friction angle range of realistic granular materials. The fact demonstrates that numerical particle gradation model is helpful for conventional DEM to model real granular behaviors. Table 5 gives friction angle, coordination number, porosity and fabric deviator at critical state of all the samples. The results for well graded particle samples are smaller than those for poorly graded ones.

Table 4 Peak friction angles of ten samples generated by the ten particle gradation curves

Item	PSD#	Dense	Medium dense	Loose	Very Loose
1	1	38	29	21	19
2	2	35	27	22	20
3	3	34	25	20	20
4	4	32	23	21	21
5	5	32	23	21	21
6	6	30	21	20	21
7	7	33	24	20	20
8	8	33	25	21	20
9	9	33	24	21	21
10	10	31	22	21	20

Table 5 Friction angle, coordination number, porosity and fabric deviator at critical state of ten samples generated by the ten particle gradation curves

Item	PSD#	Φ_{cr}	Z_{cr}	n_{cr}	F_{cr}
1	1	18.4	4.26	0.312	0.026
2	2	19.1	4.42	0.322	0.024
3	3	19.9	4.49	0.344	0.031
4	4	20.6	4.56	0.374	0.039
5	5	20.4	4.58	0.371	0.037
6	6	20.8	4.61	0.396	0.045
7	7	20.1	4.51	0.355	0.036
8	8	20.1	4.52	0.344	0.030
9	9	20.3	4.52	0.355	0.034
10	10	20.7	4.57	0.384	0.042

5. Conclusion

Incomplete Beta function is used to construct a mathematical model of particle gradation curve in this study. A series of packing and triaxial compression tests of samples generated by the ten different particle gradation curves were produced using discrete element simulation. The results show that the mathematical model is able to generate varieties of particle size distribution. C_u and C_c are found to be easily coordinated merely by the adjustment of parameters a and b. Packing samples generated by well graded particles exhibit denser solid packing with higher friction angle than those of poorly graded particles. Furthermore, friction angle, coordination number, porosity and fabric deviator of samples of well graded particle samples at critical state is smaller than those of poorly graded particle samples. In summary, the mathematical particle gradation model is shown to be a very useful tool for conventional DEM to simulate behavior of realistic granular materials.

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