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Mechanisms for non-ideal flow in low-power arc-heated supersonic nozzles

Cheng-Kang Wu[1] · Wen-Xia Pan[1] · Xian Meng[1] · Hai-Xing Wang[2]

Abstract The flow in a low-powered arc gas heater combined with a supersonic nozzle of throat diameter less than 1 mm is quite complicated and difficult to describe in quantitative detail. Experiments on arc-heated supersonic jet thrusters of monatomic gases argon and helium have been carried out and their performance measured. The flow characteristics are analyzed with the help of numerical simulation. Results show that the viscous effect is the most important factor causing the large difference between ideal and real performance. A large outer section of the exit flow is slow-moving. This is especially pronounced in helium, where 70 % of the exit area of the nozzle might be in subsonic flow. Friction forces can be much larger than the net thrust, reaching several times higher in helium, resulting in very low efficiencies. Other factors causing the differences between ideal and real flow include: complex flow in the throat region, electric arc extending to the nozzle expansion section, heat transfer to the inlet gas and from the hot plasma, and environmental pressure in the vacuum chamber. It is recognized that the ordinary concepts of supersonic nozzle flow must be greatly modified when dealing with such complicated situations. The general concepts presented in this paper could be helpful in guiding the design and operation of this equipment.

Keywords Arc-heated supersonic jet · Low-power · Real flow · Viscosity effect · Low Reynolds number

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Nomenclature

A^*	Throat area, m²
C	Nozzle flow coefficient
c_p	Specific heat, J/(kg K)
F	Thrust, N
F_0	Cold thrust, N
h_0	Total enthalpy, J/kg
I	Arc current, A
I_{sp}	Specific impulse, m/s (or s)
$\dot{m}$	Mass flow rate, kg/s
Ma	Mach number
p_0	Inlet pressure, Pa
p_{vc}	Vacuum chamber pressure, Pa
r	Radial distance from axis, m
R	Gas constant, J/(kg K)
Re	Reynolds number
T_0	Total temperature, K
u	Axial velocity, m/s
U	Arc voltage, V
z	Axial distance from the inlet plane, m
V_e	Exhaust velocity, m/s
W_{sp}	Specific power, J/kg
γ	Specific heat ratio
η_{Isp}	Specific impulse efficiency
η_T	Thrust efficiency
ρ	Mass density, kg/m³

1 Introduction

The combination of a low-powered arc gas heater and a supersonic nozzle has applications in space propulsion, high-velocity materials deposition, gas dynamic experimental

facilities, etc. [1–8]. Flow in these nozzles is quite complicated. A clearer knowledge of the physical nature of the various factors involved will help understand characteristics of flow and performance of the equipment.

Take the space thruster, the arcjet, as an example. There has been much development work and actual applications, but analyses on the flow inside the nozzles with various specific propellants are still scarce [9–11]. Performance parameters are often much below the ideal situation. In our laboratory, a wide range of experiments with various propellants: argon, helium, hydrogen, nitrogen, ammonia, and their mixtures have been performed. In order to better understand the non-ideal factors in the flow, the simpler case of monatomic gases, argon (Ar) and helium (He), is considered first to avoid the complications of chemical dissociation. Also, these gases do not ionize appreciably below a certain range of high temperature, say on the order of 7000–10000 K. To give an explanation for the experimental results, the trends of the parameters obtained in the experiments are considered in general physical terms. Some results from numerical simulation are also used to help establish the general picture of flow and to provide at least order-of-magnitude values of the parameters.

In the arc-heated thruster, the ideal case would be that all electric energy input to the gas is transformed into an increase in uni-directional kinetic energy of the exhaust gas, producing an increase in thrust over the thrust produced by the cold gas. The "thrust efficiency" in such case is defined as 100 %. Another way of defining an ideal case is that the total enthalpy of the gas, including that of the incoming gas and the electric energy input from the arc, is completely and uniformly converted into exhaust kinetic energy, i.e., maximum exhaust velocity or "specific impulse" is achieved. The ratio of the specific impulse in the real case to this ideal value is called the "specific impulse efficiency". The departure of the real case from these ideals is called the "losses".

It must be emphasized that the purpose of this paper is not to obtain a set of exact quantitative results or to provide detailed guides for design, but only to generally analyze and discuss the variation in trends of experimental parameters and the relative importance of the influencing factors, so that a better understanding of the main physical processes occurring in the nozzle can be obtained. These concepts might provide some general guidance in the design and parameter selection for the equipment in order to achieve the desired characteristics of the flow.

2 Ideal situation and measurement results

The ideal case of nozzle flow would be that the total enthalpy of the gas is completely converted through expansion into exhaust velocity in the axial direction, so that the thrust pro-

duced is a maximum. This condition is realized in adiabatic, frictionless, fully expanded one-dimensional flow in complete thermodynamic equilibrium. Ideal gas flow equations can be used to calculate the parameters in the nozzle and performance of the thruster. For instance, given the nozzle dimensions, gas flow rate, inlet gas temperature, and electric arc power input, if perfect gas with a constant specific heat is used, and assuming that the entire electric energy input is converted into enthalpy of the gas in the plenum chamber of the arc heater, then the relationship among the plenum chamber pressure, mass flow, etc., can be expressed by the ideal critical flow nozzle equation [12]:

$$\dot{m} = \frac{p_0 A^*}{\sqrt{RT_0}} \sqrt{\gamma \left(\frac{2}{\gamma + 1}\right)^{(\gamma+1)/(\gamma-1)}}, \qquad (1)$$

where T_0 can be calculated from the gas inlet temperature, electric power input, and specific heat. For a nozzle with large enough expansion ratio, the exhaust gas velocity (specific impulse) is

$$V_e = I_{sp} = \sqrt{2h_0}, \qquad (2)$$

where h_0 is the enthalpy of the inlet gas plus the electric energy input per unit mass (specific power). If the nozzle area ratio is not large enough for full expansion, then the parameters at the nozzle exit can be calculated with ideal compressible flow equations.

However, in the real situation, many factors cause the flow relations and the thruster performance to differ from the ideal case, often significantly. Two experimentally-measured parameters, show up the difference between real and ideal cases when compared with calculated ideal values with the same nozzle construction, gas flow rate and input electric power. These are the upstream total pressure and the produced thrust. It is important to find out what caused these differences, at least the underlying physical mechanisms and their relative importance. This is the main purpose of the present work.

3 Experimental

Experiments have been performed in the Aerospace Plasma Dynamics Facility (Fig. 1) of the Institute of Mechanics, Chinese Academy of Sciences. The facility consists of a vacuum chamber, 2 m dia. by 4 m long, ultimate vacuum 10^{-4} Pa. There are two set of vacuum pumps, one with a roots blower and a mechanical pump for higher operational flow rates at 10–20 Pa chamber pressure, another with an additional diffusion pump and molecular pumps for lower flow rates at $\sim$ 1 Pa. Power, cooling water supplies, and a complete set of measuring instrumentation are equipped. The thruster is
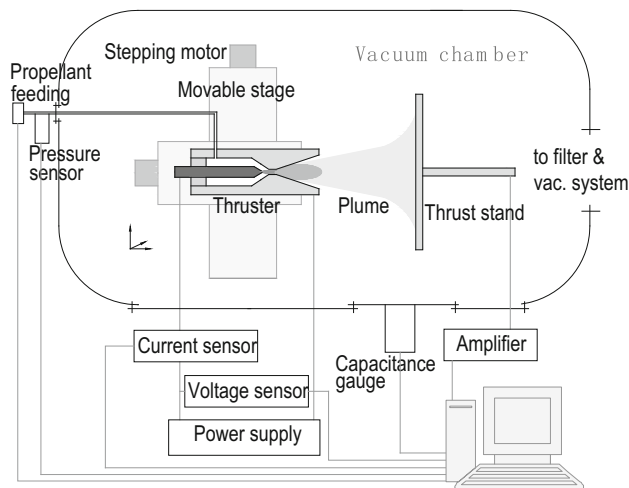
Fig. 1 Schematic diagram of the experimental setup

mounted on a 3D moveable table driven by stepping motors. The produced thrust is measured indirectly by the impulse method [13]. The mass flow rate, arc current and voltage, pressures, etc., are measured by transducers. Optical instruments measure the nozzle surface temperatures and other signals from the jet plume. All data have been collected and stored on a computer.

Two types of anode/nozzle have been used, their schematic diagrams are shown in Fig. 2. In the regeneratively-cooled nozzle, the inlet gas passes first through the passage along the inner surface inside the nozzle body, absorbing heat transferred from the hot gas and cooling the nozzle. This raises the efficiency while improving the service life of the nozzle. This nozzle worked fine with Ar, because this gas is easily ionized and has a low thermal conductivity, resulting in stable arc, easy ignition, and non-constricted anode arc root. But with He, with its higher ionization potential and thermal conductivity, it is not so easy to maintain a stable arc and a diffused arc root on the anode surface. Thus, a simple radiation-cooled nozzle operating at a higher temperature was used. The heat loss is higher, efficiency lower, but operation was stable without burning out of the anode over a long period of time. The dimensions of the two nozzles are: regeneratively-cooled nozzle (Ar), throat dia. 0.7 mm, constrictor length 2.3 mm, expansion half angle 15°, expansion

area ratio 239; for the radiation-cooled nozzle (He), throat dia. 0.6 mm, constrictor length 0.5 mm, half angle 15°, and area ratio 220.

The experiments were run within the range of stable operation of the arc and where a long runtime could be maintained without burning out or damage of the nozzle. This limited the range of arc current to an order of 15 A and corresponding volume rate of flow of the gases where a stable arc could be maintained. In the experiments, the set conditions were: nozzle type and geometry, gas type and flow rate, arc current, and vacuum system used. From the measured parameters, the specific power input, specific impulse, and thrust efficiency are calculated by the following formulae:

$$W_{sp} = UI/\dot{m} \text{ (J/kg)}, \tag{3}$$

$$I_{sp} = F/\dot{m},$$

(m/s; divide by 9.81 to obtain I_{sp} in the popular unit, s)

$$\tag{4}$$

$$\eta_T = \frac{F^2 - F_0^2}{2\dot{m}UI}. \tag{5}$$

4 Results and analyses

4.1 Experimental data and observed trends

Experimental data about the inlet pressure, specific impulse, and thrust efficiency are shown in Fig. 3. Some observed trends are that the inlet pressure rises with flow rate and specific power input. Also, Ar and He exhibit quite different ranges of operating parameters; if similar conditions are considered, Ar runs at much higher inlet pressure than He (deduced from data in Fig. 3a, d), and the specific impulse is much lower in Ar (Fig. 3b, e). At a given flow rate, with the increase in specific power, the specific impulse rises but the efficiency falls (Fig. 3b, c, e, f). At the same specific power, a higher flow rate gives a higher specific impulse and efficiency; however, this trend is only obvious in He (Fig. 3b, c, e, f). In these experiments, the general trend is lower efficiency with higher specific power, and the general level of efficiency is quite low, reaching downward to an order of 20 %. In the two sets of experiments with He,
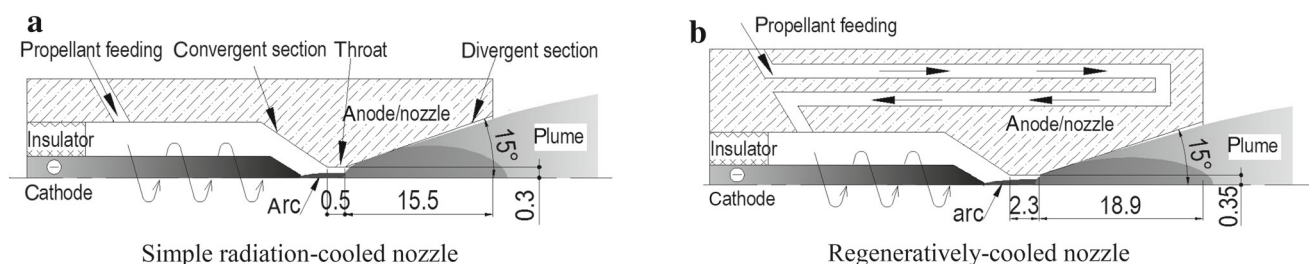

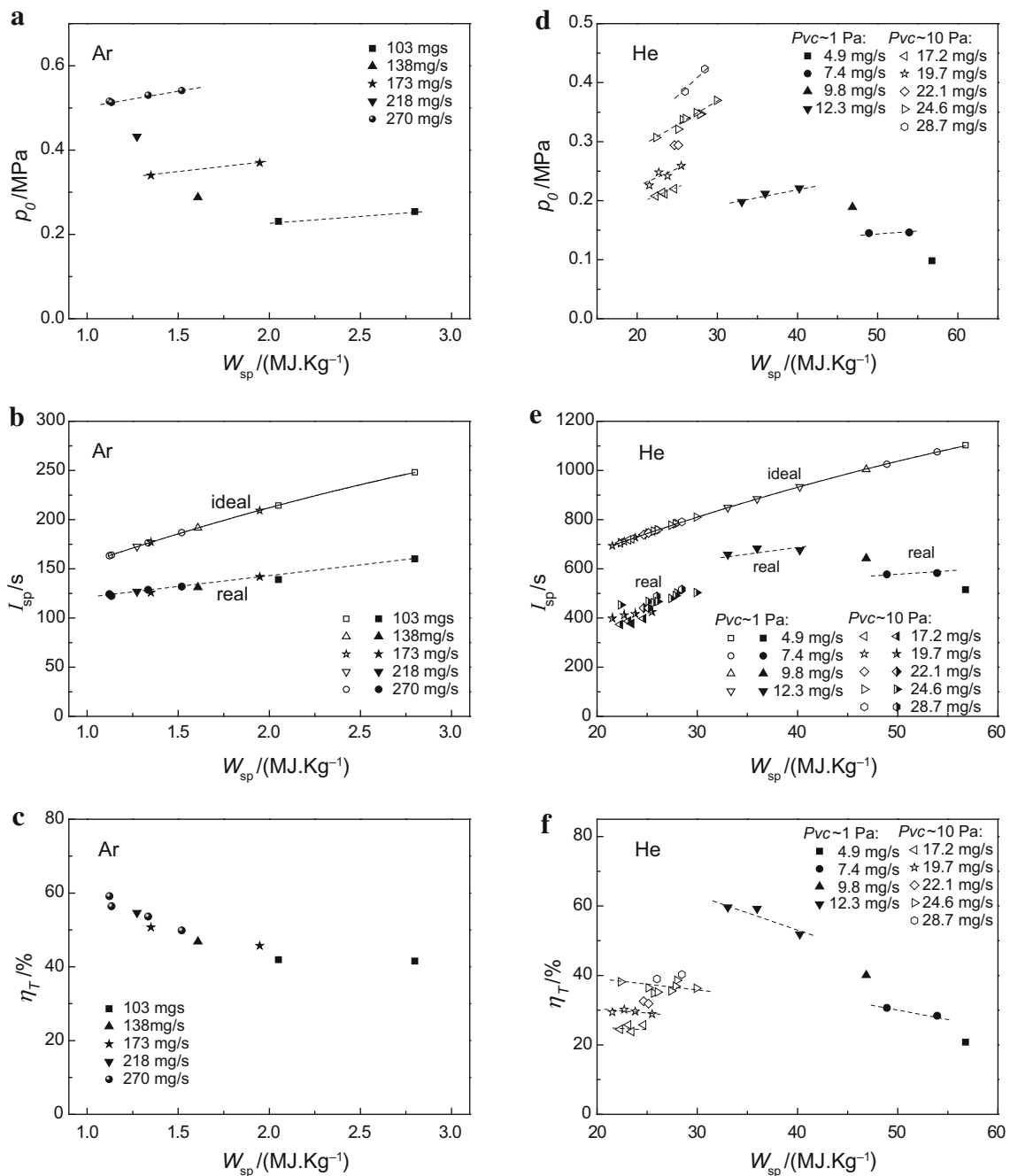**Fig. 2** Schematic diagram of the nozzles

Fig. 3 Variations of inlet pressure, specific impulse, and thrust efficiency against specific power. **a–c** Ar, **d–f** He

the vacuum chamber pressure shows a definite effect on the parameters, especially the efficiency (Fig. 3e, f). Some of the trends, e.g., higher inlet pressure with higher flow rates, lower specific impulse with heavier gas (Ar), etc., are easily explained by ordinary concepts of nozzle flow, but many others need more detailed explanation, at least on physical terms. Therefore, data on upstream total pressure and specific-impulse/efficiency are analyzed and discussed with the purpose of understanding the differences between real and ideal cases.

The comparison of experimentally measured upstream total pressure and specific-impulse/efficiency with those calculated results for the ideal case are two different, though not unrelated, problems. The mechanisms causing the differences are all there, but their relative importance might be different in the two kinds of comparison. In the first case, the factors causing the difference in total pressure/flow rate relationship mainly operate in the contracting portion and the throat region of the nozzle, although the downstream condition can also exert some influence. While in the second case,

the factors operate throughout the entire nozzle, and even the exhaust chamber pressure is important. These cases are analyzed respectively in Sects. 4.2 and 4.3.

4.2 Total pressure in arc chamber–ideal and real cases

The total pressure in the arc chamber is a directly measured quantity, its difference from the ideal value could be the result of a combination of several factors. In the ideal case, a relationship between the parameters is given by Eq. (1). It is clear that if $p_0/\dot{m}$ and T_0 are used as coordinates in plotting, all the points under various conditions should fall on a single curve for a given gas. For a gas with nearly constant specific heat, T_0 is nearly proportional to the total enthalpy, which is related to the specific power input from the arc. In the arc-heated thrusters, especially in the simple radiation-cooled nozzle, the enthalpy of the cold inlet gas is much smaller than the arc input, thus the specific power can be roughly used to represent the total enthalpy. In the real case, the curve of $p_0/\dot{m}$ against W_{sp} (specific power) would have a similar shape as the ideal curve, but with different values. This is indeed demonstrated in Fig. 4a, b. With the regeneratively cooled nozzle, the inlet gas is heated by the hot nozzle to a higher temperature than in the radiation-cooled nozzle, so the error by neglecting the inlet gas enthalpy is larger. But still it is convenient to do so for order-of-magnitude analysis.

In Eq. (1), if $\dot{m}$ is moved to the right side, then the combination on the right equals 1.0 in the ideal case. The gas constant and specific heat of He are 2077 and 5193 (J/kg)K, those for Ar are 208 and 520 (J/kg)K (in the temperature range of nearly constant specific heat), and both have $\gamma = 1.67$. Assuming that T_0 can be approximately represented by W_{sp}/c_p, and putting in the corresponding numbers, the combination $1.15 p_0^* A^*/(\dot{m} W_{sp}^{1/2})$ should also be nearly 1.0 for both Ar and He (p_0^* is the arc chamber pressure in the ideal case). If p_0 for the real case is used, then the combina-

tion shows the difference between it and the ideal one. The ratio between the two cases (real/ideal) is about 0.65–0.8 for Ar and 0.8–0.9 for He. The reciprocal of this ratio can actually be considered as a kind of "effective flow coefficient" for the nozzle, which in the present cases is greater than 1.0, caused by a combination of factors as will be discussed below.

The real flow is quite different from that in the ordinarily recognized critical-flow nozzle. From both actual observation and numerical modeling, it is clear that the arc extends from the tip of the cathode in the upstream chamber through the throat (constriction channel), and the anode arc root is attached to the expansion portion of the nozzle. The flow is far from being uniform and one-dimensional. Viscosity effect is prominent and the boundary layer may fill a major part of the nozzle, as the Reynolds number is quite low. Heat transfer effects are present, and high temperature effects such as dissociation, ionization, and even electromagnetic effects might be significant. In the present cases of Ar and He gas under a certain range of temperature, the last three factors are not being considered.

Based on the observations and modeling results, as shown in Figs. 5, 6 and 7–9, where the temperature and flow fields of the arcjet thruster are obtained with solving the governing equations under the local thermodynamic equilibrium assumption, which take into account the effects of compressibility, Joule heating, and the Lorentz force, as well as the temperature and pressure dependence of the gas properties, and detailed descriptions of the method of calculation have been given in Refs. [14,15], the following possible factors are now discussed:

4.2.1 Heat transfer in the arc chamber

The inlet gas at 300 K is heated by the hot nozzle/anode and its temperature will rise, though not too significantly. When the nozzle is not regeneratively cooled, the rise in inlet
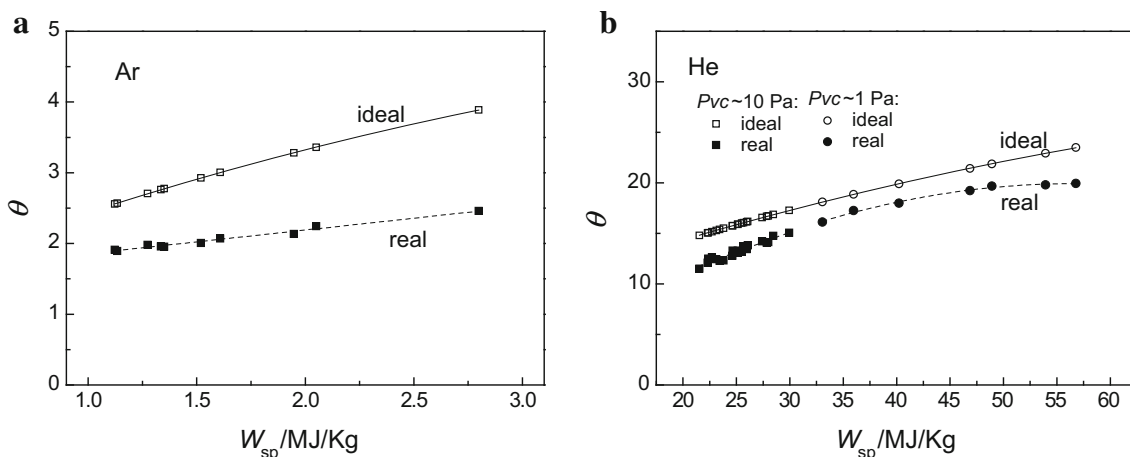

Fig. 4 Variations of $\theta = p_0/\dot{m}$, kPa/(mg/s), against specific power. **a** Ar, **b** He
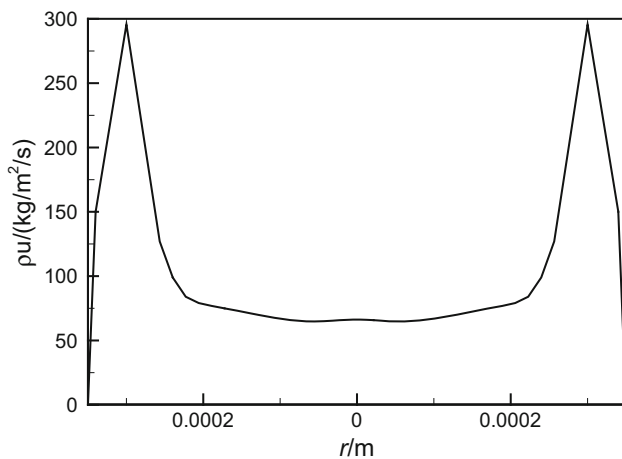
Fig. 5 Computed mass flow density distribution at the constriction channel exit, Ar

gas temperature might be a couple of 100 K. The main rise occurs in the arc, the average temperature will reach many 1000 K. However, the tip of the cathode is near the entrance to the throat or the constriction channel, and in the initial portion of the arc, the hot core is surrounded by cool gases. So not too much heat loss occurs in the subsonic portion of the flow. This is also shown in the numerical calculation [14], which shows negligible heat loss in this part of flow. Heat loss mainly occurs in the expansion portion of the nozzle, where the anode arc root is also attached. Thus, for the simple radiation-cooled nozzle, heat transfer in the arc chamber upstream of the throat should have little effect on the relationship between total pressure and flow rate.

In the regeneratively cooled nozzle, the inlet gas is heated by the hot nozzle to perhaps around 1000 K. This is an appreciable rise in the enthalpy of the gas, and may account for up to an order of 10 % of total energy input to the gas. This is equivalent to an increase of power input in the upstream chamber, and will result in a corresponding increase of upstream total pressure for a given flow rate.

4.2.2 Non-uniformity of upstream gas parameters

An ideal critical flow nozzle assumes that the inlet gas is uniform and the flow is isentropic. A sonic condition is reached at the throat. If the upstream gas is not uniform, the critical-flow relationship between total pressure and flow rate might not be the same as the ideal one. Suppose the flow is composed of a very hot core surrounded by cool gas, as might be the case in a longitudinal arc-heated nozzle, could the flow be somehow "blocked" by the low-density core and the flow reduced at a given upstream total pressure? Or in other words, for a given flow rate, the upstream pressure must be raised?

However, through the calculation of one example, it can be seen that this effect may not be very large. The example assumes Ar at an upstream total pressure of 0.25 MPa, $\gamma = 1.67$, throat dia. 0.7 mm. The ideal case has a uniform total temperature of 5000 K, giving a flow rate of 68.5 mg/s. the non-uniform case assumes half of the mass flow is at 8000 K, surrounded by another half of the mass flow at 2000 K, the average total enthalpy is the same in both cases. Both the hot core and the cool surrounding gases reach $Ma = 1$ at the throat. Then by the ideal critical flow equation, the portions of throat area occupied by the hot and cool gases are $(8000/2000)^{1/2} = 2:1$. For the total throat area of 0.385 mm^2, that occupied by the hot and cool gases are 0.257 and 0.128 mm^2, respectively. This gives flow rates of 36.0 mg/s each, or a total flow rate of 72.0 mg/s. This number is not too greatly different ($\sim$ 5 %) from the uniform case. The reason for this situation is that, although the surrounding cool gas has a lower velocity and occupies less portion of the throat area, its density is high, so the mass flow rate is still high. Numerical calculation also shows such a trend in Fig. 5. Therefore, the effect of upstream gas non-uniformity on the nozzle flow coefficient may not be too great.

4.2.3 Part of the arc extends past the throat into the downstream part of nozzle

As seen in experimental observations of the anode arc root attachment position and also numerical modeling, the arc originates from the cathode tip and extends past the throat or constrictor passage to the downstream side. Thus, the electric power input to the gas in the form of Joule heating before the throat is only a part of the total arc power. This causes the temperature rise in the upstream chamber to be much less than the total amount. The exact number is hard to determine, but can be estimated from the position of the anode arc root or numerical modeling. In an example of numerical modeling for Ar, the upstream portion of arc power is 54 %, in another example for He, the number is 51 %. Assuming constant specific heats, for a given mass flow, the upstream total pressure will be lower than the ideal case where all the arc power is dissipated upstream of the throat. The ratio is by the square root of total temperature, or approximately by the square root of electric power input upstream to the total power. Using the above numbers for order-of-magnitude estimates, the total pressure upstream will be 0.73 and 0.71 of the ideal case, for Ar and He respectively. Thus, this factor is quite important for the difference between the real and ideal case.

4.2.4 Non-ideal flow in the throat region

In the ideal case, the uniform gas flows one-dimensionally through the narrowest area at $Ma = 1$. But in the real case, the flow is quite complicated, having non-uniformity in all parameters, a thick boundary layer at low Reynolds num-

bers, strong gas swirl, etc. The overall effect is that the ideal picture cannot describe the flow, even approximately. Figure 6 shows an example of numerical modeling (swirl not yet considered), in which the surface of $Ma = 1$ is not at the narrowest section, but rather extends to a much wider area downstream. If an "effective throat area" representing the $Ma = 1$ surface is considered, its value would be much larger than the geometrical throat area. The ratio is hard to define, but a value of order of 2 might not be unreasonable. With this effect in mind, the upstream total pressure would be much lower than in the ideal case, for a given mass flow.

4.2.5 Viscosity effect

Low-power arc-heated thrusters have small size, low mass flow rate, and very high gas temperature with correspondingly high viscosity, thus the flow is at a low Reynolds number, with a dominant viscous effect. This results in laminar flow with very thick boundary layers and high frictional resistance. These have pronounced effects on the nozzle flow, and are the major cause of losses in these nozzles. For the present study, the effect of Re on the nozzle flow coefficient can be seen in Fig. 7, data taken with cold He flow through the nozzle. The values of Re in this study ranges from several tens (in He) to several thousands (in Ar). The effect on the nozzle flow coefficient is especially pronounced at low Reynolds numbers. The characteristic values used to calculate the Reynolds number in this study are: nozzle throat diameter, gas mass flow rate, and viscosity at the total temperature of the gas in the ideal case. These will give a representative number for the flow Reynolds number for the purpose of comparison. Low Reynolds numbers will lower the nozzle flow coefficient, and if the mass flow rate is given, the upstream total pressure will be higher than in the ideal non-viscous case.

If the combination $1.15 p_0 A^*/(\dot{m} W_{sp}^{1/2})$ is plotted against Re, it could show the effect of viscosity on the total-pressure/flow relationship, as shown in Fig. 8. The ordinate
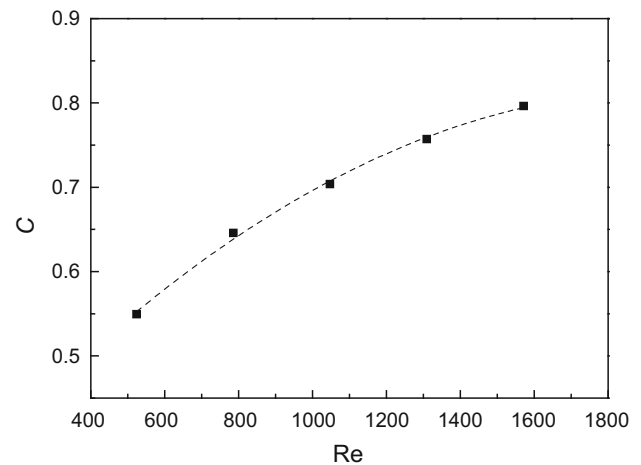
Fig. 7 Nozzle flow coefficient C variation with Reynolds number, cold He

value of the curve includes all the effects due to the various factors discussed above, but the variation with Re should be an indication of the importance of viscous effects. It can be seen that at high Re, the effect becomes somewhat less pronounced. In the He experiments, the level of exhaust chamber pressure has an effect on the total-pressure/flow-rate relationship. This is not possible in ideal non-viscous critical flow. This observed phenomenon is probably the result of a pressure effect felt through the thick subsonic boundary layer at extremely low Reynolds numbers. From the example of the numerical calculation shown in Fig. 9, it can be seen that the subsonic boundary layer occupies a large portion of the flow cross section, especially in He where Re is extremely low, and the subsonic flow takes up 70 % of the exit area. In the numerical calculation, the exhaust environment is a vacuum, so the effect of back pressure is not considered. But in the real experimental condition, the back pressure will exert its influence through the subsonic boundary layer and influence the flow upstream.
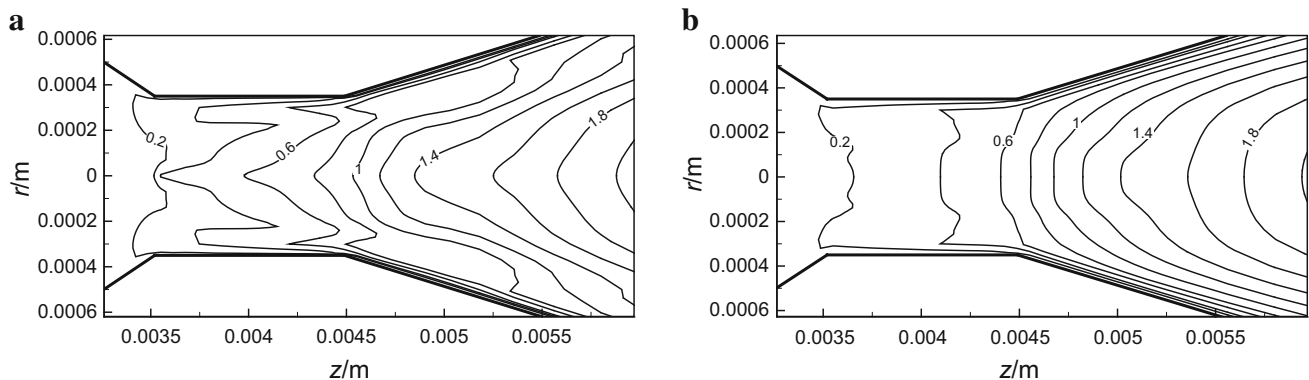
Fig. 6 Computed Mach number distribution near the exit of constriction channel, **a** Ar, **b** He
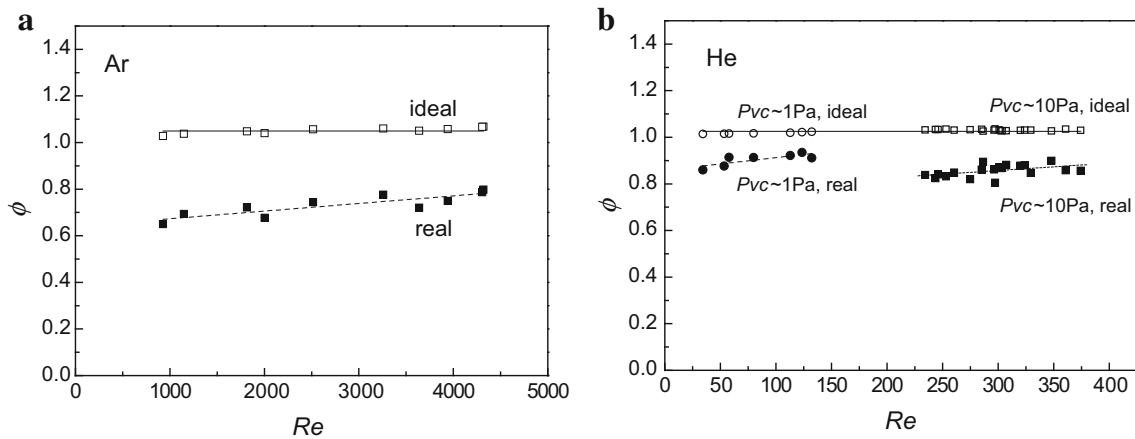
Fig. 8 Variations of $\phi = 1.15 p_0 A^* / (\dot{m} W_{sp}^{1/2})$ against Reynolds number. **a** Ar, **b** He
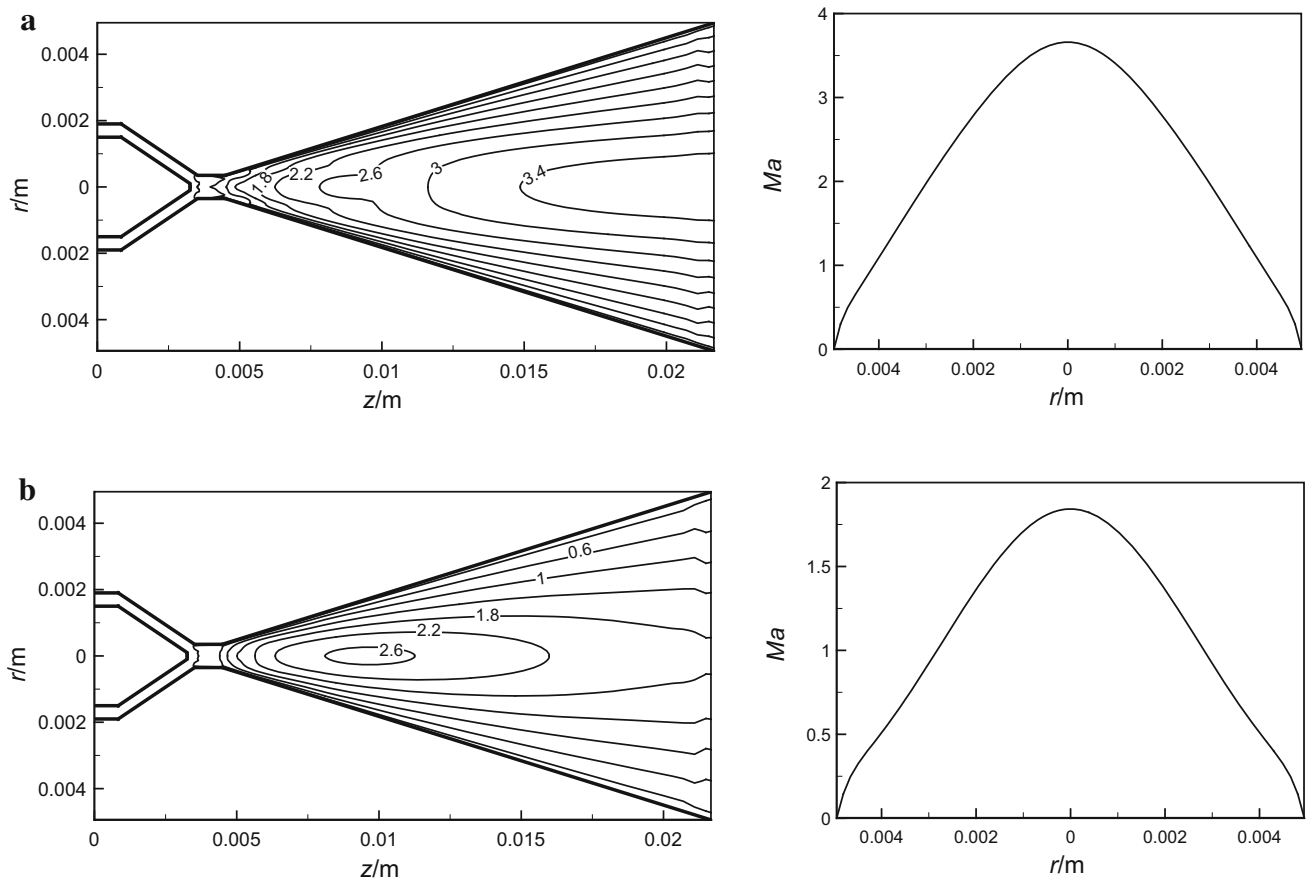

Fig. 9 Mach number distribution in the nozzle (*left*) and on the exit plane (*right*). **a** Ar, **b** He

4.2.6 Discussion on total pressures in Ar and He

The relative magnitude of upstream total pressure in Ar and He can be roughly estimated by considering the possible parameters in the arc chamber. To maintain a stable electric arc without causing damage of the nozzle, the average gas temperature should be in the range of several 1000 K to over 10000 K. If the volumetric flow of the two gases is the same, then the mass rates of flow is the ratio 10:1 (Ar:He). If the total temperature is about the same, then for the same A^* and same nozzle flow coefficient, the upstream total pressure in the two gases would be proportional to $(\dot{m} R^{1/2})$, so the ratio would be $10 \times 10^{-1/2} = 3.16$. This order of magnitude is not unreasonable when compared with the experimental

data, taking into consideration the facts that the nozzles are of different types, throat diameters and nozzle flow coefficients are not quite the same, and inlet gas temperatures are not the same in the experiments for Ar and He.

4.2.7 A brief summary

With the above analyses and the observed trends of the combined experimental data, it can be said that the relationship among upstream total pressure, mass flow rate, and specific power input can be explained qualitatively by considering the physical factors. The most important factors causing the difference between real and ideal cases are: arc extending into expanding portion of the nozzle, non-ideal flow pattern in the throat region, and viscous effects. In the regeneratively cooled nozzle, heating of the inlet gas by the hot nozzle can also cause a difference.

4.3 Specific impulse and thrust efficiency

For monatomic gases such as Ar and He, there is no chemical dissociation, and at the general level of average temperatures (e.g., 7000–15000 K) in these facilities, the energy related to ionization is still a small portion of the total energy. Thus, for thrust performance with these gases, the non-equilibrium or frozen flow effects are not important. Those factors causing non-ideal phenomena discussed in the above section are still topics for consideration.

The production of thrust depends on the conversion of the gas enthalpy into directional velocity at the exhaust through the expansion process. To achieve high efficiency of conversion, the gas should experience full expansion while keeping the "losses" to a minimum. The factors discussed in the above section and their importance in influencing specific impulse and thrust efficiency are analyzed below. It was found that viscosity is the most important factor causing the difference between real and ideal cases; therefore, it is discussed first.

4.3.1 Viscosity effects

In the low-power arc-heated thrusters, the flow Reynolds numbers are very low, meaning that the viscosity effects are prominent. Similar to the above discussion about the viscous effect on the total-pressure/flow-rate relationship, its effect on the thrust performance is also very important. The importance of a low Reynolds number flow in small nozzles has been recognized in the early literature, e.g. Refs. [16–18], and is again demonstrated in the present work. Low Re implies thick, laminar boundary layers and high frictional forces. The nozzle flow may be wholly filled by boundary layer (fully viscous). In the numerical example, it is seen that in both the constriction channel and divergent section of the nozzle to the exit, the velocity distribution shows the characteristics of

a boundary layer. A large portion of the nozzle is filled with subsonic flow (Fig. 9). For instance, in Ar, exit flow in 30 % of the exhaust area is subsonic, and in He, the number reaches 70 %. This also explains why vacuum chamber pressure can affect flow rate in these "supersonic" nozzles, as shown in the previous section.

In a low Reynolds number flow, the wall friction is proportional to the reciprocal of the square root of Re. Thus, the frictional loss is much greater with the decrease in Re. In an example of a numerical calculation, the axial component of the wall friction integrated over the nozzle wall is quite large. In the Ar example, the net thrust is 130 mN, and the friction is 81 mN, while in the He example, the net thrust is 38 mN, and the friction is 126 mN! Such figures may not be quantitatively exact, but the order of magnitude clearly shows the significance of the problem. This also agrees with the experimental results. This further suggests that it is important to minimize the nozzle surface area to reduce total friction, while keeping enough expansion ratio, indicating that there might exist some optimum angle for the expanding part of the nozzle.

The viscous effects also cause the expansion process in the nozzle to be incomplete, even though the geometric expansion ratio is large. This effect is also seen in the numerical results shown in Fig. 9. Thus, the exhaust gas exits the nozzle at a lower Mach number and relatively high average temperature, bringing with it a loss in the form of high thermal energy.

Figure 10 shows the trends of specific impulse and thrust efficiencies with varying Re. It can be seen that the influence is especially pronounced at low Reynolds numbers. He has low mass flow and high viscosity, so the Reynolds numbers are much lower than Ar. In Ar, the Reynolds numbers are in the thousands, thus the trend is noticeable but less steep. This fact may explain why the efficiency data of different flow rates all fall on the same curve for Ar but on different curves for He, since the He experiments were run at much lower Re where the effect of flow Reynolds number is much more pronounced. The two sets of He data were obtained with exhaust chamber pressure different by an order of magnitude, and they are on two different and separate curves. This can be explained by the fact that the thick subsonic boundary layers cause the downstream back pressure to have a significant influence on the expansion process, and the set with low exhaust pressure has much fuller expansion and thus higher efficiencies.

4.3.2 Heat transfer between gas and wall

The heat transfer from the high temperature gases to the nozzle wall occurs mainly in the downstream portion of the nozzle, which has a relatively large area and where the anode arc root is also attached. The anode arc root contributes to the heat loss through the electron work function
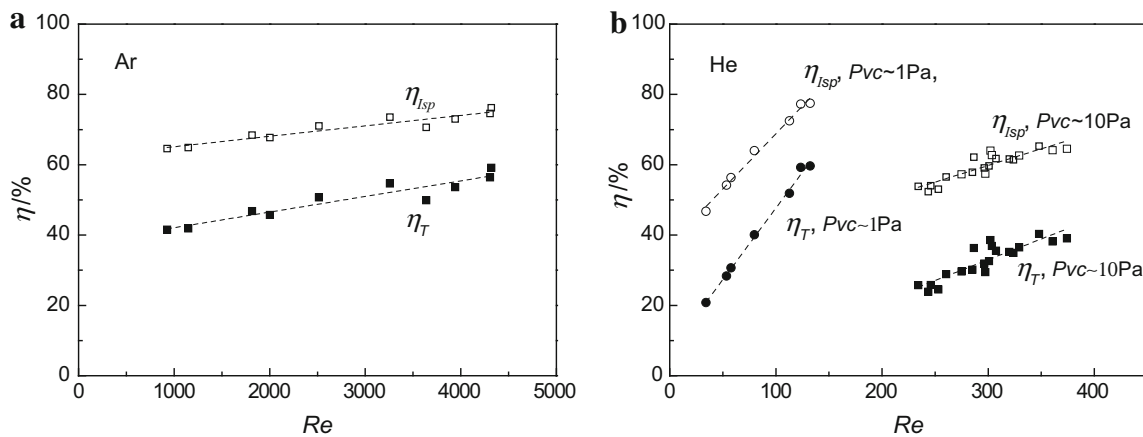
Fig. 10 Variations of specific impulse efficiency and thrust efficiency against Reynolds number. **a** Ar, **b** He

plus gaseous heat transfer. The heat dissipates into the environment through radiation from the outer surface of the anode-nozzle. The heat loss can be seen as a decrease of energy input from the arc to the gas. In an example of a numerical calculation, the heat loss is of the order of 20 % of the arc power. However, not all the heat transferred to the wall is lost, because the hot wall heats up the inlet gas, and the additional enthalpy can be converted into exhaust velocity in the case of full expansion. The recovery of heat is most effective in the regeneratively cooled anode/nozzle, so the efficiencies would be higher than the simple radiation-cooled nozzles. In an experiment with a simple radiation-cooled nozzle, the heat radiated from the nozzle surface was of the order of less than 10 %, by measurement with a radiation pyrometer. Thus, heat loss will lower the specific impulse and thrust efficiency by a definite amount, but may not be the major influencing factor, especially with the regeneratively cooled nozzle.

4.3.3 Non-uniformity in gas parameters

In the ideal case with full expansion of gases, the gas enthalpy is completely converted into exhaust velocity (kinetic energy), and thus this factor should not have an effect on the specific impulse or thrust efficiency. However, in real cases, full expansion may not be achieved even with large geometrical area ratios due to various reasons, and the non-uniformity factor may have an influence on the efficiencies.

4.3.4 Arc extends past the throat into the downstream portion of the nozzle

This means that part of the energy input to the gas occurs in the supersonic portion of the flow, and is not as efficient thermodynamically as heat addition in the upstream chamber. However, if the expansion is sufficient, the gas enthalpy can still be fully converted into exhaust velocity, and this factor

should not affect the efficiencies. But again, the expansion is not complete in the real case due to the viscosity effect, and this factor may also have an effect on the efficiencies.

4.3.5 Non-ideal flow in the throat region

Although this factor has an important effect on the total-pressure/flow-rate relationship, it may not influence the final efficiency with full expansion in the nozzle. However, since the expansion is incomplete in the real case, this factor can also cause a lowering of specific-impulse/efficiency.

4.3.6 A brief summary

Following the above discussion, it can be seen that in the present experiments, among the factors causing the loss of efficiency in specific impulse and thrust, viscosity is the most important.

5 Discussion

From the above, it can be seen that the characteristics of flow in these low-power arc-heated supersonic nozzles is much different from the concepts usually established in larger or higher Reynolds number supersonic nozzles such as rockets, wind tunnels, flow meters, plasma spray guns, etc. The special arc structure, the form of heat transfer and its distribution, the complicated flow pattern in the throat region, and especially, the low Reynolds number flow with high viscosity and its many secondary effects, all cause large departure from the usual concepts of more or less uniform flow with thin boundary layers.

In ordinary supersonic nozzles, if the area expansion ratio is over 100, then the main flow would definitely be in the high supersonic region, and the expansion would be quite com-

plete, converting most enthalpy of the gas into directional exit velocity. But as can be seen in the nozzles studied here, even at area ratios around 200, the exit Mach numbers are still quite low, with a thick boundary layer occupying a large part of the exit section, even with subsonic flow. This is especially evident in He, which has a low mass flow rate and high viscosity. This is also why vacuum chamber pressure can affect the flow and expansion process. The frictional losses are so high that it is not advisable to make a long nozzle, yet a large expansion ratio is still necessary. So there is an optimum combination of expansion angle, nozzle length and expansion ratio. Looking at arcjets in practical applications, a half angle of 20° and area ratio around 200 seem typical. In nozzles operating under realistic conditions, the expansion process is still far from complete, thus the conversion from thermal to kinetic energy is inefficient, leaving a high portion of thermal energy in the exhaust gas.

The heat loss in the expanding portion of the nozzle reduces net energy input to the gas, thus reducing the efficiency by a noticeable amount, say 10 %. A regeneratively cooled nozzle/anode can recover an appreciable part of this loss, as evidenced by the much lower temperature at the outer surface of the nozzle and correspondingly lower radiated power. However, due to the incomplete expansion process, the recovered heat is not all converted into kinetic energy at the exit.

The arc extends past the throat into the downstream area. This is one of the factors which cause the total-pressure/mass-flow relationship to depart appreciably from that of the ideal critical nozzle. However, this should not have too large an effect on the performance of the thruster. A longer arc is usually associated with higher arc voltage and higher power input. Also, the attachment of the anode root of the arc to a larger area downstream of the throat, preferably in a diffused form on a hot surface, will be highly beneficial to the operational life of the nozzle.

The final design of the thruster should take into consideration all the important factors and try to achieve an optimum balance among them. Generally speaking, the specific power input needs to be relatively high to give a high specific impulse, but efforts should be made to keep the efficiency not too low. The Reynolds number of flow should be kept relatively high, and the nozzle should be designed for minimum friction loss but sufficient expansion.

The differences between performance parameters of Ar and He can be attributed to the nature of these two monatomic gases. Their atomic weights, thermodynamic and transport properties, ionization potentials, etc., are so widely different that the arc structure, heat transfer and frictional characteristics, expansion behavior, and, thus, the performance characteristics are also quite different. Detailed and accurate numerical modeling is needed to clarify the finer points of these mechanisms.

The most important parameter demonstrating thruster performance is specific impulse. It represents the total impulse (thrust × time) that can be provided by a given amount of propellant. For chemical propellants, the maximum specific impulse is determined by their chemical composition. For arc-heated thrusters, besides the thermo-chemical nature of the propellant, the specific impulse depends on the energy input from the electric arc. In the ideal case, the specific impulse is nearly proportional to the square root of specific power input, whereas in the real case, I_{sp} is reduced by the factor of efficiency. As seen from the above discussion, efficiency depends on specific power, flow rate, and Reynolds number, and its value can vary over a wide range. If the electric power comes from solar energy, and if the weight of the equipment used for generating electricity is negligible, then low efficiency might not be a serious problem. However, the weight and volume of the equipment is never a small issue, especially on a small satellite. It is always a part of the payload, and must be sent into orbit at great expense. Thus, high efficiency is also very important for the thruster system. From the above analysis, it is seen that there exist contradictory trends among several factors such as specific power and efficiency, small thruster size, and low viscous losses, etc. Thus, the optimum design should take into consideration tradeoff among various factors, under the limitations of the given task. As a general principle, the flow rate and specific power need to be relatively high, in order to operate at a Reynolds number which is not too low, while obtaining a high specific impulse. The objective is the optimum combination of weight and total impulse for the given task. In applications other than space propulsion, other considerations are needed to evaluate the effect of nozzle flow on the design and operation of the equipment.

6 Concluding remarks

In aerospace thruster applications of low-power arc-heated supersonic nozzles, the flow differs widely from the ideal case, causing the performance to be much lower. The flow process is so complicated that a detailed and accurate quantitative description is very difficult. Experiments of monatomic gases Ar and He are arranged and results are analyzed with a purpose of clarifying the effect of various factors on the real flow and understanding the basic physical picture. Some results of numerical modeling are also used to help with the analysis. It can be said that a better understanding has been obtained on the characteristics of the flow and the main causes affecting the performance.

For the thrusters of throat dia. around 0.7 mm in the present study using Ar and He as propellants, the main factors causing the upstream total pressure to differ from ideal are: the arc passing through the throat and attaching to the expansion part

of the nozzle; the non-ideal flow in the constriction channel; preheating of the inlet gas in the case of a regeneratively-cooled nozzle; and the viscosity effect on the nozzle flow coefficient at low Reynolds numbers. In these nozzles, the viscosity effect produces a very thick boundary layer, causing high friction, non-uniform flow field and insufficient expansion, thus playing the most important role in reducing the specific impulse and efficiency. Therefore, in applications where small size arc-heaters are used for producing supersonic gas flow, the design should consider operation at higher specific power but with Reynolds numbers that are not too low, and nozzles which will minimize friction yet providing enough of an expansion ratio.

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