Vol. XXIV No. 2 SCIENTIA SINICA February 1981

A STATISTICAL CRITERION FOR THE FOLLOW-UP BEHAVIOUR OF PARTICLES SUSPENDED IN A STATIONARY TURBULENT MEDIUM AT LARGE REYNOLDS NUMBER

LI YINTING (李荫亭) AND GUAN DEXIANG (关德相)

(Division of Fundamental Research, Institute of Mechanics, Academia Sinica)

Received September 10, 1979.

ABSTRACT

The study of the motion of particles suspended in a turbulent medium is of great significance to the research on laser velocity-measuring technique, atmosphere pollution, formation of clouds, cause of rain and the deposition of mud and sand. In this paper, the mean values have been obtained of the velocity and the square velocity of particles suspended in a stationary turbulent medium at large Reynolds number, and a discussion has been devoted to the "follow-up" behaviour of suspended particles, which is contingent on the properties of the turbulent medium and those of the suspended particles themselves. At the same time, a statistical criterion has been given for the complete "follow-up", i.e. equivalence of the velocity of suspended particles to that of the fluid.

I. INTRODUCTION

In recent years, a new method using laser has been developed for measuring the flow velocity of a fluid. By this method, it is the usual practice that the velocity of suspended particles in fluid, such as dust particles in a gaseous flow, is first measured and the results thus obtained are made to represent the velocity of the fluid. Yet, strictly speaking, as a result of inertial effect of the particles, their velocity does not quite tally with that of the fluid. This is especially so in the case of a turbulent flow, where the suspended particles generally cannot follow up the random perturbation motion.

Therefore, in introducing the laser velocity measuring technique, an answer has to be given for the following questions first: How large is the difference between the velocity of the suspended particles and that of the fluid? What conditions must be met so that the suspended particles can serve as an indicator of the flow velocity? To clarify these two points, a meticulous study has to be made of the behaviour of particles suspended in a turbulent medium and a criterion found for the complete "follow-up" of the suspended particles with the surrounding medium. Besides, the study in question is also of practical significance to the research on atmosphere pollution, the formation of clouds, the cause of rain, and the deposition of mud and sand.

By the "follow-up" behaviour of suspended particles is generally meant this:

from the known fluctuating velocities in a turbulence field to find out the velocities of the suspended particles, and by comparing the former with the latter to determine their relative magnitudes. In Reference [3], the transfer probability function is given for the motion of fluid particle in a stationary turbulence field at large Reynolds number, which makes it possible to give a complete statistical description to Lagrange's turbulence field. It is the purpose of the present paper to analyse the "follow-up" behaviour of particles suspended in the known turbulence field, and to provide a statistical criterion for the complete "follow-up" cf suspended particles with the surrounding medium on the basis of the results obtained in [3].

II. Motion Equation Governing a Particle Suspended in a Turbulent Medium

It was C. M. Tchen[4] who first examined the motion of a particle suspended in a turbulent medium and gave the general form of the motion eqnation governing a spherical particle, with a diameter less than the smallest scale of turbulence eddies, suspended in a homogeneous field of non-steady velocity. Later on, Corrsin and Lumley[5] pointed out that in applying Tchen's equation to a turbulence field, the inhomogeneity in space of a flow field must be taken into account. Finally, Hinze[6] obtained the following equation governing the motion in a certain direction of a particle suspended in a turbulent medium:

$$\frac{dv_p}{dt} + av_p = av_F + b\frac{dv_F}{dt} + c\int_{t_0}^{t} \frac{\frac{dv_F}{dt} - \frac{dv_p}{dt}}{\sqrt{t-t'}} dt' + f^2, \tag{2.1}$$

$$a = \frac{36\nu\gamma}{(2+\gamma)d^2} = \alpha\gamma, \tag{2.2}$$

$$b = \frac{3\gamma}{2+\gamma} = k\gamma, \tag{2.3}$$

$$c = \frac{18\gamma}{(2+\gamma)d}\sqrt{\frac{\nu}{\pi}}. \tag{2.4}$$

In these equations, v_p represents the velocity of a suspended particle; v_F, the velocity of a fluid particle; r, the density ratio of the suspended particle to the fluid medium; d, the diameter of a suspended particle; f_e, the external force existing in the said direction such as the gravitational force, the electromagnetic force, and ν, the viscosity of the fluid. The integral term on the right side of Eq. (2.1) is known as Basset's term. Фукс[7] pointed out that when f_e stands for a constant force, the gravitional force for example, the presence of Basset's term slightly reduces the acceleration of the suspended particles, and the amount of reduction in acceleration is proportional to the density ratio r. (The present study concentrates only on conditions where $r \leqslant 1$). The acceleration change is not larger than 2% when $r \leqslant 0.25$. In the case of solid or liquid particles suspended in an airflow, whose density ratio r is generally smaller than 0.001, Basset's term can be neglected. Thus, the motion equation becomes

$$\frac{dv_p}{dt} + \alpha\gamma v_p = \alpha\gamma v_F + k\gamma \frac{dv_F}{dt} + f_e, \tag{2.5}$$

or

$$\frac{dv_R}{dt} + \alpha\gamma v_R = (k\gamma - 1)\frac{dv_F}{dt} + f_e, \tag{2.6}$$

where $v_R = v_p - v_F$.

Assuming that the concentration of suspended particles is so low that the reaction of suspended particles on the turbulent medium can be neglected, in other words, the presence of the suspended particles does not affect the turbulence field, v_F in the above equation is therefore given as a known velocity in the turbulence field, and as a result, the evaluation of the "follow-up" behaviour of the suspended particles in relation to the fluctuating motion in the turbulence field is reduced to finding out v_p or v_r from the given v_F.

As in Reference [8], the motion of a fluid particle in a turbulence field can be described by Langevin's equation as:

$$\frac{dv_F}{dt} + \beta v_F = f(t), \tag{2.7}$$

where β is the friction coefficient. With a homogeneous and isotropic turbulence, $\beta = 5\nu/\lambda^2$, where λ represents the smallest scale of turbulent eddies. In Eq. (2.7), $f(t)$ is the random force acting on a fluid particle. According to the results obtained in Reference [3], for a stationary turbulence at large Reynolds number, the following expressions are valid:

$$\langle f(t)\rangle_u = \overline{f(t)} = 0, \tag{2.8}$$

$$\langle f(t)f(t+\tau)\rangle_u = \overline{f(t)f(t+\tau)} = K\delta(\tau), \tag{2.9}$$

where "$\langle\ \rangle_u$" represents the average over ensemble with velocity u; "—" represents the average over the complete ensemble; $\delta(\tau)$ is the delta function and $K = 2\beta\bar{v}_F^2$. In the case of a stationary turbulence, K is a constant.

The solution of Eq. (2.7) is

$$v_F(t) = v_{F_0}e^{-\beta t} + e^{-\beta t}\int_0^t e^{\beta\xi} f(\xi)d\xi, \tag{2.10}$$

where v_{F_0} represents the value of $v_F(t)$ when $t = 0$.

III. Solution of Motion Equation of Suspended Particles

First of all, let us take into consideration the conditions where $f_e = 0$. In that case, the solution of Eq. (2.5) can be given as follows:

$$v_p(t) = v_{p_0}e^{-\alpha\gamma t} + k\gamma e^{-\alpha\gamma t}\int_0^t e^{\alpha\gamma\xi}\left[f(\xi) + \left(\frac{\alpha}{k} - \beta\right)v_F(\xi)\right]d\xi, \tag{3.1}$$

where v_{p_0} is the value of $v_p(t)$ at $t = 0$. Substituting Eq. (2.10) into Eq. (3.1), we have

$$v_p(t) = v_{p_0}e^{-\alpha\gamma t} + k\gamma e^{-\alpha\gamma t}\left(\frac{\alpha}{k} - \beta\right)\int_0^t e^{\alpha\gamma\xi}e^{-\beta\xi}v_{F_0}d\xi$$
$$+ k\gamma e^{-\alpha\gamma t}\int_0^t e^{\alpha\gamma\xi}f(\xi)d\xi$$
$$+ k\gamma e^{-\alpha\gamma t}\left(\frac{\alpha}{k} - \beta\right)\int_0^t e^{\alpha\gamma\xi}e^{-\beta\xi}\left[\int_0^\xi e^{\beta\xi'}f(\xi')d\xi'\right]d\xi. \tag{3.2}$$

Making partial integration, we can easily obtain

$$\int_0^t e^{\alpha\gamma\xi}e^{-\beta\xi}\left[\int_0^\xi e^{\beta\xi'}f(\xi')d\xi'\right]d\xi = \frac{1}{\alpha\gamma - \beta}\left[e^{(\alpha\gamma-\beta)t}\int_0^t e^{\beta\xi}f(\xi)d\xi\right.$$
$$\left. - \int_0^t e^{\alpha\gamma\xi}f(\xi)d\xi\right]. \tag{3.3}$$

Substituting Eq. (3.3) into Eq. (3.2), we have

$$v_p(t) = v_{p_0}e^{-\alpha\gamma t} + k\gamma e^{-\alpha\gamma t}\left(\frac{\alpha}{k} - \beta\right)\int_0^t e^{(\alpha\gamma-\beta)\xi}v_{F_0}d\xi$$
$$+ \left[k\gamma e^{-\alpha\gamma t} - \frac{k\gamma\left(\frac{\alpha}{k} - \beta\right)}{\alpha\gamma - \beta}e^{-\alpha\beta t}\right]\int_0^t e^{\alpha\gamma\xi}f(\xi)d\xi$$
$$+ \frac{k\gamma\left(\frac{\alpha}{k} - \beta\right)}{\alpha\gamma - \beta}e^{-\beta t}\int_0^t e^{\beta\xi}f(\xi)d\xi. \tag{3.4}$$

When we calculate the mean values of the velocity and the square velocity of suspended particles, we take the average over the ensemble with initial velocity at v_{p_0}, which is represented by $\langle\ \rangle v_{p_0}$. Thus, we have

$$\langle v_p\rangle_{v_{p_0}} = v_{p_0}e^{-\alpha\gamma t}, \tag{3.5}$$

where we have made use of Eq. (2.8) and the condition, $\langle v_{F_0}\rangle_{v_{p_0}} = \bar{v}_{F_0} = 0$.

$$\langle v_p^2\rangle_{v_{p_0}} = v_{p_0}^2e^{-2\alpha\gamma t} + \bar{v}_{F_0}^2G_p^2e^{-2\alpha\gamma t}[e^{(\alpha\gamma-\beta)t} - 1]^2$$
$$+ E_p^2e^{-2\alpha\gamma t}\left\langle\int_0^t e^{\alpha\gamma\xi}f(\xi)d\xi\int_0^t e^{\alpha\gamma\eta}f(\eta)\,d\eta\right\rangle_{v_{p_0}}$$
$$+ G_p^2e^{-2\beta t}\left\langle\int_0^t e^{\beta\xi}f(\xi)d\xi\int_0^t e^{\beta\eta}f(\eta)d\eta\right\rangle_{v_{p_0}}$$
$$- 2E_pG_pe^{-(\alpha\gamma+\beta)t}\left\langle\int_0^t e^{\beta\gamma\xi}f(\xi)d\xi\int_0^t e^{\beta\eta}f(\eta)d\eta\right\rangle_{v_{p_0}}, \tag{3.6}$$

where

$$E_p = \frac{k\alpha\gamma\left(\gamma - \frac{1}{k}\right)}{\alpha\gamma - \beta}, \tag{3.7}$$

$$G_p = -\frac{k\gamma\left(\frac{\alpha}{k} - \beta\right)}{\alpha\gamma - \beta}. \tag{3.8}$$

In calculating Eq. (3.6), we have made use of the condition of statistical independence of v_{F_0} and $f(\xi)$ or $f(\eta)$[3].

The first integral term on the right side of Eq. (3.6) can be written as,

$$\left\langle \int_0^t e^{\alpha\gamma\xi} f(\xi) d\xi \int_0^t e^{\alpha\gamma\eta} f(\eta) d\eta \right\rangle_{v_{p_0}} = \int_0^t \int_0^t e^{\alpha\gamma\xi} e^{\alpha\gamma\eta} \langle f(\xi) f(\eta) \rangle_{v_{p_0}} d\xi d\eta.$$

By using Eq. (2.9), we can change the above into

$$\left\langle \int_0^t e^{\alpha\gamma\xi} f(\xi) d\xi \int_0^t e^{\alpha\gamma\eta} f(\eta) d\eta \right\rangle_{v_{p_0}} = \int_0^t \int_0^t e^{\alpha\gamma(\xi+\eta)} K\delta(\xi - \eta) d\xi d\eta. \tag{3.9}$$

By introducing the transforms, $\sigma = \xi + \eta$, $\tau = \xi - \eta$, it follows that

$$\int_0^t \int_0^t e^{\alpha\gamma(\xi+\eta)} K\delta(\xi \cdot \eta) d\xi d\eta = \int_0^t e^{\alpha\gamma\sigma} \left[\int_{-\sigma}^{\sigma} K\delta(\tau) D d\tau \right] d\sigma$$
$$+ \int_t^{2t} e^{\alpha\gamma\sigma} \left[\int_{-(2t-\sigma)}^{2t-\sigma} K\delta(\tau) D d\tau \right] d\sigma, \tag{3.10}$$

where $D = \left| \frac{\partial\xi}{\partial\tau} \frac{\partial\eta}{\partial\sigma} - \frac{\partial\eta}{\partial\tau} \frac{\partial\xi}{\partial\sigma} \right| = \frac{1}{2}$, thereupon Eq. (3.10) becomes

$$\int_0^t \int_0^t e^{\alpha\gamma(\xi+\eta)} K\delta(\xi \cdot \eta) d\xi d\eta = \frac{K}{2\alpha\gamma} [e^{2\alpha\gamma t} - 1]$$
$$= \frac{\beta \bar{v}_F^2}{\alpha\gamma} [e^{2\alpha\gamma t} - 1], \tag{3.11}$$

in which the condition, $K = 2\beta\bar{v}_F^2$, has been made use of.

As regards the other two integral terms on the right side of Eq. (3.6), results similar to Eq. (3.11) can easily be obtained. Thus, we have

$$\langle v_p^2 \rangle_{v_{p_0}} = v_p^2 e^{-2\alpha\gamma t} + \beta \bar{v}_{F_0}^2 \left\{ \left[\frac{E_p^2}{\alpha\gamma} + \frac{G_p^2}{\beta} - \frac{4E_pG_p}{\alpha\gamma + \beta} \right] \right.$$
$$\left. - \left(\frac{E_p^2}{\alpha\gamma} - \frac{G_p^2}{\beta} \right) e^{-2\alpha\gamma t} + \left[\frac{4E_pG_p}{\alpha\gamma + \beta} - \frac{2G_p^2}{\beta} \right] e^{-(\alpha\gamma+\beta)t} \right\}, \tag{3.12}$$

where the stationary condition, $\bar{v}_F^2 = \bar{v}_{F_0}^2$, has been made use of.

Giving a treatment similar to the above to $v_R(t)$, we have

$$v_R(t) = v_{R_0} e^{-\alpha\gamma t} - (k\gamma - 1) e^{-\alpha\gamma t} \beta v_{F_0} \int_0^t e^{(\alpha\gamma - \beta)\xi} d\xi$$
$$+ \left[(k\gamma - 1) e^{-\alpha\gamma t} + \frac{(k\gamma - 1)\beta}{(\alpha\gamma - \beta)} e^{-\alpha\gamma t} \right] \int_0^t e^{\alpha\gamma\xi} f(\xi) d\xi$$
$$- \frac{(k\gamma - 1)\beta}{\alpha\gamma - \beta} e^{-\beta t} \int_0^t e^{\beta\xi} f(\xi) d\xi, \tag{3.13}$$

$$\langle v_R \rangle_{v_{p_0}} = v_{p_0} e^{-\alpha\gamma t}, \tag{3.14}$$

$$\langle v_R^2 \rangle_{v_{p_0}} = v_{p_0}^2 e^{-2\alpha\gamma t} + \beta \bar{v}_{F_0}^2 \left\{ \left[\frac{E_R^2}{\alpha\gamma} + \frac{G_R^2}{\beta} - \frac{4E_RG_R}{\alpha\gamma + \beta} \right] \right.$$

$$+ \left[\frac{(1-G_R)^2}{\beta} - \frac{E_2^R}{\alpha\gamma} e^{-2\alpha\gamma t} + \left[\frac{4E_R G_R}{\alpha\gamma+\beta}\right.\right.$$

$$\left.\left. + \frac{2G_R(1-G_R)}{\beta}\right] e^{-(\alpha\gamma+\beta)t}\right\}, \tag{3.15}$$

where

$$E_R = \frac{(k\gamma-1)\alpha\gamma}{\alpha\gamma-\beta}, \tag{3.16}$$

$$G_R = \frac{(k\gamma-1)}{\beta\alpha\gamma-\beta}, \tag{3.17}$$

$$v_{R_0} = v_{p_0} - v_{F_0}. \tag{3.18}$$

At this point, we shall proceed to solve the motion equation under the condition where $f_e = 0$. The study here is restricted only to questions involving f_e as the gravitational force. If the x-axis is assumed to be vertical with what goes upward as positive, while the y-axis, horizontal, then $f_{ex} = -g(1-\gamma)$, $f_{ey} = 0$, where f_{ex} and f_{ey} are the components of the gravitational force in the x- and y-directions respectively. Now we shall make u and v represent velocity components in the x- and y-directions respectively. As the mean value and the square mean value of the component of velocity in the y-direction have already been given in Eqs. (3.5), (3.12), (3.14) and (3.15), it is only necessary, here, for us to find the mean and the square mean values of the component of velocity in the x-direction.

Substituting $f_{ex} = -g(1-\gamma)$ into Eqs. (2.5) and (2.6), and noting that u is substituted for v in the two equations, we obtain

$$u_p = -\frac{g(1-\gamma)}{\alpha\gamma}(1-e^{-\alpha\gamma t}) + u_{p_0}e^{-\alpha\gamma t}$$

$$+ k\gamma e^{-\alpha\gamma t}\int_0^t e^{\alpha\gamma\xi}\left[f(\xi) + \left(\frac{\alpha}{k}-\beta\right)u_F(\xi)\right]d\xi. \tag{3.19}$$

$$u_R = -\frac{g(1-\gamma)}{\alpha\gamma}(1-e^{-\alpha\gamma t}) + u_{R_0}e^{-\alpha\gamma t}$$

$$+ (k\gamma-1)e^{-\alpha\gamma t}\int_0^t e^{\alpha\gamma\xi}[f(\xi) - \beta u_F(\xi)]d\xi. \tag{3.20}$$

Repeating the above operation, we obtain

$$\langle u_p\rangle_{u_{p_0}} = -\frac{g(1-\gamma)}{\alpha\gamma}(1-e^{-\alpha\gamma t}) + u_{p_0}e^{-\alpha\gamma t}, \tag{3.21}$$

$$\langle u_R\rangle_{u_{p_0}} = -\frac{g(1-\gamma)}{\alpha\gamma}(1-e^{-\alpha\gamma t}) + u_{p_0}e^{-\alpha\gamma t}, \tag{3.22}$$

$$\langle u_p^2\rangle_{u_{p_0}} = \left[\frac{-g(1-\gamma)}{\alpha\gamma}(1-e^{-\alpha\gamma t}) + u_{p_0}e^{-\alpha\gamma t}\right]^2$$

$$+ \beta\bar{u}_{F_0}^2\left\{\left[\frac{E_p^2}{\alpha\gamma} + \frac{G_2^p}{\beta} - \frac{dE_pG_p}{\alpha\gamma+\beta}\right]\right.$$

$$-\left(\frac{E_p^2}{\alpha\gamma}-\frac{G_p^2}{\beta}\right)e^{-2\alpha\gamma t}+\left[\frac{4E_pG_p}{\alpha\gamma+\beta}-\frac{2G_p^2}{\beta}\right]e^{-(\alpha\gamma+\beta)t}, \tag{3.23}$$

$$\langle u_R^2\rangle_{u_{p_0}}=\left[\frac{-g(1-\gamma)}{\alpha\gamma}(1-e^{-\alpha\gamma t})+u_{p_0}e^{-\alpha\gamma t}\right]^2$$
$$+\beta\bar{u}_{F_0}^2\left\{\left[\frac{E_R^2}{\alpha\gamma}+\frac{G_R^2}{\beta}-\frac{4E_RG_R}{\alpha\gamma+\beta}\right]+\left[\frac{(1-G_R)^2}{\beta}-\frac{E_R^2}{\alpha\gamma}\right]e^{-2\alpha\gamma t}\right.$$
$$\left.+\left[\frac{4E_RG_R}{\alpha\gamma+\beta}+\frac{2G_R(1-G_R)}{\beta}\right]e^{-(\alpha\gamma+\beta)t}\right\}. \tag{3.24}$$

IV. Discussion of the Results of Calculation

Using the formulae obtained in the above section, we shall now discuss how the "follow-up" behaviour of suspended particles stands in relation to the parameters of the properties of the turbulent medium and suspended particles.

(1) When random force $f(\xi)$ and the fluctuating velocities $v_F(\xi)$ and $u_F(\xi)$ in a turbulence field all equivalent to 0, Eqs. (3.1) and (3.19) change respectively to:

$$v_p(t)=v_{p_0}e^{-\alpha\gamma t}, \tag{4.1}$$

$$u_p(t)=-\frac{g(1-\gamma)}{\alpha\gamma}(1-e^{-\alpha\gamma t})+u_{p_0}e^{-\alpha\gamma t}. \tag{4.2}$$

These are the well known formulae in an exponential form, which are valid in a laminar field.

(2) When $t\to\infty$, Eqs. (3.12), (3.15), (3.23) and (3.24) asymptotically go over into the following respectively:

$$\lim_{t\to\infty}\langle v_p^2\rangle_{v_{p_0}}=\langle v_p^2\rangle_\infty=\beta\bar{v}_{F_0}^2\frac{1}{(\alpha\gamma-\beta)^2}\left[\alpha\gamma(k\gamma-1)^2\right.$$
$$\left.+\frac{r^2}{\beta}(\alpha-k\beta)^2+\frac{4\gamma^2\alpha(k\gamma-1)(\alpha-\beta k)}{\alpha\gamma+\beta}\right], \tag{4.3}$$

$$\lim_{t\to\infty}\langle v_R^2\rangle_{v_{p_0}}=\langle v_R^2\rangle_\infty=\beta\bar{v}_{F_0}^2\frac{(kr-1)^2}{(\alpha\gamma+\beta)}, \tag{4.4}$$

$$\lim_{t\to\infty}\langle u_p^2\rangle_{u_{p_0}}=\langle u_p^2\rangle_\infty=\left[-\frac{g(1-\gamma)}{\alpha\gamma}\right]^2+\beta\bar{u}_{F_0}^2\frac{1}{(\alpha\gamma-\beta)^2}\left[\alpha\gamma(k\gamma-1)^2\right.$$
$$\left.+\frac{\gamma^2}{\beta}(\alpha-k\beta)^2+\frac{4\gamma^2\alpha(k\gamma-1)(\alpha-\beta k)}{\alpha\gamma+\beta}\right], \tag{4.5}$$

$$\lim_{t\to\infty}\langle u_R^2\rangle_{u_{p_0}}=\langle u_R^2\rangle_\infty=\left[-\frac{g(1-\gamma)}{\alpha\gamma}\right]^2+\beta\bar{u}_{F_0}^2\frac{(k\gamma-1)^2}{(\alpha\gamma+\beta)}. \tag{4.6}$$

From Eqs. (3.12), (3.15), (3.23) and (3.24), we can see that $t\to\infty$ means $t\gg 1/2\alpha\gamma$ and $t\gg 1/(\alpha\gamma+\beta)$, with $\alpha\gamma$ as defined in Eq. (2.2) and β representing the friction coefficient in Langevin's equation. For a homogeneous and isotropic turbulence field, $\beta=5\nu/\lambda^2$, where λ is the smallest scale of turbulence eddies. We can now estimate the magnitudes of $\alpha\gamma$ and β on the basis of the following condition which is usually present in laser velocity-measuring technique: at a room temperature, with the atmospheric pressure at 1, ν in a turbulent airflow is about 0.15 cm^2/sec; if the density of suspended particles is taken to be 1 g/cm^3, then r is about 10^{-3}; if the diameter of suspended particles is taken to be 0.5 μ[1], $\alpha\gamma$ is about 1×10^6 sec^{-1}; if λ is taken to be 0.1 mm, β is about 0.75×10^4 sec^{-1}. Thus $\alpha\gamma \gg \beta$, and at this point $t\to\infty$ means $t \gg 1/\alpha\gamma$, i.e., $t \gg 10^{-6}$ sec.

(3) The effect of the gravitational force on the "follow-up" behaviour can be seen from Eq. (4.6). Let

$$Q = \frac{\beta \overline{u}^2_{F_0} \dfrac{(k\gamma-1)^2}{\alpha\gamma+\beta}}{\left[-\dfrac{g(1-\gamma)}{\alpha\gamma}\right]^2} = \frac{\beta \overline{u}^2_{F_0}\alpha^2\gamma^2(k\gamma-1)^2}{g^2(1-\gamma)^2(\alpha\gamma+\beta)}. \tag{4.7}$$

When $Q \gg 1$, the effect of the gravitational force can be neglected. When $Q \ll 1$, the effect of the turbulence field can be neglected. If we still take $\gamma = 10^{-3}$, $\alpha\gamma = 1\times10^6$ sec^{-1}, $\beta = 0.75\times10^4$ sec^{-1} and even if we take $\sqrt{\overline{u}^2_{F_0}} = 1$ cm/sec, Q will also approximate to 0.75×10^4, which means that the effect of the gravitational force may well be neglected here.

(4) When the effect of the gravitational force is neglected, Eq. (4.4) has the same form as Eq. (4.6). Therefore, we might as well discuss the "follow-up" behaviour of suspended particles in terms of Eq. (4.4) alone. Eq. (4.4) can be written as

$$\frac{\langle v_R^2\rangle_\infty}{\overline{v}^2_{F_0}} = \beta\frac{(k\gamma-1)^2}{(\alpha\gamma+\beta)}. \tag{4.8}$$

With the introduction of the dimensionless parameter

$$Br = \frac{\alpha\gamma}{\beta}, \tag{4.9}$$

Eq. (4.8) becomes

$$\frac{\langle v_R^2\rangle_\infty}{\overline{v}^2_{F_0}} = (k\gamma-1)^2\frac{1}{Br+1}. \tag{4.10}$$

Fig. 1 shows the dependence of $\langle v_R^2\rangle_\infty/\overline{v}^2_{F_0}$ on Br, with the solid line denoting $\gamma\to0$, the dot-and-dash line, $\gamma = 0.1$ and the dotted line, $\gamma = 0.5$. When $\gamma = 1$, the line coincides with the abscissa axis.

For solid or liquid particles suspended in a turbulent airflow, the condition, $\gamma \ll 1$, is usually satisfied. Thus we have approximately

$$\frac{\langle v_R^2\rangle_\infty}{v^2_{F_0}} = \frac{1}{Br+1}, \quad Br = \frac{\rho_F\lambda^2}{\rho_p r^2},$$

where ρ_F and ρ_p are the densities of the fluid and the suspended particles respectively; r is the radius of the suspended particles, i.e. $r = d/2$.

When $Br \ll 1$, $\langle v_R^2\rangle_\infty/\overline{v}^2_{F_0} \approx 1$, in other words, the suspended particles do not

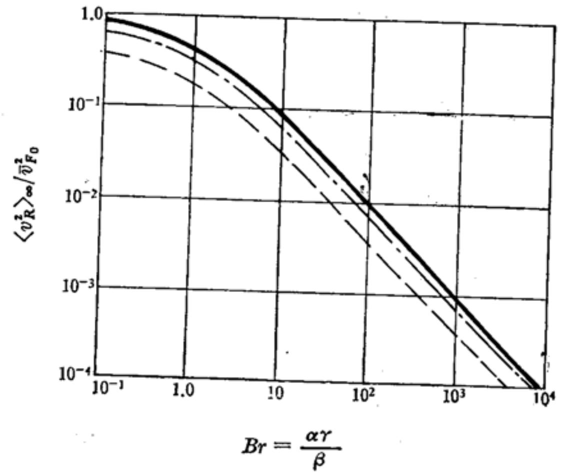

Fig. 1. The change of $\langle v_R^2\rangle_\infty/\bar{v}_{F_0}^2$ with Br.
——, $\gamma\to 0$; —·—, $\gamma = 0.1$; – – –, $\gamma = 0.5$

follow up the fluctuating motion in a turbulence field at all. If we take $Br = 0.1$, $\sqrt{\langle v_R^2\rangle_\infty/\bar{v}_{F_0}^2}$ is about 95%.

When $Br \gg 1$, $\langle v_R^2\rangle_\infty/\bar{v}_{F_0}^2 \approx 0$, which means that there is a complete follow-up of the suspended particles with the fluctuating motion in a turbulence field. If we take $Br = 10^3$, $\sqrt{\langle v_R^2\rangle_\infty/\bar{v}_{F_0}^2}$ is about 3%. At this point, if we assume $\lambda = 0.1$ mm, $d = 0.2\,\mu$. This value is found to be in complete agreement with empirical results stated in Reference [1].

V. Concluding Remarks

Based on a theoretical study of the law governing the motion of the suspended particles in a turbulent medium, the present paper has made an analysis of and established a statistical criterion, Br, for the "follow-up" behaviour of particles suspended in a stationary turbulent medium at large Reynolds number. When $Br \gg 1$, i.e., $r\sqrt{\rho_p} \gg \lambda\sqrt{\rho_F}$, the suspended particles will follow up the fluid motion completely. This is found to be in agreement with the empirical results achieved by the laser velocity-measuring technique[1].

The authors wish to make a cordial acknowledgement to Prof. Tan Haosheng (H. S. Tan) for his advice and encouragements.

References

[1] Pfeifer, H. J., *Journal of Physics E, Scientific Instruments*, 8(1975), 245—252.
[2] Hidy, G. M. & Brock, J. R., *The Dynamics of Aerocolloidal Systems*, Pergamon Press, 1970.
[3] 曾岳元,李荫亭,关德相,张 彬,《中国科学》. (1974), No. 2. 148—157.
[4] Tchen, C. M., Mean value and correlation problem connected Eluid, Ph. D. Thesis, Delft, 1947.
[5] Corrsin, S. & Lumley, J., *Appl. Sci. Research*, **6A**(1956), 114—116.
[6] Hinze, J. O., *Turbulence*, McGraw-Hill, 1959.
[7] Фукс, H. A., 《气溶胶力学》, 顾震潮等译, 科学出版社, 北京, 1960.
[8] Lin, C. C. & Reid, W. H., in *Handbuch der Physik*, (Ed. S. Flügge), VIII/2, 1963, 509—520.