Supplementary Information

# A Generalized Formula for Inertial Lift on a Sphere in Microchannels

Chao Liu,[a] Chundong Xue,[a] Jiashu Sun,[b] Guoqing Hu*[a]

[a] *State Key Laboratory of Nonlinear Mechanics, Beijing Key Laboratory of Engineered Construction and Mechanobiology, Institute of Mechanics, Chinese Academy of Sciences, Beijing 100190, China. E-mail: Guoqing.hu@imech.ac.cn*

[b] *Beijing Engineering Research Center for BioNanotechnology & CAS Key Labtorary for Biological Effects of Nanomaterials and Nanosafety, National Center for Nanoscience and Technology, Beijing 100190, China.*

**Pseudocode of the UDF**

```
Lift_calculation (){
    define global parameters;
    define fitting constants;
    calculate shear rates and shear gradient; //call built-in macros after solving the continuous flow field without particles.
    if (straight_channel){
        if (coordinate_x){
            return component_x = 0;        //The lift along the main flow direction is vanishing.
        }
        if (coordinate_y){
            use fitting constants for AR = 1;
            calculate component_y of the lift vector;       //Eqn. 13 in the main text
            return component_y;
        }
        if (coordinate_z){
            use fitting constants for AR = W/H;
            calculate component_z of the lift vector;       //Eqn. 13 in the main text
            return component_z;
        }
    if (curved_channel){       //using cylindrical coordinate system
        if (coordinate_theta){            //the tangential direction
            return component_theta = 0;   //The lift along the main flow direction is vanishing.
        }
        if (coordinate_r){       //the radial direction
            use fitting constants for AR = W/H;
            calculate component_r of the lift vector;       //Eqn. 13 in the main text
            return component_r;
        }
        if (coordinate_y){
            use fitting constants for AR = 1;
```

```
            calculate component_y of the lift vector;       //Eqn. 13 in the main text
            return component_y;
        }
}
```

**Data fitting**

The fitting constants $c = [C_1\ C_2\ C_3\ C_4]^{\mathrm{T}}$ by solving the matrix equation:

$$\mathbf{A}c = \mathbf{C}$$

$$\mathbf{A} = \begin{bmatrix} F_w(x_1) & F_s(x_1) & F_{ss}(x_1) & F_c(x_1) \\ F_w(x_2) & F_s(x_2) & F_{ss}(x_2) & F_c(x_2) \\ \vdots & \vdots & \vdots & \vdots \\ F_w(x_N) & F_s(x_N) & F_{ss}(x_N) & F_c(x_N) \end{bmatrix}, c = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} C_L(x_1) \\ C_L(x_2) \\ \vdots \\ C_L(x_N) \end{bmatrix} \tag{S1}$$

where matrix **A** is constructed by the contributions of $F_w$, $F_s$, $F_{ss}$, and $F_c$ to the $C_L$ and vector **C** by the $C_L$ calculated by DNS at positions $x_1$, $x_2 \ldots x_N$. $c$ is determined as the least square solution of Eqn. S1, which can be easily solved using MATLAB.

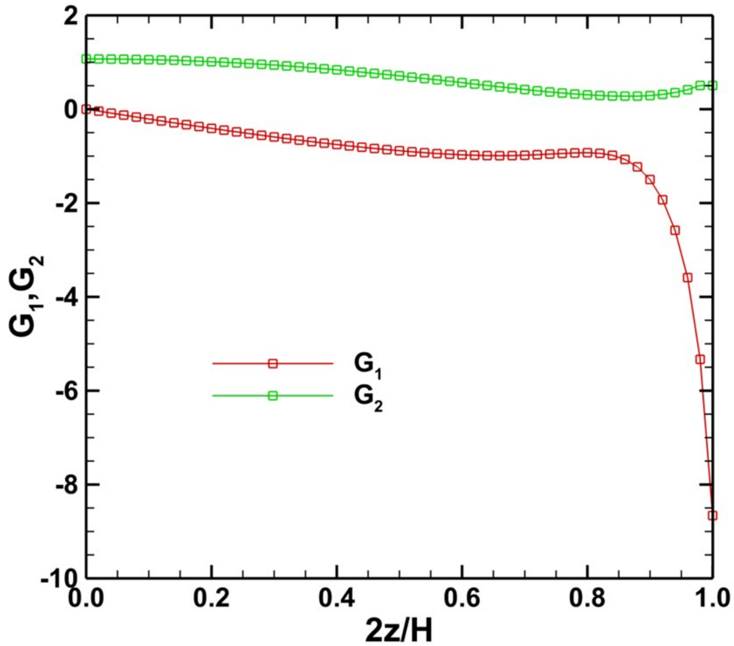


**Figure S1.** The functions $G_1$ (red) and $G_2$ (green) calculated by Ho & Leal.[1]

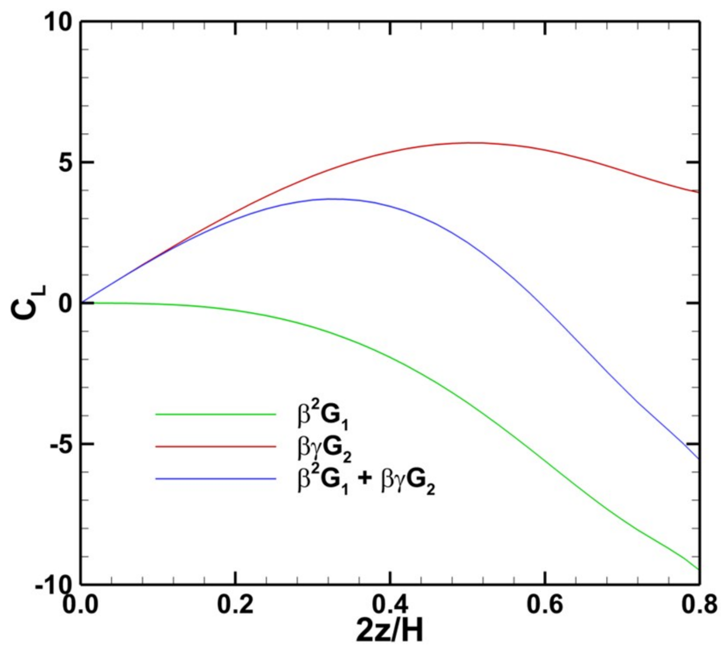


**Figure S2.** The lift coefficients $C_L$ theoretically predicted by Ho & Leal.[1] Green: the contribution of wall-induced lift, red: the contribution of shear-gradient-induced lift, and blue: the $C_L$ for the net lift force.

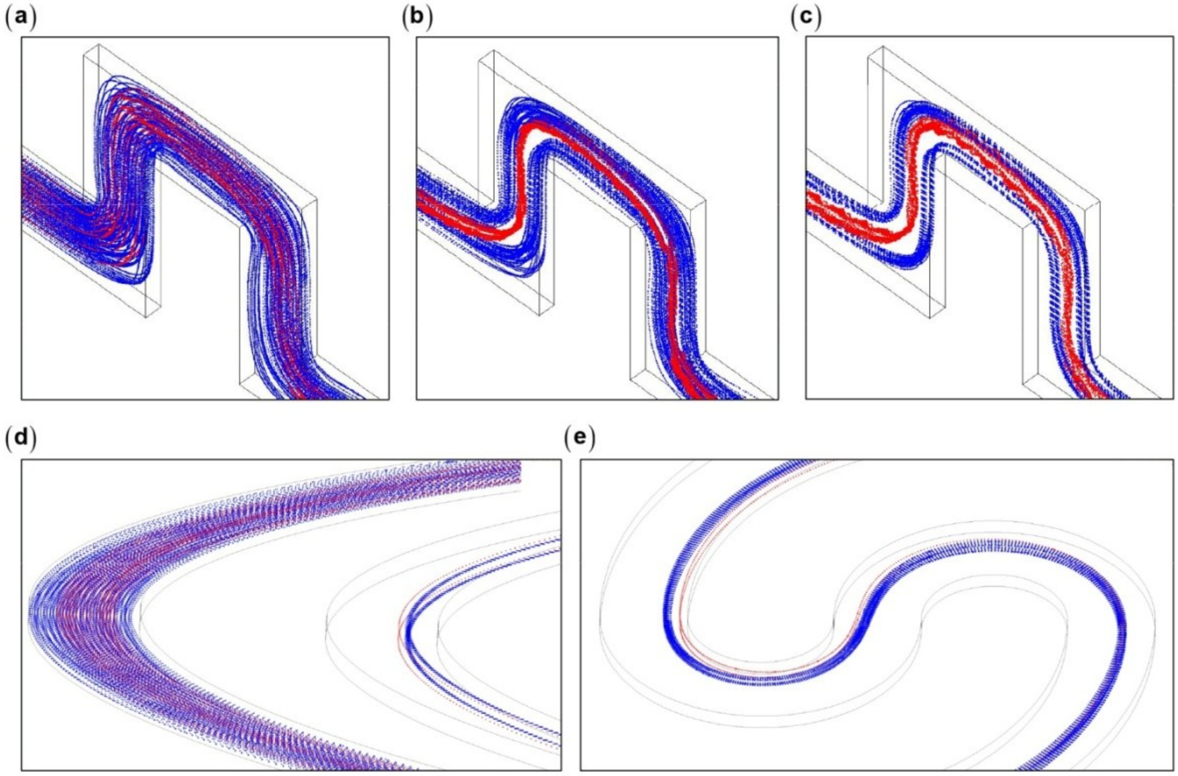


**Figure S3.** The 3D particle trajectories of 5- (blue) and 15-µm (red) particles are shown at the (a) 1st unit, (b) 10th unit, and (c) 20th unit of the serpentine microchannel and at (d) the inlet and the loop followed by the outlet and (e) the S-shaped junction of the double spiral microchannel. The simulation conditions are identical to those for Figure 9 and 10.

1 B. P. Ho and L. G. Leal, *J. Fluid Mech.*, 1974, **65**, 365-400.