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Instabilities of thermocapillary flows between counter-rotating disks

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Abstract

Thermocapillary flows are important in many applications, such as the floating-zone and Czochralski crystal growth techniques. In production of crystals by the floating zone method, the feed and crystal rods are often rotating in order to suppress the azimuthal asymmetry. We perform the linear stability analysis of the thermocapillary flows between counter-rotating disks. The basic flow and temperature solutions are obtained by using the pseudo-spectral Chebyshev method. The perturbation equations are solved with Chebyshev polynomial expansions in the radial and vertical directions. When no rotation is applied, the instability depends on the Prandtl number. For small Prandtl number liquids ($Pr <= 0.1$), the first instability of the axisymmetric flow is a stationary secondary flow. When a rotation is applied, the bifurcation is from the axisymmetric state to an oscillatory state. The most unstable mode is a traveling wave. The critical frequencies changes with the rotation Reynolds number significantly, and the direction of wave propagation can be opposite for high rotation Reynolds numbers. The flow is destabilized by weak rotation but stabilized by strong rotation. As the rotation Reynolds number increases, the appearance of the secondary vortex in the basic flow can decrease the growth rate of perturbation significantly. Energy analysis shows that the perturbation energy consists of the viscous dissipation, the work done by Marangoni forces and the interaction between the perturbation flow and the basic flow, respectively. For Prandtl numbers lower than 0.01, the perturbation energy mainly comes from the third part, which suggests that the perturbation is hydrodynamic. When Prandtl number is larger than 0.1, the second part becomes more important, and the perturbation consists of hydrothermal waves, which shows that the thermocapillary effects are important for large Prandtl number. The work done by Marangoni forces decrease with the rotation Reynolds number.

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Keywords: instabilities; thermocapillary flow; liquid bridge; counter-rotating

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1. Introduction

Thermocapillary flows are important in many applications, such as the floating-zone method and Czochralski crystal growth techniques. In the production of crystals by the floating zone method, the feed and crystal rods are often rotating in order to suppress the azimuthal asymmetry. In Figure 1, a model of liquid bridge with counter-rotating disks is proposed [1]. We numerically simulate the basic flow and perform the linear stability analysis. Energy analysis is used to study the mechanism of the instability.

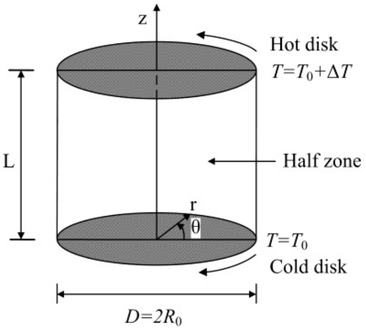


Fig. 1. Schematic of a liquid bridge between counter-rotating disks.

Nomenclature

R_0	disk radius
L	the distance of two disks
ρ_0	liquid density
ν	kinematic viscosity
α	thermal diffusivity
k	thermal conductivity
σ_T'	surface tension derivative with respect to temperature
Ω	angular velocity of rotation for the disks
ΔT	temperature difference of two disks

Typical dimensionless parameters such as aspect ratio, Reynolds number, Marangoni number and Prandtl numbers are defined as

$$A = L/2R_0,\ \mathrm{Re} = U_0 R_0/\nu,\ \ \mathrm{Pr} = \nu/\alpha, \quad (1)$$

where the reference velocity is $U_0 = \left|\sigma_T'\right| \Delta T/\rho_0 \nu$. $\mathrm{Re}_\Omega = \Omega R_0^{\ 2}/\nu$ is the Reynolds number for the disk rotation.

2. Governing equations

The dimensionless governing equations of the flow are as follows [2],

$$\nabla \cdot \mathbf{u} = 0, \quad \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \frac{1}{\mathrm{Re}} \nabla^2 \mathbf{u}, \quad \frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = \frac{1}{\mathrm{Re}\,\mathrm{Pr}} \nabla^2 T \tag{2}$$

Here, $\mathbf{u}, T, p$ are velocity, temperature and pressure, respectively. At solid disks, non-slip boundary conditions are imposed for velocity, $T = 0,1$ for the cold and hot disks, respectively. The boundary conditions at the free surface are

$$\mathbf{u} \cdot \mathbf{n} = 0\,, \mathbf{t} \cdot (\mathbf{S} \cdot \mathbf{n} + \nabla T) = 0\,, \mathbf{n} \cdot \nabla T = 0 \tag{3}$$

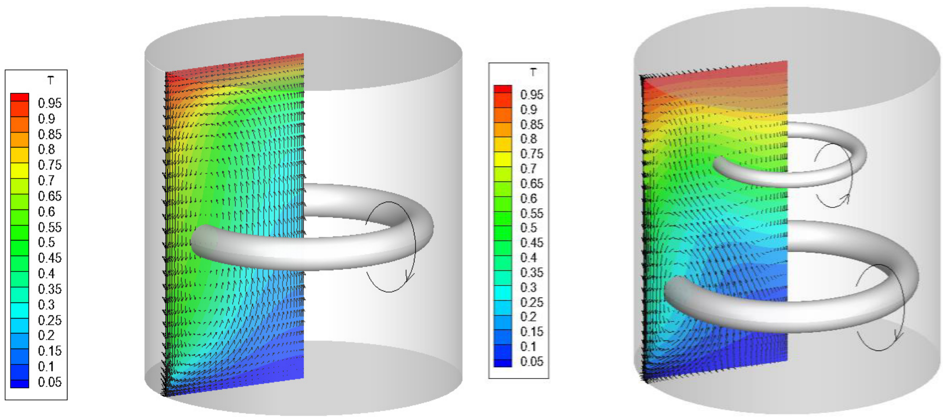
where **n** denotes the normal unit vector at the free surface, **t** denotes the tangential unit vectors in the vertical cross-sections, and $\mathbf{S}$ denotes the rate-of-strain tensor.

The basic flow and temperature solutions are obtained by using the pseudo-spectral Chebyshev method. The perturbation quantities can be expanded as a sum of normal modes, and their equations are solved with Chebyshev polynomial expansions in the radial and vertical directions.

3. Numerical results

3.1. Basic flow

Figure 2 shows the flow pattern in liquid bridge with Pr = 0.1, *A* = 1, Re = 4000 and different rotation Reynolds numbers. When no rotation is applied, the flow pattern has a primary vortex, which is caused by thermocapillary forces. As the rotation Reynolds number increases, a secondary vortex appears in the upper core region, whose streamtraces rotate in the opposite direction. This vortex caused by rotating disks has a great impact on flow stability.



(a) (b)

Fig. 2 The basic flow and temperature field for
$\mathrm{Pr} = 0.1, \mathrm{Re} = 4000, \mathrm{A} = 1$, (a) $\mathrm{Re}_\Omega = 0$; (b) $\mathrm{Re}_\Omega = 80$.

When no rotation is applied, the instability depends on the Prandtl number. For small Prandtl number liquids (Pr <= 0.1), the first instability of the axisymmetric flow is a stationary secondary flow. When a rotation is applied, the bifurcation is from the axisymmetric state to an oscillatory state. The most unstable mode is a traveling wave. The critical frequencies changes with the rotation Reynolds number significantly, and the direction of wave propagation can be opposite for high rotation Reynolds numbers. The flow is destabilized by weak rotation but stabilized by strong rotation.

3.2. Energy analysis

We study the instability of the most unstable mode by energy analysis. The perturbation energy can be obtained as follows.

$$\frac{\partial E_{kin}}{\partial t} = -\frac{1}{2Re}\int (\mathbf{S}:\mathbf{S})d^3r + \frac{1}{Re}\int \mathbf{u}\cdot\mathbf{S}\cdot\mathbf{n}\,d^2r + \int \mathbf{u}\cdot((\mathbf{u}\cdot\nabla)\mathbf{u}_0)d^3r = -D_v + M + I_v \qquad (4)$$

Here, $\mathbf{u}_0$ is the velocity of the basic flow, $\mathbf{u}$ and $\mathbf{S}$ are the velocity and the rate-of-strain tensor of the perturbation flow, D_v is viscous dissipation, I_v is the interaction between the perturbation flow and the basic flow, M are the work done by Marangoni forces on the surface, respectively [3]. As the viscous dissipation is always positive, it is set as a unit.

In Table 1, we list the perturbation energy for various Prandtl numbers and rotation Reynolds numbers. For Prandtl numbers lower than 0.01, the perturbation energy mainly comes from the interaction between the perturbation flow and the basic flow, which suggests that the instability is hydrodynamic. When Prandtl number is larger than 0.1, the work done by Marangoni forces become more important, the perturbation consists of hydrothermal waves. So the thermocapillary effects are more important for the larger Prandtl number.

Table 1. The perturbation energy for various Prandtl numbers and rotation Reynolds numbers.

Pr	A	Re	Re_Ω	D_v	M	I_v
0.001	0.6	1500	0	1	-0.0006	1.0222
0.01	1.2	800	0	1	0.0005	1.1136
0.1	0.8	4500	100	1	0.0713	0.9420
1	0.5	2500	50	1	0.6314	0.3484

4. Conclusions

In this paper, we study the linear stability of the thermocapillary flows between counter-rotating disks. The Chebyshev-collocation method is used to solve the basic state and perturbation equations. As the rotation Reynolds number increases, the secondary vortex appears in the basic flow, which can decrease the growth rate of perturbation significantly.The perturbation energy consists of the viscous dissipation, the work done by Marangoni forces and the interaction between the perturbation flow and the basic flow, respectively. When Prandtl number is larger than 0.1, the second part becomes more important.

Acknowledgements

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