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The realization of non-reflecting boundaries for compressible Rayleigh-Taylor flows with variable acceleration histories

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Abstract

The Navier-Stokes characteristic boundary conditions (NSCBC) approach is extended to compressible Rayleigh-Taylor flow (CRT-F) with variable acceleration histories, whose time- and space- dependent open boundaries challenge the available methods. The non-reflecting boundary conditions are realized by combining CRTF's physical boundary conditions and NSCBC's idea, and by appending a dissipation region, where physics-consistent viscous terms are introduced to realize non-reflection without additional effect. Numerical tests confirm the effectiveness and robustness of newly proposed schemes.

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1. Introduction

Compressible Rayleigh-Taylor flow (CRTF) occurs when a compressible fluid of heavy density is accelerated or supported against gravity by a compressible fluid of light density [1], and is of fundamental importance in applications from combustion, to inertial confinement fusion, and to astrophysics[2]. Traditionally, CRTFs are studied under constant acceleration histories[3]. Due to the nature of the processes, it is necessary to study CRTF with variable acceleration histories $g(t)$[1,4–7]. Moreover, owing to the limitation of theoretical analysis and the difficulty of experimental measure, numerical simulation has became the most important tool in studying CRTFs[2]. In this aspect, the time- and space- dependent open boundaries in CRTF with variable acceleration histories challenge the available boundary treatments and consequently the realization of numerical simulations.

As for boundary treatments, mostly used approaches are to specify the boundary values of (i) primitive/conservative variables $\boldsymbol{U}/\widetilde{\boldsymbol{U}}$ or (ii) the amplitudes of incoming characteristic waves $\mathcal{L}_{in}$. The latter is the well-known Navier-Stokes characteristic boundary conditions (NSCBC) approach[8], and has been systematically developed in 1-D[8],2-D[9],3-D[10,11] and reactive[9,11,12] flows due to its speciality in controlling incoming waves using $\mathcal{L}_{in}$[8-12] and/or additional relaxation terms $\mathcal{R}_{in}$[13–15]. However, for CRTF with variable acceleration histories, the unsteady and unknown $\boldsymbol{U}(t)$ or $\widetilde{\boldsymbol{U}}(t)$ excludes approach (i), and prevents a straightforward application of the available[8–15] approach (ii), too.

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In this paper, NSCBC-based approach for CRTF with variable acceleration histories will be proposed to address the mentioned challenges with the additional introduction of physics-consistent viscous terms to realize non-reflection. We named this approach as physical-boundaries based characteristic boundary conditions (PBCBC) approach.

2. Methods

We first introduce some rules adopted in this paper: (1) using characters like $\boldsymbol{F}, \tilde{\boldsymbol{F}}, \mathcal{F}$ and $\mathbf{F}$ to denote terms in NS equations written with primitive, conservative, characteristic and arbitrary variables, respectively; (2) using $\mathbf{U}, \mathbf{T}, \mathbf{L}, \mathbf{D}$, $\mathbf{s}$ and $\mathbf{S}$ $(= \mathbf{D} + \mathbf{s} - \mathbf{T})$ to denote the terms of equation variables, transverse convection, normal-to-boundary convection, viscous diffusion, source and generalized source, respectively; (3) using x_i, $u_i (i = 1, 2, 3)$ to denote respectively the spatial coordinate and velocity component in the ith direction, and subscript n the acceleration or normal-to-boundary direction and is assumed to be 2 in this paper; (4) defining the primitive, conservative and differential characteristic variables as $\boldsymbol{U} = (\rho, u_1, u_2, u_3, T, Y)^T$, $\tilde{\boldsymbol{U}} = (\rho, \rho u_1, \rho u_2, \rho u_3, \rho e, \rho Y)^T$, $\partial\mathcal{U} = (\partial p - \rho c \partial u_2, u_1, c^2\partial\rho - \partial p, \partial u_3, \partial p + \rho c \partial u_2, \partial Y)^T$, where ρ, p, T, e, c and Y are density, pressure, temperature, total energy, sound speed and concentration of heavy fluid, respectively. Using the rules, NS equations can be rewritten as:

$$\partial_t \mathbf{U} + \mathbf{A}_n \partial_{x_n} \mathbf{U} = \mathbf{S}, \tag{1}$$

where $\mathbf{A}_n$ is the Jacobian/eigenvalue matrix, referring Refs.[10–12,16] for details.

NSCBC procedures start from the characteristic form of Eq.(1):

$$\partial_t \mathcal{U} + \mathcal{L} = \mathcal{S}, \tag{2}$$

where t is time, $\mathcal{L} = \mathcal{A}_n \partial_{x_n} \mathcal{U}$, $\mathcal{A}_n = \text{diag}\, \lambda_\ell$ ($\ell = 1, 2, ...6$, $\lambda_1 = u_n - c$, $\lambda_2 = \lambda_3 = \lambda_4 = \lambda_6 = u_n$, $\lambda_5 = u_n + c$). The $\mathcal{L}_\ell$ ($= \lambda_\ell \partial_{x_n} \mathcal{U}_\ell$, no summation) quantify the wave amplitude variations of incoming/outgoing (determined by the sign of λ_ℓ) characteristic waves $\mathcal{L}_{in}/\mathcal{L}_{out}$[8–12]. The task of NSCBC is to determine the unknown $\mathcal{L}_{in}$, leaving $\mathcal{L}_{out}$ determined by one-sided upwind difference[10–12,16]. Previous NSCBC approaches are developed mostly for steady boundary and/or vanishing $\mathcal{S}$, obviously not suitable for CRTF. To overcome the difficulty of nonzero $\mathcal{S}$ by nonzero acceleration, Reckinger *et al.*[16] attempted to decompose instantaneous $\boldsymbol{U}$ into the part of initial state (denote with $\boldsymbol{U}^0$) and the part of deviation to $\boldsymbol{U}^0$ (denote with $\boldsymbol{U}'$), leading to $\mathcal{L} = \mathcal{L}^0 + \mathcal{L}'$. For CRTF with constant acceleration, the initial hydrostatic field is steady and one has $\mathcal{L}^0 \approx \mathcal{S}$. In consequence, one only need to specify $\mathcal{L}'_{in}$ with similar procedures of NSCBC. Since only $\mathcal{L}'_{in}$ are specified, we named this approach as fluctuating-quantities based characteristic boundary conditions (FBCBC).

Following the logic presented above, FBCBC doesn't suitable for unsteady boundaries, e.g. CRTF with variable acceleration histories. To resolve this problem, we notice that the boundary conditions of $u_1 = 0, u_3 = 0, \partial_{x_n} Y = 0, \partial_{x_n} T|_{t=0} = 0$ always work for CRTF under constant or variable acceleration histories. With the conditions, energy equation can be simplified to give

$$\rho C_V(\partial_t T + u_n \partial_{x_n} T) - \partial_{x_n}(k \partial_{x_n} T) = (4/3)\partial_{x_n}[\mu u_n \partial_{x_n}(u_n)] - p\partial_{x_n}(u_n), \tag{3}$$

where C_V, μ and k are heat capacity at constant volume, dynamic viscosity and thermal conductivity, respectively. At boundary of CRTF, $\partial_{x_n} T|_{t=0} = 0$, Eq.(3) has a physics-consistent solution of $\partial_{x_n}(u_n, T) = 0$. Collecting the results, one obtains

$$\partial_{x_n}(u_1, u_n, u_3, T, Y) = 0. \tag{4}$$

With Eq.(4), now $\mathcal{L}_\ell$ can be simplified to give

$$\mathcal{L}_{2,4,6} = 0, \mathcal{L}_1/\lambda_1 = (\gamma - 1)^{-1}\mathcal{L}_3/\lambda_3 = \mathcal{L}_5/\lambda_5, \tag{5}$$

where γ is specific heat ratio. Following NSCBC, the unknown $\mathcal{L}_{in}$ can either be imposed as zero or be expressed as a function of $\mathcal{L}_{out}$ (calculated by one-sided upwind difference) by utilizing Eq.(5), and hence closing boundary treatments. Since this approach is derived from primitive or induced physical boundary conditions, we named this approach as PBCBC.

In PBCBC, the temperature at boundaries is fixed. This is no problem before the disturbed waves, generated from interface of heavy and light fluids, arrive at boundaries. After that, however, unphysical waves will be reflected into the mixing region of CRTF, similar to the discussions in Ref.[13] about fixed pressure or velocity. Since the interactions between waves and flows have a potential influence on the evolution of CRTF, especially for lately developed turbulent mixing, it is necessary to diminish the reflected waves. In this aspect, a series of techniques have been developed, including grid-stretching and filtering[17], numerical sponge layers[18] and perfectly matched layers[19]. Unfortunately, the filtering is found to be instable for CRTF, and other techniques requires the expected boundary values of $\boldsymbol{U}$, unknown and consequently inappropriate for CRTF with variable acceleration histories.

To realize the non-reflection with PBCBC, we suggest to append dissipation regions at the two ends of computational domain, and to introduce physics-consistent artificial viscous terms $\mathrm{A_v}$ in the regions to attenuate disturbed waves. The $\mathrm{A_v}$ is = $(0, 0, f(x_n)\mu\partial_{x_n x_n}(u_n), 0, f(x_n)k\partial_{x_n x_n}T, 0)^T$ is added in the right hand side of primitive or conservative form of Eq.(1), and $f(x_n)$ is carefully constructed with hyperbolic tangent function (*tanh*) to smoothly transit from 0 at the starting location of dissipation region x_n^{start} to maximum b at the ending location of dissipation region x_n^{end}:

$$f(x_n) = b\frac{tanh[a(\eta - 0.5)] + 1}{2}, \eta = \frac{x_n - x_n^{start}}{x_n^{start} - x_n^{end}}, \tag{6}$$

where a and b characterize the increase rate and maximum amplitude of $f(x_n)$, and are set as $a = 4, b = 200$ in this paper. Since the viscous terms are added only to normal-to-boundary momentum equation and energy equation, whose physical solutions near boundaries are $\partial_{x_n}(u_n, T) = 0$ (see Eq.(4)) for undisturbed CRTF boundaries. This means artificial viscous terms do not work ($\mathrm{A_v} = \mathbf{0}$) if this is no disturbed waves. In other words, the one and only effect of introducing $\mathrm{A_v}$ is to attenuate disturbed waves.

3. Results

Now we test the proposed approaches. The NS equation[16] in non-dimensional form are solved with finite difference code OpenCFD[20]. The non-dimensional parameters based on referenced quantities (denote with subscript r) are $Re = \rho_r U_r L_r/\mu_r = 6626$, $Fr = U_r^2/(L_r g_r) = 1.2$, $Pr = \gamma C_{V r}\mu_r/k_r = 1$, $Sc = \mu_r/(\rho_r D_r) = 10$, $\gamma = 1.4$, where L_r and D_r are referenced length scale and mass diffusion coefficient. Without loss of essence, coarse 3-D grids are used to test the boundary treatments of NSCBC, FBCBC and PBCBC under variable acceleration histories, with procedures similar to NSCBC[8–15]. The non-reflecting NSCBC approach is implemented by imposing $\mathcal{L}_{in} = 0$ following Ref.[8]. Two grid-stretching dissipation regions are appended in $-7.8 \le x_n \le -5$ and $3.5 \le x_n \le 4.3$. The acceleration histories are controlled by the non-dimensional parameter $Fr[g(t)]$: $Fr_{var.} = Fr_{con.} + 0.4sin\,[5mod(t, 3)]$. In this formula, $Fr_{con.} = 1.2$ is used because only this value gives a steady initial field with parameters given above, and this formula will produce a variable and discontinued acceleration to challenge the robustness of PBCBC. To startup the evolution of CRTF, a 3-D perturbation is imposed at the interface of $x = 0$. All tests are lasting to $t = 6.0$.

Fig.1 shows the test results. This figure plots the u_n distributions along a line parallel to x_n direction. The figure shows that only PBCBC give the physical results of constant and nonzero u_n nearby the two ends of boundaries, while other approaches produce unphysical oscillations. In inset, we also plot the evolutions of normal-to-boundary Mach number $M_n(= u_n/c)$ at bottom boundary to validate the non-reflecting performance of artificial viscous terms. This variations of M_n^{bottom} show that the boundary is indeed unsteady, and switches frequently between inflow ($M_n^{bottom} > 0$) and outflow ($M_n^{bottom} < 0$). For case without $\mathrm{A_v}$, the disturbed waves arrive at bottom boundary at $t \approx 2.25$ and are reflected until $t \approx 2.75$. In contrast, a smooth variation of M_n^{bottom} can be seen in case with $\mathrm{A_v}$ between $t = 2.25 \sim 2.75$, hence successfully removing disturbed waves.

4. Discussions

Now we make some discussions. The PBCBC approach only applies to CRTF with subsonic open boundaries because there are six $\mathcal{L}_{in}$ and zero $\mathcal{L}_{out}$ for supersonic inflow, and naturally $\mathcal{L}_{in}$ can not be expressed as a functions of $\mathcal{L}_{out}$, hence PBCBC fails. Finally, it is worthy noting that our numerical practices found that the stability of PBCBC

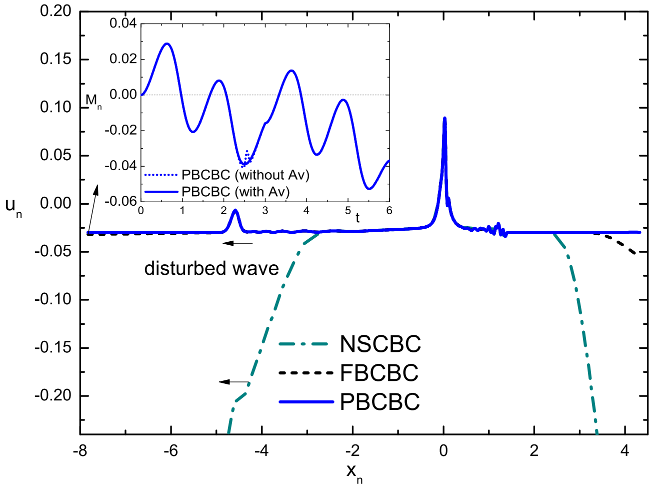

Fig. 1. (color online) The u_n distributions in a line parallel to x_n direction for CRTF under variable acceleration histories and at time $t = 1.52$. Inset shows the evolutions of M_n at bottom boundary with time t for implementations with or without artificial viscous terms A_v, respectively. The transition at $t = 3$ is caused by the imposed discontinued acceleration.

can be greatly enhanced by imposing $u_{1,3} = 0$ at boundaries with reasons as follows: in this case of nonzero u_n, the numerical error of u_n by one-sided difference would interact with $u_{1,3}$ in nonlinear ways to lower the stability.

5. Conclusions

In summary, we have proposed an approach to extend previous NSCBC approaches to CRTF with variable acceleration histories. For the flows, previous developed NSCBC approaches do not work because of the lack of a way to specify unknown incoming waves under unsteady boundary, and specified in this paper by outgoing waves (extrapolated from the interior) through relations in Eq.(5). The relations are derived by combining CRTF's physical boundary conditions and NSCBC's idea, and named as PBCBC. Finally, the non-reflection of PBCBC is realized by appending dissipation regions, where artificial but physics-consistent viscous terms are introduced without additional effect. Numerical tests and comparisons confirmed the effectiveness and robustness of newly proposed method.

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References

[1] G. Dimonte, P. Ramaprabhu, M. Andrews, Rayleigh-taylor instability with complex acceleration history, Physical Review E 76 (2007) 046313.
[2] D. Livescu, Numerical simulations of two-fluid turbulent mixing at large density ratios and applications to the rayleigh–taylor instability, Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences 371 (2013) 20120185.
[3] W. H. Cabot, A. W. Cook, Reynolds number effects on rayleigh–taylor instability with possible implications for type ia supernovae, Nature Physics 2 (2006) 562–568.
[4] G. Dimonte, M. Schneider, Turbulent rayleigh-taylor instability experiments with variable acceleration, Physical Review E 54 (1996) 3740.
[5] G. Dimonte, M. Schneider, Density ratio dependence of rayleigh-taylor mixing for sustained and impulsive acceleration histories, Physics of Fluids 12 (2000) 304–321.
[6] J. D. Ramshaw, Simple model for linear and nonlinear mixing at unstable fluid interfaces with variable acceleration, Physical Review E 58 (1998) 5834.
[7] J. D. Ramshaw, Simple model for mixing at accelerated fluid interfaces with shear and compression, Physical Review E 61 (2000) 5339.

[8] T. J. Poinsot, S. Lelef, Boundary conditions for direct simulations of compressible viscous flows, Journal of computational physics 101 (1992) 104–129.
[9] C. S. Yoo, Y. Wang, A. Trouvé, H. G. Im, Characteristic boundary conditions for direct simulations of turbulent counterflow flames, Combustion Theory and Modelling 9 (2005) 617–646.
[10] G. Lodato, P. Domingo, L. Vervisch, Three-dimensional boundary conditions for direct and large-eddy simulation of compressible viscous flows, Journal of Computational Physics 227 (2008) 5105–5143.
[11] A. Coussement, O. Gicquel, J. Caudal, B. Fiorina, G. Degrez, Three-dimensional boundary conditions for numerical simulations of reactive compressible flows with complex thermochemistry, Journal of computational physics 231 (2012) 5571–5611.
[12] J. C. Sutherland, C. A. Kennedy, Improved boundary conditions for viscous, reacting, compressible flows, Journal of computational physics 191 (2003) 502–524.
[13] W. Polifke, C. Wall, P. Moin, Partially reflecting and non-reflecting boundary conditions for simulation of compressible viscous flow, Journal of Computational Physics 213 (2006) 437–449.
[14] D. H. Rudy, J. C. Strikwerda, A nonreflecting outflow boundary condition for subsonic navier-stokes calculations, Journal of Computational Physics 36 (1980) 55–70.
[15] S. Pirozzoli, T. Colonius, Generalized characteristic relaxation boundary conditions for unsteady compressible flow simulations, Journal of Computational Physics 248 (2013) 109–126.
[16] S. Reckinger, D. Livescu, O. Vasilyev, Adaptive wavelet collocation method simulations of rayleigh–taylor instability, Physica Scripta 2010 (2010) 014064.
[17] T. Colonius, S. K. Lele, P. Moin, Boundary conditions for direct computation of aerodynamic sound generation, AIAA journal 31 (1993) 1574–1582.
[18] A. Mani, Analysis and optimization of numerical sponge layers as a nonreflective boundary treatment, Journal of Computational Physics 231 (2012) 704–716.
[19] F. Q. Hu, X. Li, D. Lin, Absorbing boundary conditions for nonlinear euler and navier–stokes equations based on the perfectly matched layer technique, Journal of computational physics 227 (2008) 4398–4424.
[20] L. Xin-Liang, F. De-Xun, M. Yan-Wen, Direct numerical simulation of a spatially evolving supersonic turbulent boundary layer at ma= 6, Chinese Physics Letters 23 (2006) 1519.