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Sediment concentration in steady rill flow: An analytical study

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Abstract

Accurate estimation of the amount of rill erosion, which is generally the main erosion pattern on the hill-slope, is essential for hill-slope erosion studies. Rose et al. (2007) developed an approximate analytical solution for over land erosion processes. It is based on the averaged overland flow concept that does not consider the difference between rill flow and inter-rill processes. Focusing on the dynamic erosion characteristics of the rill, this paper expands the work of Rose and derives an improved approximate analytic solution for fully develop drill flow. This solution considers the influence of the bare soil erosion at the rill channel sides and the change of hydraulic radius, and it can return to Rose's solution automatically in the case of inter-rill erosion or generalize overland erosion situation. Laboratory experimental data on rill erosion by Lei et al. (2009) is employed to verify the solution, showing a good agreement except some very steep situations. The contribution of lateral erosion from the rill is studied and the influence of slope and flow discharge on it is discussed.

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*Keywords:*Soil erosion; rillflow; analytic solution; sediment concentration

1. Introduction

Soil erosion often causes many environmental problems, such as water and soil loss, desertification and debrisflow. Rill erosion caused by overland flow is a vital process of soil erosion on hill-slope, so understanding the sediment transport processes in rill is essential for forecasting soil erosion and the design of soil conservation practice. Rose et al. [1] proposed an approximate sediment concentration analytical solution describing the dynamic erosion of soil in steady sheet flow on slopes. This solution could describe the time and space variation of the sediment concentration, which provides a useful tool for hill-slope scale water erosion study. However, this research

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only considered generalized over land processes. For complex structure consist of rill and inter-rill areas, while the rill flow is generally believed as the key factor in hill-slope erosion. Different from inter-rill sheet flow, the rill flow, which is different from inter-rill sheet flow has approximately fixed channel and receives contribution from inter-rill sheet flow. All these unique characteristics are not considered in Rose's solution.

The objective of this paper is to derive an approximate sediment concentration analytical solution to investigate the difference between rill and sheet flows. And then, the mechanism of rill erosion in hill-slope water erosion processes with this solution is studied.

2. Solution of the Sediment Transport Equation in the Rill

2.1. Sediment transport equation in the rill

Raindrop fallen on hill-slope will accumulate on the inter-rill area, and then converge into an adjacent rill. As the rill flow is highly concentrate than inter-rill flow, the rill flow and erosion dominate the overland transport processes. Assuming the flow equation is already solved, the sediment transport process in rill can be expressed by the following sediment transport equation, containing only one dependent variable c:

$$\frac{\partial(cA)}{\partial t}+\frac{\partial(cQ)}{\partial x}=D_r+q_{s,y_{in}} \quad (1)$$

In which, c is the sediment concentration in the rill (kg·m^{-3}); A is the cross-sectional area of the rill (m^2); Q is the volumetric flow rate per rill(m^3·s^{-1}); D_r is net erosion rate per unit area in a rill (kg·m^{-1}·s^{-1}), $q_{s,y_{in}}$ is the lateral sediment flux to the rill in mass rate per unit rill length,(kg·m^{-1}·s^{-1}).

The suspended sediment particle will bedeposed on therill bottom which is addressed as deposited layer in this paper. As only sediment deposition contribute to the growth of this layer, the sediment in the layer will be non-cohesive,which is more easily removed than the original un-eroded cohesive soil. This difference will result in different erosion process when the rill bottom is covered by different material. Raudkivi and Tan's experiment also showed that, for cohesive sediment, the effect of cohesion is fargreater than that of the submerged unit weight of detached particles in inhibiting removal of soil. Due to the mutual shear stress at the soil-and-water interface, the sediment of the deposited layer will enter rill flow (re-entrainment) first when the original soil is covered by deposition layer, and the original cohesive soil mass will be eroded into rill flow (entrainment) only when it is not protect by deposition layer. Then we could rewrite eq. (1) as:

$$\frac{\partial(cA)}{\partial t}+\frac{\partial(cQ)}{\partial x}=q_{s,r}+q_{s,r_r}-q_{s,d}+q_{s,y_{in}} \quad (2)$$

in which, $q_{s,r}$, q_{s,r_r} is the product of entrainment, re-entrainment for unit length of rill in unit time (kg·m^{-1}·s^{-1}), respectively; $q_{s,d}$ is the downward sediment for unit length of rill in unit time(kg·m^{-1}·s^{-1}).This equation shows that the sediment concentration in rills is contributed from the inter-rill inflow source, the re-entrainment sedimentary layer, and the entrainment sediment from bare soil at bottom and sides of the rill channel.

Conceptually, we know that deposition occurs randomly, so does the deposited layerdistribution. Deposition does not cover all the soil-and-water interface on the steep rills channel sides, as the most of them will confine to the bottom of the rill, leaving the rill sides unshielded.While at the bottom bed, deposited layer varies dynamically by deposition and re-entrainment processes, resulting bare soil on bottom bed not fully shielded. For a trapezoidal rill (Fig.1), if α denote the relative extent of the deposited layer at the bottom in protecting the original soil at any time, then fractional exposure of the original cohesive soil to entrainment is (1-α). So, we can get:

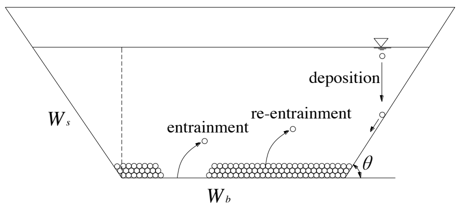

Fig.1. the movement of sediment in the rill

$$q_{s,r} = \left[(1-\alpha)W_b + W_s\right]r \qquad q_{s,r_r} = \alpha W_b r_r \qquad q_{s,d} = fW_b d \tag{3}$$

in which, W_b is the base width of the rill and W_s is the width of the soil-and-water interface wall (m); r, r_r is the rate of entrainment, and re-entrainment (kg·m^{-2}·s^{-1}) respectively; d is the rate of deposition (kg·m^{-2}·s^{-1}); f is a non-dimensional fact or describing the shape of the rill cross-section, and can be expressed as $f = 1 + 2H\cot\theta / W_b$ for trapezoidal rill.

Furthermore, deposition is connected with grain sizes, and the fraction of the total rate of entrainment, re-entrain mentand deposition for every grain size of sediment are proportional to the mass fraction of that grain size. For the sediment of grain size i, the sediment transport equation could be expressed as:

$$\frac{\partial(c_i A)}{\partial t} + \frac{\partial(c_i Q)}{\partial x} = \left[(1-\alpha)W_b + W_s\right]r_i + \alpha W_b r_{ri} - fW_b d_i + q_{s,y_{in}} \tag{4}$$

where, c_i is the sediment concentration of grain size i in the rill (kg·m^{-3}); r_i, r_{ri} is the rate of entrainment and re-entrainment of grain size i (kg·m^{-2}·s^{-1}) respectively; d_i is the rate of deposition of grain size i (kg·m^{-2}·s^{-1}).

2.2. The source of sediment transport equation

The entrainment takes place when the fluid stresses of rill flow exerts on the soil matrix counteract the resistance offered by the soil matrix. For the sediment in the deposited layer, the power of lifting it to some height in the flow originates from potential energy of rill flow. Following the principles of conservation of energy, the rate of erosion in original soil and deposition can be expressed as

$$r_i = \frac{\beta(\Omega - \Omega_0)}{J}\frac{M_i}{\sum M_i}, \qquad r_{ri} = \frac{\beta}{gH}(\Omega - \Omega_0)\frac{\rho_s}{\rho_s - \rho}\frac{M_{di}}{M_{dt}} \tag{5}$$

in which, β is the fraction of effective stream power used ineroding sediment; Ω is stream power, the rate of working of the shear stresses (W·m^{-2}); Ω_0 is threshold value of stream power, below which no erosion occurs (W·m^{-2}); J is the energy per unit mass required to entrain sediment from the soil matrix (J·kg^{-1}); M_i is the mass of grain size i entrained (kg); $\sum M_i$ is the total mass of grain size entrained (kg); g is acceleration due to gravity (m·s^{-2}); H is the height the sediment raised in the flow (m); ρ, ρ_s is the density of water and sediment (kg·m^{-3}); M_{di} is the sediment mass per unit area of previously deposited for grain size$i$(kg·m^{-2}); M_{dt} is the total of M_{di} (kg·m^{-2}).

Deposition process is continuous, and can be described by the expression:

$$d_i = v_i c_i \tag{6}$$

in which, v_i is the setting velocity representative of grain size i.

If we describe the rate of build-up of the mass per unit area as $\partial M_{di}/\partial t = fd_i - \alpha r_{ri}$, then the sediment transport equation in rill can be expressed with M_{di} as:

$$\frac{\partial(c_i A)}{\partial t} + \frac{\partial(c_i Q)}{\partial x} = \left[(1-\alpha)W_b + W_s\right]\frac{\beta(\Omega - \Omega_0)}{J}\frac{M_i}{\sum M_i} - W_b\frac{\partial M_{di}}{\partial t} + q_{s,y_{in}} \tag{7}$$

2.3. The solution of sediment transport equation

Rill is randomly growing on the slope because of heterogeneity of soil and micro landform. According to Tayfur's study [2], we assume that there are N parallel rills per unit widthon the hill-slope, and the flow of all rills is equal, then the flow in a rill (Q) at the distance of x can be expressed as $Q = \left(q_{in} + q_{int\_rill}x\right)/N$.

While sediment transport is steady in the rill, the transverse section and Q don't vary along with time, then $\partial A/\partial t = 0$, $\partial Q/\partial x = q_{int\_rill}/N$, and $\partial Q/\partial t = 0$.

At last, the sediment transport equation in rill can be expressed as:

$$A\frac{\partial c_i}{\partial t} + Q\frac{\partial c_i}{\partial x} + c_i\frac{q_{\text{int\_rill}}}{N} = \left[(1-\alpha)W_b + W_s\right]\frac{\beta(\Omega - \Omega_0)}{J}\frac{M_i}{\sum M_i} - W_b\frac{\partial M_{di}}{\partial t} + q_{s,y_{in}} \tag{8}$$

Rose [3] proposes that for sediment transport in the overland, deposition and re-entrainment is important, but the rate of deposition and re-entrainment are very closely equal compared to the magnitude of the seterms, so $\partial M_{di}/\partial t$ is very small. Thus,

$$\alpha r_{ri} - fd_i \approx 0 \tag{9}$$

Substituting from eq. (5) for r_{ri} and from eq. (6) for d_i into eq. (9) and then the sediment of grain size i in deposition layer can be expressed as,

$$M_{di} = v_i c_i M_{dt}^{\infty} / A_r \tag{10}$$

So, the changing of the deposition layer can be written:

$$\frac{\partial M_{di}}{\partial t} = \frac{\partial}{\partial t}\left(\frac{v_i c_i M_{dt}^{\infty}}{A_r}\right) = \frac{v_i M_{dt}^{\infty}}{A_r}\frac{\partial c_i}{\partial t} \tag{11}$$

Taking eq.(11) into eq.(8) and rearranging gives:

$$\frac{\partial c_i}{\partial t} + \frac{1}{E_r}\frac{\partial c_i}{\partial x} = \frac{\left[(1-\alpha)W_b + W_s\right]B_r + q_{s,y_{in}}}{E_r Q} - \frac{c_i q_{\text{int\_rill}}}{E_r(q_{in} + q_{\text{int\_rill}} x)} \tag{12}$$

in which, $A_r^{'} = A + W_b \frac{v_i M_{dt}^{\infty}}{A_r}$, $E_r = \frac{A_r^{'}}{Q}$, $B_r = \frac{\beta(\Omega - \Omega_0)}{J}\frac{M_i}{\sum M_i}$.

Eq.(12) is the sediment transport equation in dynamics equilibrium condition of deposition and re-entrainment.

Considering eq.(12), in the right term, only α varies with time. After analyzing the regular pattern of α at some distance L through solving eq.(12) when $x=L$, an obvious result can be obtained:

$$\alpha = \alpha_{\infty}(1 - e^{-t/t^*}) \tag{13}$$

in which, $t^* = (1/L\tilde{E}_r + w)^{-1}$, $\alpha_{\infty} = \frac{L\tilde{E}_r w}{1 + L\tilde{E}_r w}\left(1 + \frac{W_s}{W_b} + \frac{q_{s,y_{in}}}{W_b B_r}\right)$

Taking eq.(13) into eq.(12), we get the concentration of sediment for grain size i in rill at the condition of dynamics equilibrium condition of deposition and re-entrainment is:

$$c_i(x,t) = \frac{\left[(1-\alpha_{\infty})W_b + W_s\right]B_r + q_{sy_{in}}}{q_{in} + 2q_{\text{int\_rill}}x} Nx\left[1 - \exp\left(-\left(\frac{1}{E_r x} + \frac{q_{\text{int\_rill}}}{E_r(q_{in} + q_{\text{int\_rill}}x)}\right)t\right)\right] + \frac{\alpha_{\infty}W_b B_r}{(q_{in} + 2q_{\text{int\_rill}}x)t^* - E_r x(q_{in} + q_{\text{int\_rill}}x)} Nxt^*\left[e^{-t/t^*} - \exp\left(-\left(\frac{1}{E_r x} + \frac{q_{\text{int\_rill}}}{E_r(q_{in} + q_{\text{int\_rill}}x)}\right)t\right)\right] \tag{14}$$

Eq.(14) is the analytical solution of sediment concentration derived following the assumption that the deposition and the re-entrainment is in dynamicbalance status. We can see that the source of the sediment concentration is the bare soil erosion at the rill channel sides and bottom of the rill bed, the re-entrainment of deposition layer and the lateral sediment flux. When the sedimentary layer becomesdeep enough to fully covering the rill bottom, erosion atthe rill channel sides could be the dominant factor.

3. Verificationand Discussion

Table 1 Particle size distribution of silty-clay

grain size/mm	<0.001	0.001~0.005	0.005~0.01	0.01~0.05	0.05~0.25	0.25~1
Constitute by weight/%	12.82	3.10	7.81	56.09	20.17	0.01

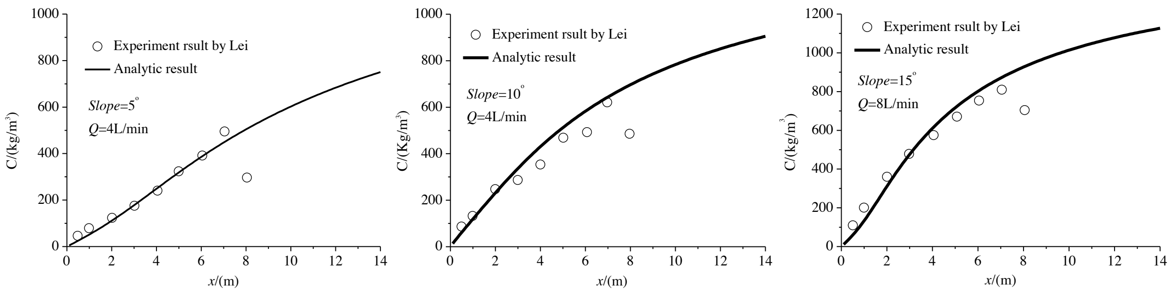

Fig.2. Acomparison of analytic solution and experimental data of T.W. Lei at different slope and Q

A set of laboratory data on rill erosion by Lei [4] is employed to validate this solution. The slope grade and flow discharge setting in the validation tests are as following: ①slopeis 5°, flow discharge (Q) is 4 L/min ②slopeis 10°,Q

is 4L/min ③slopeis 15°,Q is 8L/min, and the soil is setting to silty-clay soil (Table 1), a typical Loess Plateau soil. Good agreement between analytic result and experiment data is observed (Fig.2). And the analytical solution fully represents the sediment concentration distribution along with the slope length under different slope (except very steep situations) and Q. Along with the slope length, the sediment concentration increases more rapidly to the saturation concentration in steeper slope and larger discharge case than the gentler slope and lower discharge case.

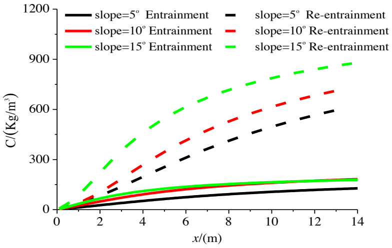

Fig.3.Sediment concentration contributed from entrainment and re-entrainment processes along with the slope length(slope=5°, 10°, 15°, Q=4L/min)

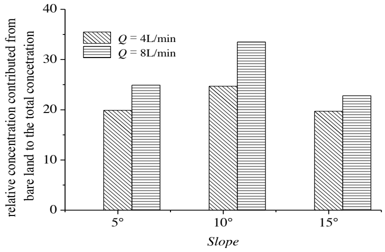

Fig.4.Sediment concentration contributed from bare soil to the total concentration (slope=5°, 10°, 15°, Q=4L/min, 8L/min)

This solution is employed to study the contribution of the rill channel sides erosion to the total erosion. The test cases are as below: slope gradevaries in 5°, 10° and 15°, rill flow discharge varies in 4L/min and 8L/min, silty-clay soil. The sediment concentration distribution along with the slope length for the cases that slopevaries in 5°, 10° and 15° and flow discharge is 4L/min are shown in Fig.3. No matter whether sediment entrained from bare soil or sediment re-entrained from deposition layer, along with the slope length, it rapidly reaches the saturation status in steep slope cases. The steeper the slope, the faster it will be. A detailed analysis (Fig.4) shows that the sediment entraining from bare soil at both sides of the rill channel is about 1/5 of that from sedimentary layer and bare soil with the assumption that the rill bottom has been fully covered. So, sediment eroded from rill channel sides, which Rose's solution doesn't considered, could not been neglected in common rill erosion situation. Considering Fig. 2, the rill channel side erosion which is related with the rill width does not increase dramatically with the slope length after a certain distance from the head of rill. That explains the phenomena that rill width is often very limited while the rill depth could vary in a larger extent.

4. Conclusions

It is essential to accurately estimate the sediment concentration of rill flow on the hill-slope. In this paper, based onsediment transport equation of rill flow an approximate analytic solution focusing on the characteristics of dynamic soil erosion for steady rill flow is derived. The solution considers the sediment from bare soil erosion at the rill channel sides that is not included in the solution proposed by Rose. A detailed analysis employed this solution points out that sediment generated from the bare soil erosion cannot be neglected even though water and soil transport reaches balance with the assumption that the rill bottom has been fully covered.

Acknowledgements

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